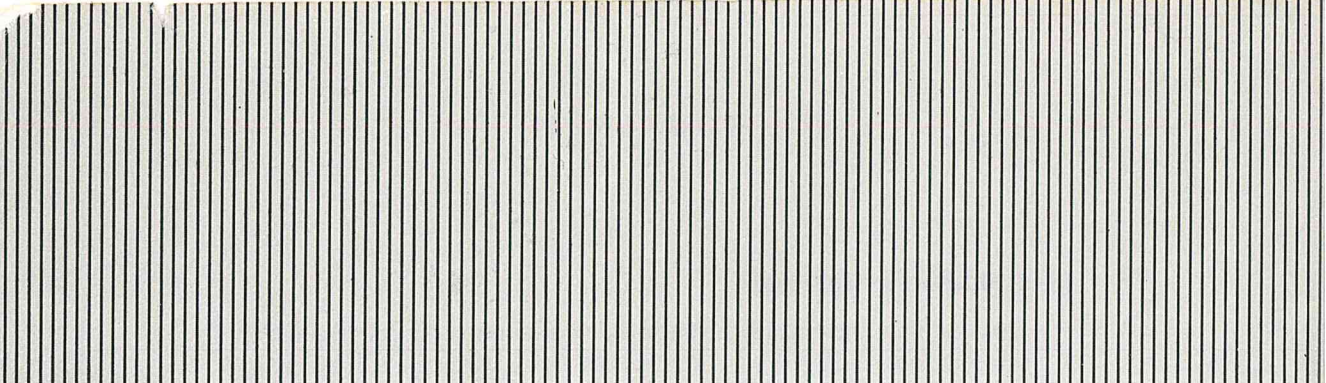


Modern School Mathematics

Structure and Use

(TEACHER'S EDITION)

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Modern School Mathematics Structure and Use

ERNEST R. DUNCAN

LELON R. CAPPS

MARY P. DOLCIANI

W. G. QUAST

MARILYN ZWENG

EDITORIAL ADVISER

ANDREW M. GLEASON

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Modern School Mathematics

Structure and Use

ABOUT THE AUTHORS

Ernest R. Duncan, Chairman of the Department of Curriculum and Instruction, Rutgers University, is responsible for mathematics education in the Graduate School of Education. Dr. Duncan has been a teacher and supervisor in elementary and secondary schools. He conducts many mathematics in-service courses for teachers and writes extensively in elementary mathematics.

Lelon R. Capps, Director of Elementary Education, University of Kansas, brings rich elementary teaching experience to the program. Dr. Capps serves as a mathematics consultant to many school districts and writes numerous articles for professional journals.

Mary P. Dolciani, Professor of Mathematics, Hunter College, New York. Dr. Dolciani has been a member of the School Mathematics Study Group and a director and teacher in numerous National Science Foundation and New York State Education Department institutes for mathematics teachers, and Visiting Secondary School Lecturer for the Mathematical Association of America.

W. G. Quast, Instructor, Curriculum and Instruction Department, Graduate School of Education, Rutgers University. Mr. Quast brings to his authorship a rich background of both mathematical study and teaching experience.

Marilyn J. Zweng, Assistant Professor of Mathematics and Education, University of Iowa, is the television teacher for the series, "Patterns in Arithmetic", sponsored by the University of Wisconsin. Dr. Zweng has written numerous professional articles on the teaching of mathematics in the elementary schools as well as being a consultant to many school districts.

EDITORIAL ADVISER

Andrew M. Gleason, Professor of Mathematics, Harvard University, is in the forefront of the new changes in mathematics. Professor Gleason was Chairman of the Advisory Board for SMSG as well as Co-chairman of the Cambridge Conference which wrote the influential report, *Goals for School Mathematics*. He is a member of the National Academy of Sciences and is a well-known research mathematician.

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CHAPTER 1 • Sets, Numbers, Numerals

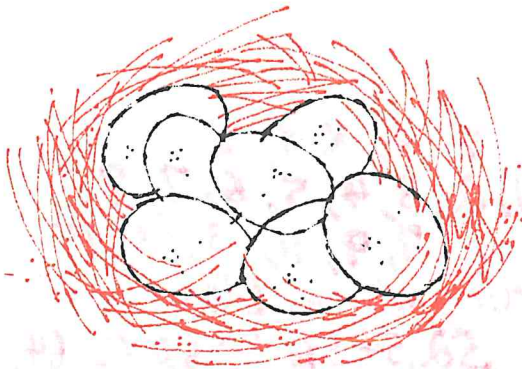
Sets



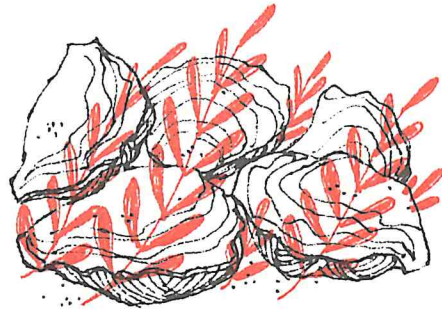
a swarm of bees



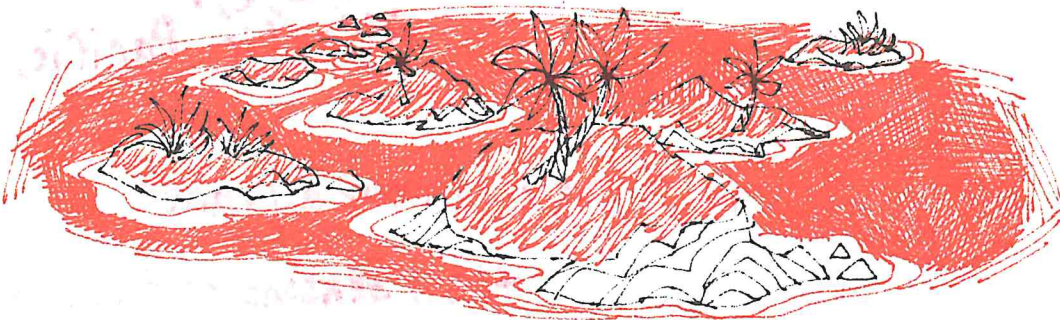
a team of horses



a clutch of eggs



a bed of oysters



an archipelago

These are all examples of **sets**. Can you think of some others?

Sets

A collection of objects is called a **set**. The objects in a set are called its **members**. Braces, { }, are usually used to show sets.

The members of a set may be **described** or **listed**. For example, if Karen, Lisa, Helen, and Sharon are the only girls in Karen's class who like to sew, then {Karen, Lisa, Helen, Sharon} and {girls in Karen's class who like to sew} are ways to show the same set. These sets are **equal** because they contain exactly the same members.

The set with no members is called the **empty set**. There is only one empty set. We write { } to show the empty set.



EXERCISES

List the objects in the sets.

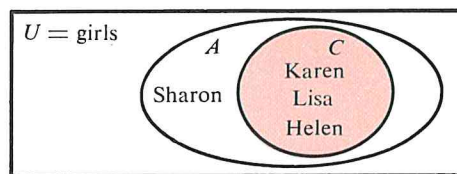
1. $A = \{\text{the last 5 letters in the alphabet}\}$ *v, w, x, y, z*
2. $B = \{\text{the first 8 counting numbers}\}$ *1, 2, 3, 4, 5, 6, 7, 8*
3. $C = \{\text{the 6 odd numbers between 6 and 18}\}$ *7, 9, 11, 13, 15, 17*
4. $D = \{\text{colors of the American flag}\}$ *red, white, blue*
5. $E = \{\text{even numbers between 51 and 71}\}$ *52, 54, 56, 58, 60, 62, 64, 66, 68, 70*
6. $F = \{\text{the first 3 months of the year}\}$ *January, February, March*
7. $G = \{\text{letters in the word "number"}\}$ *n, u, m, b, e, r*
8. $H = \{\text{names of the 4 oceans in the world}\}$ *Atlantic, Pacific, Indian, Arctic*

Describe the objects in the sets.

9. $A = \{\text{water, ice, steam}\}$
10. $B = \{\text{a, e, i, o, u}\}$ *the 5 vowels in our alphabet*
11. $C = \{\text{Michigan, Huron, Superior, Erie, Ontario}\}$
12. $D = \{\text{spring, fall, winter, summer}\}$ *the 4 seasons of the year*
13. $E = \{\text{July, August, September, October, November, December}\}$ *the last 6 months of the year*
14. $F = \{0, 2, 4, 6, 8\}$ *even numbers*
15. $G = \{\text{Washington, D.C.}\}$

Suppose $A = \{\text{Karen, Lisa, Helen, Sharon}\}$ and
 $C = \{\text{Karen, Lisa, Helen}\}$.

All the members of C are in A . We say, “ C is a **subset** of A ” and write $C \subset A$, or we say “ A is a **superset** of C ” and write $A \supset C$.



The superset we are thinking about is called the **universe**. The universe here is $\{\text{Karen, Lisa, Helen, Sharon}\}$. A larger universe would be $\{\text{girls in Karen's class}\}$. Figure 1 shows that $C \subset A$.

EXERCISES

Pair the subsets and supersets.

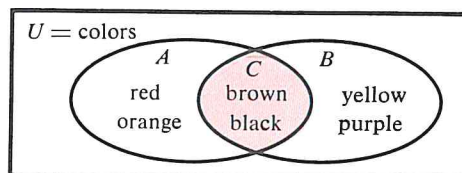
- | | |
|---|--|
| 1. $A = \{\text{Tim, Tom, Tony}\}$ | 6. $F = \{\text{even numbers less than 10}\}$ |
| 2. $B = \{2, 4, 6, 8\}$ | 7. $G = \{\text{Pacific, Atlantic, Indian, Arctic}\}$ |
| 3. $C = \{\text{Africa, Europe, Asia, Australia}\}$ | 8. $H = \{\text{Terry, Tim, Tom, Ted, Tony}\}$ |
| 4. $D = \{\text{Pacific, Atlantic, Indian}\}$ | 9. $I = \{\text{silk, cotton, rayon, wool}\}$ |
| 5. $E = \{\text{cotton, wool}\}$ | 10. $J = \{\text{Africa, Asia, Europe, North America, South America, Australia}\}$ |

Name a universe for each of the sets.

- | | |
|---|---|
| 11. $A = \{\text{January, December, March}\}$ | 15. $B = \{\text{pencil, paper, eraser, book}\}$ |
| 12. $C = \{\text{children in your classroom}\}$ | 16. $D = \{\text{the Alps, the Rocky Mountains, the Himalayas}\}$ |
| 13. $E = \{\text{counting numbers between 23 and 47}\}$ | 17. $F = \{\text{lakes in New Zealand}\}$ |
| 14. $G = \{\text{insects that have 6 legs}\}$ | 18. $H = \{\text{letters in the word "numeral"}\}$ |

Intersection of sets

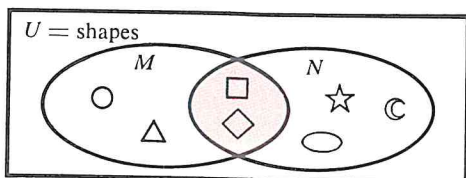
- $A = \{\text{red, orange, brown, black}\}$
 $B = \{\text{yellow, purple, brown, black}\}$
 $C = \{\text{colors common to } A \text{ and } B\}$
 $= \{\text{brown, black}\}$



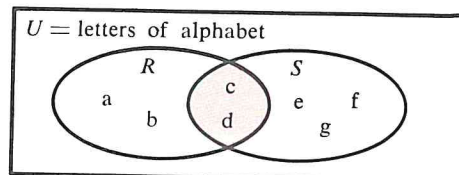
C is the **intersection** of A and B , and is written $A \cap B$. The diagram shows a universe is the set of all colors.

EXERCISES

Use the diagrams to name the intersections of the sets.



1. $M \cap N$

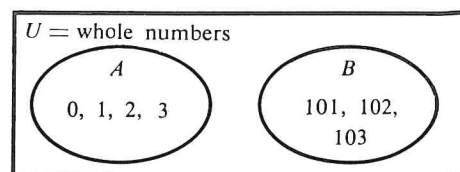


2. $R \cap S$

Name the intersections of the sets.

- $\{\text{Karen, Lisa, Sharon, Margie, Belle}\}$ and $\{\text{Karen, Pat, Joan, Barbara, Lisa}\}$
- $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ and $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $\{\text{so, of, to, on, in, at, oh}\}$ and $\{\text{we, to, at, saw, of, so, oh}\}$
- $\{\text{two, four, six, eight, ten, twelve}\}$ and $\{\text{zero, four, eight, twelve, fourteen}\}$

Disjoint sets are sets that have no member in common. A and B are disjoint sets. The diagram shows that the intersection of A and B is the empty set. $A \cap B = \{ \}$.



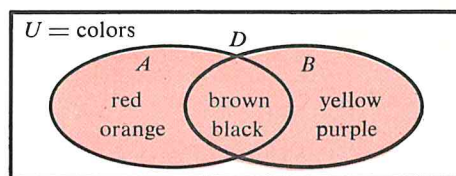
Draw diagrams to show the intersections of the sets.

- $U = \{\text{letters of the alphabet}\}$
 $A = \{\text{a, b, c, d, e, f}\}$
 $B = \{\text{a, e, i, o, u}\}$
- $U = \{\text{whole numbers}\}$
 $A = \{2, 9, 13, 4, 21\}$
 $B = \{1, 3, 6, 11, 10\}$

Complete the sentence.

- The intersection of 2 sets is a ____ of either set.

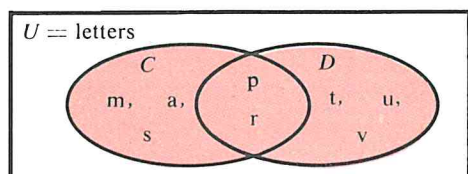
$A = \{\text{red, orange, brown, black}\}$
 $B = \{\text{yellow, purple, brown, black}\}$
 $D = \{\text{red, orange, yellow, purple, brown, black}\}$
 $= \{\text{colors that are in } A \text{ or } B\}$



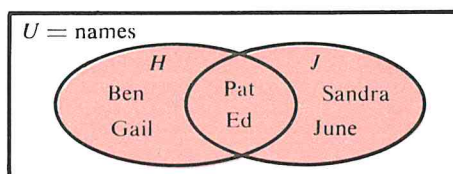
D is the **union** of A and B , and is written $A \cup B$. Since we do not repeat members of a set, brown and black appear only once in $A \cup B$. The diagram shows that we do not repeat members of a set.

EXERCISES

Use the diagrams to name the unions of the sets.



1. $C \cup D$



2. $H \cup J$

Name the unions of the sets.

3. $A = \{0, 2, 4, 6, 8, 10\}$ and $D = \{1, 3, 5, 7, 9\}$
4. $B = \{\text{Sunday, Tuesday, Wednesday, Friday}\}$ and $E = \{\text{Tuesday, Monday, Thursday, Saturday}\}$
5. $C = \{\text{trousers, socks, shoes}\}$ and $F = \{\text{belt, shirt, tie, coat}\}$

Draw diagrams to show the unions of the sets.

6. $U = \{\text{fruits}\}$
 $A = \{\text{apple, orange, pear, peach}\}$
 $B = \{\text{grape, banana, apple, cherry}\}$
7. $U = \{\text{whole numbers}\}$
 $A = \{\text{even numbers}\}$
 $B = \{\text{odd numbers}\}$

Complete the sentences.

8. The union of 2 sets is a ? of either set.
- *9. If U is a universe for any set A , then $U \cup A = \underline{\hspace{1cm}}$ and $U \cap A = \underline{\hspace{1cm}}$
- **10. If C is the empty set, then for any set B , $B \cup C = \underline{\hspace{1cm}}$ and $B \cap C = \underline{\hspace{1cm}}$.

Equivalent sets

Set A has 3 members. The **cardinal number** of A is 3. We write $n(A) = 3$. Each member of A is matched with one and only one member of B . A is matched **one-to-one** with B . $n(A) = n(B)$.

$$\begin{array}{l} A = \{ \text{red square}, \text{red circle}, \text{red triangle} \} \\ \quad \downarrow \quad \downarrow \quad \downarrow \\ B = \{ \text{red letter C}, \text{red diamond}, \text{red star} \} \end{array}$$

Sets which are matched one-to-one have the same cardinal number. They are **equivalent**.

EXERCISES

Study the sets, then name the cardinal numbers.

$A = \{\text{maple, oak, spruce, fir, walnut}\}$

$B = \{\text{milk, cheese, butter, cream}\}$

$C = \{\text{months of the year}\}$

$D = \{\text{letters in the alphabet}\}$

$E = \{\text{a, e, i, o, u}\}$

$F = \{\text{boys over 10 feet tall}\}$

- | | | | | |
|-----------|-----------|-----------|------------------|-------------------|
| 1. $n(A)$ | 3. $n(C)$ | 5. $n(E)$ | 7. $n(A \cup B)$ | 9. $n(D \cup E)$ |
| 2. $n(B)$ | 4. $n(D)$ | 6. $n(F)$ | 8. $n(C \cup F)$ | 10. $n(D \cap E)$ |

Study the sets, then answer the questions.

$H = \{0, 2, 4, 6, 8\}$

$J = \{\text{a, e, i, o, u}\}$

$K = \{\text{June, July, January}\}$

$L = \{\text{months whose names begin with J}\}$

$M = \{8, 0, 4, 2, 6\}$

- Name the sets which are equal.
- Name the sets which are equivalent.
- Are equal sets equivalent? Are all equivalent sets equal?

$C = \{1, 2, 3, 4, 5, \dots\}$ is the set of **counting numbers**.

$W = \{0, 1, 2, 3, 4, 5, \dots\}$ is the set of **whole numbers**.

The 3 dots mean "and so on in the same manner."

Complete the sentences.

- Set C is not equal to set W because W contains _____ which is not a member of C .
- The union of C and W is _____; the intersection of C and W is _____.
- The cardinal number of the empty set is _____.



It would be difficult to list all the members of

$$M = \{\text{all people in the world}\}.$$

It could be done, and we could name the cardinal number of the set as a whole number. This is a **finite set**.

It would be impossible to list all the members of

$$W = \{0, 1, 2, 3, 4, 5, \dots\}.$$

We cannot name its cardinal number as a whole number. This is an **infinite set**.

EXERCISES

Next to each numeral write *Yes* or *No* to tell whether it is possible to list all the members of the sets.

1. {letters of the alphabet}
2. {names for the number 9}
3. {capital cities in the world}
4. {even numbers}
5. {five lettered English words}
6. {designs that can be drawn}

7-12. In exercises 1-6, tell whether each set is finite or infinite.

Tell whether the following sets are finite or infinite.

- | | |
|---|--|
| 13. {counting numbers} | 16. {grains of sand on the nearest beach} |
| 14. {brick houses in the world} | 17. {noises that can be made} |
| 15. {animals that live in woods} | 18. {fractional numbers} |

Answer the questions.

- *19. Can $W = \{0, 1, 2, 3, \dots\}$ and $C = \{1, 2, 3, 4, \dots\}$ be matched one-to-one?
- *20. Is $C = \{1, 2, 3, 4, \dots\}$ infinite? Explain.

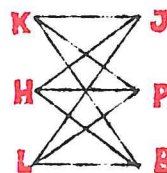
Cross products

Karen, Helen, and Lisa are on the red basketball team. Joanne, Pat, and Barbara are on the striped team. If each girl on the red team passes the ball once to each girl on the striped team, there are $3 \times 3 = 9$ passes.



$$A = \{K, H, L\} \quad \text{and} \quad B = \{J, P, B\}$$

Each member of A is matched with each member of B . This set of pairs is called the **cross product** of A and B , written $A \times B$. $A \times B$ may be written as:



$$\{(K, J), (K, P), (K, B), (H, J), (H, P), (H, B), (L, J), (L, P), (L, B)\}$$

$$n(A \times B) = 9$$

EXERCISES

Using the given sets, write the cross products.

$$A = \{1, 2, 3, 4, 5\}$$

$$C = \{w, x, y\}$$

$$B = \{a, b, c\}$$

$$D = \{\text{Ted}, \text{Joe}\}$$

1. $A \times B$

2. $A \times D$

3. $C \times B$

4. $C \times D$

Name these cardinal numbers for the sets above.

5. $n(A)$

8. $n(C)$

11. $n(A)$

14. $n(C)$

6. $n(B)$

9. $n(D)$

12. $n(D)$

15. $n(B)$

7. $n(A \times B)$

10. $n(C \times D)$

13. $n(A \times D)$

16. $n(C \times B)$

Name the two sets which form each cross product.

17. $A \times B = \{(2, a), (2, b), (2, c), (6, a), (6, b), (6, c)\}$

18. $C \times D = \{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3), (z, 1), (z, 2), (z, 3)\}$

Solve the problem.

19. If there are 4 kinds of pie — apple, cherry, apricot, and peach, and 5 kinds of ice cream — vanilla, chocolate, strawberry, maple, and butter pecan, how many different desserts can you form using one kind of pie and one kind of ice cream?

We may show two sets and the cross product as in the diagram. We see that the cross product is a set of ordered pairs. The first member of each pair is from A , the second is from B .

Bill	(Bill, ball)	(Bill, bat)	(Bill, glove)
Freda	(Freda, ball)	(Freda, bat)	(Freda, glove)
Linda	(Linda, ball)	(Linda, bat)	(Linda, glove)
	ball	bat	glove

$$A = \{\text{Bill, Freda, Linda}\}$$

$$B = \{\text{ball, bat, glove}\}$$

$$n(A \times B) = n(A) \times n(B) = 3 \times 3 = 9$$

EXERCISES

Name the ordered pairs to complete the sets of cross products.

1.

Brenda	(Brenda, dog)	e	n
Jose	a	(Jose, cat)	m
Mike	c	r	(Mike, horse)
	dog	cat	horse

2.

Bernie	(Bernie, skating)	m	n
Bruce	r	s	u
Belinda	v	c	e
	skating	movies	dancing

Name the two sets which form each cross product.

3. $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

4. $M \times N = \left\{ \left(f, \frac{1}{2}\right), \left(f, \frac{1}{3}\right), \left(f, \frac{1}{4}\right), \left(g, \frac{1}{2}\right), \left(g, \frac{1}{3}\right), \left(g, \frac{1}{4}\right) \right\}$

5. $R \times T = \{(t, 0), (t, 1), (t, 2), (t, 3), (t, 4), (t, 5)\}$

Answer the questions.

6. If $A = \{a, b, c\}$ and $B = \{1, 2\}$,
is it true that
 $A \times B = B \times A$?

7. If $A = \{a, b, c\}$ and $B = \{1, 2\}$,
is it true that
 $n(A \times B) = n(B \times A)$?

Practice with sets

Name the sets by listing the members.

1. $M = \{\text{the last 8 letters in the alphabet}\}$
2. $N = \{\text{people in your room over 10 feet tall}\}$
3. $O = \{\text{days in the week whose name begins with S}\}$
4. $P = \{\text{the digits}\}$

Name the sets by describing the members.

5. $F = \{a, e\}$
6. $G = \{241, 242, 243, 244\}$
7. $H = \{1, 3, 5\}$
8. $I = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday}\}$

List the members of these sets from exercises 1–8.

- | | | | |
|----------------|----------------|----------------|--------------------|
| 9. $M \cup F$ | 12. $M \cap N$ | 15. $O \cap I$ | 18. $H \otimes F$ |
| 10. $M \cap F$ | 13. $N \cup O$ | 16. $P \cup H$ | 19. $O \otimes H$ |
| 11. $M \cup N$ | 14. $O \cup I$ | 17. $P \cap H$ | *20. $H \otimes H$ |

Name the cardinal numbers for these sets from exercises 1–20.

- | | | | |
|-------------------|-------------------|-------------------|----------------------|
| 21. $n(M)$ | 24. $n(O)$ | 27. $n(P \cap H)$ | 30. $n(G \cup H)$ |
| 22. $n(N)$ | 25. $n(O \cup I)$ | 28. $n(N \cup O)$ | 31. $n(H \otimes F)$ |
| 23. $n(M \cup F)$ | 26. $n(O \cap I)$ | 29. $n(G)$ | 32. $n(O \otimes H)$ |

Write T or F to tell whether each of the statements is *true* or *false*.

33. The empty set has no members.
34. $W = \{\text{whole numbers}\}$ is an infinite set.
35. The largest set has a million members.
36. The intersection of the empty set and any set is the empty set.
37. The empty set is a subset of every set.
38. The union of sets C and D is a *subset* of both C and D .
39. The intersection of sets C and D is a *superset* of both C and D .
40. A universe for the set of counting numbers is the set of whole numbers.



We use many different kinds of symbols in mathematics.
Study this chart.

	symbol	name	use
S E T S	{ }	braces	to show a set of elements
	$\cup$ $\cap$	union intersection	to show an operation with sets
	$\subset$ $\supset$	subset superset	to state a relation between sets
	A letter like n	placeholder	to hold the place for a numeral as in $n + 4 = 9$
N U M B E R S	$+$ $-$ $\times$ $\div$	addition (plus) subtraction (minus) multiplication (times) division	to show an operation with numbers
	$>$ $<$ $=$	greater than less than equals	to state a relation between numbers

Use symbols to help you write the sentences as briefly as possible.

1. Set A is a subset of set B .
2. Set C is the union of set D and set E .
3. Three is less than four.
4. The sum of 17 and 13 is 30.
5. The numerals $13 + 6$ and $9 + 10$ name the same number.
6. When 1924 is divided by 37, the quotient is 52.
7. Set M is the intersection of set R and set S .
8. Set D is a superset of set G .
9. W is the set of whole numbers.
10. The product of 23 and 92 is 2116.
11. The sum of 8 and 9 is greater than 13.
12. The sum of m and 47 is 79.
13. The difference between 33 and n is 16.

Sentences

	word sentences	number sentences
open	It is the sum of 73 and 16. It is less than 100.	$n = 73 + 16$ $x < 100$
closed: true	Eighty-nine is the sum of 73 and 16. Sixty-three is less than 100.	$89 = 73 + 16$ $63 < 100$
closed: false	Eighty-five is the sum of 73 and 16. One hundred seventeen is less than 100.	$85 = 73 + 16$ $117 < 100$

We cannot tell whether an open word sentence is true until we replace the **pronoun** by a **noun**.

We cannot tell whether an open number sentence is true until we replace the **placeholder** by a **numeral**.

A number whose name makes a number sentence true is a **solution** of the number sentence. An open equation, like $n = 5 + 6$, has one solution; an open inequality, like $n < 4$, has many solutions.

EXERCISES

Write a noun to make each of the statements true.

1. It blows from the north, the south, the east, or the west.
2. When heated, it becomes steam; when cooled, it becomes ice.
3. He was one of the first men to fly an airplane.

Replace each placeholder with one or more numerals from $\{0, 1, 2, 3, 4, 5, 6\}$ to make the open sentence true.

- | | | | |
|----------------|----------------------|------------|-------------|
| 4. $n + 5 = 9$ | 6. $2 \times n = 12$ | 8. $n < 1$ | 10. $n < 2$ |
| 5. $n - 2 = 3$ | 7. $n - 6 = 0$ | 9. $n > 4$ | 11. $n > 5$ |



We must know the set from which we can choose our solution before we make our choice of solutions. If a solution of the inequality $n < 4$ has to be a whole number, our **set of solutions** is $\{0, 1, 2, 3\}$. If the set from which we can choose our solution is the set of fractional numbers, we have an infinite set of solutions. The set from which we can choose a number to replace the placeholder is a *universal set* (or superset).

EXERCISES

Choose a member from the set on the left to replace the pronoun and make the sentence true.

- | | |
|---|--|
| 1. $U = \{\text{Alice in Wonderland, Florence Nightingale, Queen Elizabeth I}\}$ | She was a famous nurse. |
| 2. $U = \{\text{Paul Revere, George Washington, Ben Franklin, Abraham Lincoln}\}$ | He was the first President of the United States. |
| 3. $U = \{\text{an airplane, a boat, a bicycle, a car, a bus}\}$ | It usually can be ridden by only one person. |
| 4. $U = \{\text{a pear, a strawberry, an apple, a tomato, an orange}\}$ | It is usually peeled before eating. |
| 5. $U = \{\text{Marco Polo, William Tell, Buffalo Bill}\}$ | He shot an apple off his son's head. |
| 6. $U = \{\text{missile, message, mess, mister, mixture}\}$ | It is a word for a kind of rocket. |
| 7. $U = \{\text{Ben Franklin, Julius Caesar, Tom Paine}\}$ | He was a famous Roman. |

Name a number from the set on the left to solve the number sentence.

- | | |
|--|-------------------|
| 8. $U = \{0, 1, 2, 3, 4, 5, 6\}$ | $n + 4 = 10$ |
| 9. $U = \{4, 6, 8, 10, 12, 14\}$ | $n \div 2 = 5$ |
| 10. $U = \{2, 6, 10, 14, 18\}$ | $n < 5$ |
| 11. $U = \{1, 3, 5, 7, 9, 11\}$ | $n > 10$ |
| 12. $U = \{244, 246, 248, 250\}$ | $n = 8 \times 31$ |
| 13. $U = \{\text{even numbers less than 20}\}$ | $n > 17$ |
| *14. $U = \{0, \frac{1}{2}, \frac{1}{3}, 1, 2\}$ | $n \times 2 = 1$ |

Properties of the operations

There are 11 properties of the number operations that we use in arithmetic.

The Closure Property

A. of addition

The sum of 2 whole numbers is a whole number.

B. of multiplication

The product of 2 whole numbers is a whole number.

The Commutative Property

C. of addition

The order of the addends does not affect the sum.

$$6 + 18 = 18 + 6 = 24$$

D. of multiplication

The order of the factors does not affect the product.

$$6 \times 18 = 18 \times 6 = 108$$

The Associative Property

E. of addition

The way the addends are grouped does not affect the sum.

$$9 + (1 + 5) = (9 + 1) + 5$$

$$9 + 6 = 10 + 5$$

$$15 = 15$$

F. of multiplication

The way the factors are grouped does not affect the product.

$$3 \times (4 \times 10) = (3 \times 4) \times 10$$

$$3 \times 40 = 12 \times 10$$

$$120 = 120$$

G. Zero as an addend

The sum of 0 and another number is equal to the other number.

$$37 + 0 = 37$$

$$37 - 37 = 0$$

H. One as a factor

The product of 1 and another number is equal to the other number.

$$37 \times 1 = 37$$

$$37 \div 37 = 1$$



I. Subtraction is the opposite of addition.

$$\begin{aligned} 8 + 9 &= 17 \\ 17 - 9 &= 8 \end{aligned}$$

J. Division is the opposite of multiplication.

$$\begin{aligned} 8 \times 9 &= 72 \\ 72 \div 9 &= 8 \end{aligned}$$

K. The distributive (common factor) property of multiplication. The sum (or difference) of 2 addends with a common factor may be named by multiplying the sum (or difference) of the other factors by the common factor or by naming the 2 products and then adding (or subtracting).

$$\begin{aligned} &(8 + 7) \times 4 = 15 \times 4 = 60 \\ \text{or} \quad &(8 \times 4) + (7 \times 4) = 32 + 28 = 60 \end{aligned}$$

EXERCISES

Write the letters C to K. Next to each letter write the numerals for those equations that show examples of that property of the number operations, for example, (C, 1).

1. $3 + 12 = 12 + 3$
2. $3 \times (6 \times 9) = (3 \times 6) \times 9$
3. $3 \times \frac{1}{3} = \frac{1}{3} \times 3$
4. $81 \times 1 = 81$
5. $9 \times (8 + 3) = 72 + 27$
6. $\frac{1}{4} \times (3 + 1) = \frac{3}{4} + \frac{1}{4}$
7. $15 + 4 = 10 + (5 + 4)$
8. $48 - 48 = 0$
9. $24 \div 24 = 1$
10. $6 \times 89 = (6 \times 90) - (6 \times 1)$
11. $42 + 7 = 40 + (2 + 7)$
12. $3 \times 20 = (3 \times 2) \times 10$
13. $(2 + 7) + 8 = 2 + (7 + 8)$
14. $30 + 40 = (3 + 4) \times 10$
15. $640 + 3 = 600 + 43$
16. $\frac{3}{8} + \frac{5}{8} = (3 + 5) \times \frac{1}{8}$
17. $26 + 0 = 26$
18. $4 \times 36 = (4 \times 30) + (4 \times 6)$
19. $38 - 38 = 0$
20. $45 \div 3 = (30 \div 3) + (15 \div 3)$
21. $28 \times 7 = 7 \times 28$
22. $\frac{7}{9} - \frac{2}{9} = (7 - 2) \times \frac{1}{9}$
23. $6 \times 21 = (6 \times 20) + 6$
- *24. $6 \times \frac{1}{6} = 1$

A decimal system of numerals

Our system of numerals is based on a set of **ten**. The Latin word for ten is “decem”, so we call our system a **decimal** system. Using the ten **digits** 0–9 we can write a **compact numeral** for a number of any size.

In the numeral 46, the **face** value of the 4 is 4; the **place** value of the 4 is tens; the **total** value of the 4 is 4×10 , or 40.

EXERCISES

For each digit in the numeral 4689, name

1. the face value.
2. the place value.
3. the total value.

For each digit in the numeral 8037, name

4. the face value.
5. the place value.
6. the total value.

Using the digits 8, 2, 4, 5, write the numeral

7. with thousands' digit 2, hundreds' digit 4, tens' digit 5 and ones' digit 8.
8. with thousands' digit 8, hundreds' digit 5, tens' digit 2 and ones' digit 4.
9. for the greatest number you can name.
10. for the least number you can name.
11. for the least number between 2000 and 3000.
12. for the greatest number between 4000 and 5000.
13. for the least number between 8000 and 9000.



Write compact numerals for the expanded numerals

14. $1000 + 400 + 70 + 6$
15. $4000 + 700 + 60 + 5$
16. $7000 + 800 + 6$
17. $8000 + 800 + 99$
18. $9000 + 80 + 9$
19. $4000 + 4$
20. the greatest counting number with 4 digits.
21. the least counting number with 4 nonzero digits.
22. the number which is 1 greater than the greatest number you can name with 5 digits.



A multiple of ten has ten as a factor.	$30 = 3 \times 10$
A power of ten has only factors of ten.	$100 = 10 \times 10$ $1000 = 10 \times 10 \times 10$
An exponent shows how many factors of ten.	$10^1 = 10$ $10^2 = 10 \times 10$
The third power of 10 is 10^3 or 1000.	$10^3 = 10 \times 10 \times 10$

EXERCISES

Name the exponents to solve the equations.

- $2 \times 2 \times 2 \times 2 = 2^a$
- $3 \times 3 \times 3 = 3^c$
- $4 \times 4 = 4^e$
- $10 \times 10 = 10^n$
- $10 \times 10 \times 10 \times 10 = 10^r$
- $10 \times 10 \times 10 \times 10 \times 10 = 10^s$

Solve the equations.

- $2^2 = a$
- $2^4 = c$
- $3^2 = e$
- $10^3 = g$
- $10^5 = i$
- $2^3 = b$
- $2^5 = d$
- $10^2 = f$
- $10^4 = h$
- $10^6 = j$

Rename the numbers as compact numerals.

- 3×10^1
- 4×10^2
- 4×10^3
- 9×10
- $(2 \times 10) + 5$
- $(3 \times 10^2) + (6 \times 10) + 3$
- $(6 \times 10^3) + (4 \times 10^2) + (5 \times 10) + 6$
- $(9 \times 10^3) + (2 \times 10^2) + (4 \times 10) + 8$

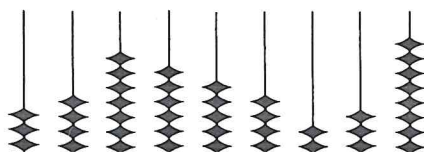
Rename these numbers as whole numbers.

- the third power of 2
- the fourth power of 2
- the sixth power of 2
- the second power of 3
- the third power of 3
- the fourth power of 10

A decimal system of numerals

In our system of numeration the first place to the left of ones' place is the place for *tens* (10^1), the second is for *hundreds* (10^2), the third is for *thousands* (10^3) and so on.

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
3	4	7	6	5	4	2	3	8



Each rod on our abacus can hold only 9 counters. If it has more than 9 counters, it is overloaded and 10 counters have to be removed and exchanged for 1 counter to go on the next rod to the left.

The places are grouped in periods of three digits for ones, thousands, millions, billions, and so on.



On our electric counter each disc has 10 digits; when 9 is passed, 1 is added to the place on the left.

3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
3	4	7	6	5	4	2	3	8	

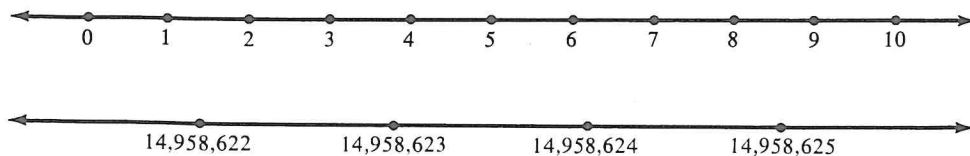


Write the compact numerals for the numbers.

1. $(7 \times 10^4) + (8 \times 10^3) + (6 \times 10^2) + (5 \times 10) + 9$
2. $(3 \times 10^5) + (4 \times 10^4) + (3 \times 10^3) + (9 \times 10^2) + 8$
3. $(4 \times 10^6) + (7 \times 10^5) + (8 \times 10^4) + (5 \times 10^3) + (4 \times 10^2) + (6 \times 10) + 2$
4. $(4 \times 10^7) + (8 \times 10^6) + (9 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (8 \times 10) + 5$
- *5. $(13 \times 10^6) + (28 \times 10^4) + (15 \times 10^2) + (3 \times 10) + 7$
6. 473 million, 286 thousand, 947
7. 829 million, 371 thousand, 456
8. 727 billion, 351 million, 865 thousand, 74
9. 890 billion, 9 million, 750 thousand, 623
10. 437,000,000 + 286,000 + 296
11. 335,000,000 + 526,000 + 780
12. 25,000,000,000 + 569,000,000 + 464,000 + 139
13. 234,000,000,000 + 357,000 + 278
14. 669,000,000 + 58,000 + 594
15. seven hundred twenty-four thousand,
three hundred seventy-six
16. nine hundred fifty-four million, eight hundred thirty-nine
thousand, two hundred five
17. forty-three million, one hundred fifty-six thousand,
three hundred seven
18. eight hundred forty million, nine hundred forty thousand,
seven hundred forty
19. 1200 + 120 + 12 + 2
20. 1700 million, 1700 thousand, 1700
21. 8 million, 1898 thousand, 2746
22. 7 million, 7000 thousand, 7
23. 10 million, 4500 thousand, 49
24. 21 billion, 24,500 thousand, 345
- *25. 3 trillion, 52 million, 482 thousand, 482.



Ordering numbers



The top number line shows that $3 < 4$, $4 < 5$, $5 < 6$, and so on. We can show that a number is between two other numbers by using the sign $<$ twice. For example $3 < 4 < 5$ may be read “4 is greater than 3 and less than 5” or “4 is between 3 and 5.”

The lower number line shows that the same ideas can be applied to very great numbers. We see that

$$14,958,623 < 14,958,624 < 14,958,625.$$

EXERCISES

Write $<$ or $>$ to tell whether the first number of the pair is less than or greater than the second.

1. (5200; 5098) $>$

2. (104,824; 104,830) $<$

3. (16,200; 7200) $>$

4. (104,824; 105,824) $<$

5. (984,400; 984,288) $>$

6. (984,059; 990,059) $<$

7. (42,006,982; 43,600,780) $<$

8. (7,900,000; 7,090,000) $>$

Use the sign $<$ to arrange each set of 3 numbers in order.

For example, we write : (7854 ; 7809 ; 7900) as

$$7809 < 7854 < 7900.$$

9. (5769; 5035; 5722)

10. (62,054; 61,983; 62,500)

11. (6400; 6040; 6404)

12. (128,900; 130,000; 128,887)

13. (250,800; 258,000; 250,880)

14. (724,985; 724,900; 749,000)

Name the least whole number and the greatest whole number between the numbers.

15. $4824 < n < 4965$

16. $2000 < n < 3000$

17. $6500 < n < 7500$

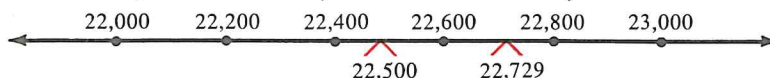
18. $23,000 < n < 23,800$

19. $1,546,000 < n < 1,546,500$

20. $99,999 < n < 150,000$

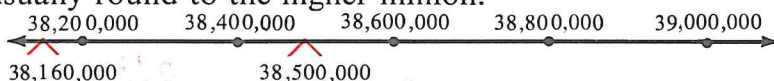


To round a number such as 22,729 to the **nearest thousand** we name the multiple of 1000 that is nearest to 22,729 on the number line. For example, to round 22,729 we notice that $729 > 500$ and 500 is halfway between 0 and 1000. So 729 is nearer 1000, therefore 22,729 rounds to 23,000.



If a number is halfway between two multiples of 1000, we usually round to the greatest number. For example 22,500 rounds to 23,000.

To round a number to the **nearest million**, we name the multiple of 1,000,000 that is closest to it on the number line. If a number is halfway between two multiples of one million we usually round to the higher million.



To the nearest million, 38,160,000 rounds to 38,000,000; 38,500,000 rounds to 39,000,000.

EXERCISES

1. You can tell from the digit in the hundreds' place whether each of the following numbers rounds to 7000 or 8000.
 7700 7465 7624 7573 7075
 - a. For which digits in the hundreds' place will you round to 7000?
 - b. For which digits in the hundreds' place will you round to 8000?
2. There are 5280 feet in a mile. Round this number to the nearest thousand.
3. The greatest ocean depth occurs in the Marianas Trench in the Pacific Ocean. Rounded to the nearest thousand the depth of the Marianas Trench is 36,000 feet. The actual depth of the Marianas Trench is between what two numbers?
4. Rounded to the nearest million the area of the Pacific Ocean is 64,000,000 square miles. The actual area of the Pacific Ocean is between what two numbers?

The Roman system of numerals

The Romans used the 7 basic symbols I, V, X, L, C, D, M to write their numerals. These were the symbols for 1, 5, 10, 50, 100, 500, 1000. The Roman system did not use *place value*, and each of the basic symbols was used as many times as needed to name a number. For example, 3 was written III, 300 was written CCC, 3000 was written MMM. Numbers were named by addition. For example,

$$CCCXXVIII = 300 + 20 + 5 + 3 = 328.$$

However, when a symbol for a lesser number was written to the left of a symbol for a greater number, the lesser number was subtracted. For example, XIV = 10 + 5 - 1 = 14.

EXERCISES

Write a decimal numeral for the number named by each of the Roman numerals.

- | | | | | |
|------------------|-------------------|-----------------------|------------------------|-------------------------|
| 1. VI 6 | 6. CX 110 | 11. XXXIV 34 | 16. MMCXXX 2130 | 21. LXXXVIII 88 |
| 2. IV 4 | 7. XC 90 | 12. CXXVII 127 | 17. MMIX 2009 | 22. XCIX 99 |
| 3. XI 11 | 8. XXI 21 | 13. LXXIX 79 | 18. MXC 1090 | 23. MCMLXX 1970 |
| 4. IX 9 | 9. XIX 19 | 14. CCXL 240 | 19. DCCL 750 | 24. MCDXCIV 1494 |
| 5. XIV 14 | 10. CM 900 | 15. XVI 16 | 20. MC 1100 | 25. XXIX 29 |

Write a Roman numeral for the number named by each of the following decimal numerals.

- | | | | | | |
|--------|--------|--------|---------|---------|----------|
| 26. 35 | 28. 58 | 30. 86 | 32. 236 | 34. 638 | 36. 246 |
| 27. 24 | 29. 72 | 31. 39 | 33. 329 | 35. 297 | 37. 2948 |

Answer the questions.

38. In any Roman numeral, the symbols I, X and C are usually not used more than three times. Explain why this is so.
39. There is no symbol for zero in the Roman numeral system. Why is a zero not needed?

Name the sums as decimal numerals.

- | | | | |
|------------|--------------|-------------|---------------|
| 40. V + II | 41. XI + III | 42. MCC + D | 43. LIX + XLV |
|------------|--------------|-------------|---------------|

Another system of numerals which has been in use for many centuries is the Hebrew system. It uses the letters of the Hebrew alphabet as numerals, like this:

our numeral	the Hebrew numeral	the Hebrew name	our numeral	the Hebrew numeral	the Hebrew name
1	א	alef	30	ל	lamed
2	ב	bays	40	מ	mem
3	ג	gimel	50	נ	nun
4	ד	dalet	60	ס	sameh
5	ה	hay	70	ז	ayin
6	ו	vov	80	ח	pay
7	ז	zayin	90	ט	tzadee
8	ח	hess	100	ק	koof
9	ט	tess	200	ר	resh
10	י	yood	300	ש	shin
20	כ	kaf	400	ת	tov

11 would be written yood, alef, 23 would be written kaf, gimel, and 246 would be written resh, mem, vov.

EXERCISES

Write the Hebrew names for the numbers.

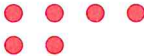
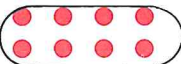
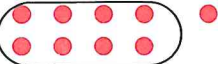
- | | | | | |
|-------|-------|-------|---------|---------|
| 1. 13 | 4. 25 | 7. 49 | 10. 86 | 13. 172 |
| 2. 17 | 5. 28 | 8. 57 | 11. 107 | 14. 203 |
| 3. 21 | 6. 44 | 9. 82 | 12. 115 | 15. 245 |

Write our numerals for the Hebrew numerals.

- | | | |
|---------------|--------------------|------------------------|
| 16. yood, vov | 19. pay, tess | 22. shin, sameh, gimel |
| 17. kaf, bays | 20. koof, kaf | 23. tov, nun, vov |
| 18. nun, hess | 21. koof, mem, hay | 24. resh, hess |
- *25. Did the Hebrews use *place value* to write their numerals?

Other place value systems

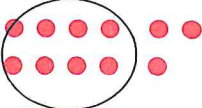
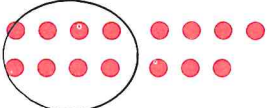
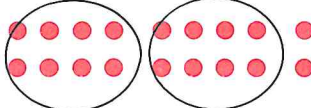
The most important properties of our system of numerals are its use of *place value* and of ten as a *base*. Bases other than ten can be used to build place value systems. For example, we could use eight as a base and our counting would begin like this:

	1	one		6	six
	2	two		7	seven
	3	three		10_{eight}	one eight
	4	four		11_{eight}	one eight one
	5	five		12_{eight}	one eight two

To distinguish between *base ten* and *base eight* numerals, we have written a small “eight” to the right of each base eight numeral.

EXERCISES

Write in digits the base eight numeral that shows these groupings.

1.  2.  3. 

We shall also write a small “ten” to the right of base ten numerals. So 14_{ten} is read “one ten four” or fourteen: 14_{eight} is read, “one eight four.” 10_{ten} represents ten, and 10_{eight} represents eight.

Name each of the numbers in words.

4. 15_{ten} 6. 23_{eight} 8. 47_{ten} 10. 56_{eight} 12. 34_{eight}
 5. 71_{ten} 7. 71_{eight} 9. 40_{ten} 11. 40_{eight} 13. 64_{eight}

Write $<$, $>$ or $=$ to compare the numbers.

14. 17_{ten} $\bigcirc$ 17_{eight} 16. 23_{eight} $\bigcirc$ 17_{ten} 18. 11_{eight} $\bigcirc$ 9_{ten}
 15. 30_{ten} $\bigcirc$ 40_{eight} 17. 42_{ten} $\bigcirc$ 57_{eight} 19. 100_{ten} $\bigcirc$ 100_{eight}

Place value in a base eight system

The places in a base eight system would show powers of eight like this:

10^4_{eight}	10^3_{eight}	10^2_{eight}	10^1_{eight}	1
-----------------------	-----------------------	-----------------------	-----------------------	---

The values of these places in our decimal system would be:

$8^4_{\text{ten}} = 4096_{\text{ten}}$	$8^3_{\text{ten}} = 512_{\text{ten}}$	$8^2_{\text{ten}} = 64_{\text{ten}}$	$8^1_{\text{ten}} = 8_{\text{ten}}$	1
--	---------------------------------------	--------------------------------------	-------------------------------------	---

EXERCISES

Name **a.** the face value, **b.** the place value and **c.** the total value of the digit 5 in each of the numerals.

1. 325_{eight} 2. 56_{eight} 3. 573_{eight} 4. 4205_{eight} 5. 5134_{eight}

Write a compact base eight numeral for each of the numbers.

6. $(1 \times 8^3) + (2 \times 8^2) + (4 \times 8) + 5$
 7. $(2 \times 8^4) + (7 \times 8^3) + (3 \times 8^2) + (6 \times 8) + 5$
 8. $(5 \times 8^4) + (4 \times 8^3) + (7 \times 8) + 6$

Write a compact base eight numeral for each of the numbers.

9. $(4 \times 4096) + (3 \times 512) + (2 \times 64) + (1 \times 8) + 3$
 10. $(5 \times 4096) + (7 \times 512) + (1 \times 64) + (5 \times 8) + 1$
 11. $(3 \times 4096) + (2 \times 512) + (7 \times 8) + 6$

Study this method for changing base eight numerals to base ten numerals.

$$\begin{aligned} 3147_{\text{eight}} &= (3 \times 8^3_{\text{ten}}) + (1 \times 8^2_{\text{ten}}) + (4 \times 8^1_{\text{ten}}) + 7_{\text{ten}} \\ &= (3 \times 512_{\text{ten}}) + (1 \times 64_{\text{ten}}) + (4 \times 8_{\text{ten}}) + 7 \\ &= 1536_{\text{ten}} + 64_{\text{ten}} + 32_{\text{ten}} + 7 \\ &= 1639_{\text{ten}} \end{aligned}$$

Write the base eight numerals as base ten numerals.

12. 37_{eight} 13. 263_{eight} 14. 546_{eight} 15. 215_{eight} *16. 2406_{eight}

Denominate numbers

units of length

12 inches — 1 foot
3 feet — 1 yard

units of capacity

2 pints — 1 quart
4 quarts — 1 gallon

units of weight

16 ounces — 1 pound
2000 pounds — 1 ton

units of time

60 seconds — 1 minute
60 minutes — 1 hour

In our system of measures we do not use *place value*. Instead, we name the units. For example, in 7 feet 5 inches, the 7 names the number of feet and the 5 names the number of inches.

EXERCISES

Write the names of the units to complete the sentences.

- Thirty-eight inches are equivalent to 1 _____, 2 _____.
- Twenty-six pints are equivalent to 3 _____, 1 _____.
- Eighty-seven ounces are equivalent to 5 _____, 7 _____.
- Two-hundred four minutes are equivalent to 3 _____, 24 _____.

Complete the sentences.

- Fourteen pints are equivalent to _____ gallons _____ quart.
- Sixty-seven inches are equivalent to _____ yards _____ inches.
- Five thousand pounds are equivalent to _____ tons _____ ounces.

Complete the table.

8.	1 yd.	2 yd.	3 gal.	1 ton	2 hr.	<i>n</i> yd.	2 gal.	1 hr.
	2 ft.	4 ft.	2 qt.	4 lb.	3 min.	4 ft.	<i>r</i> qt.	4 min.
	5 in.	2 in.	3 pt.	3 oz.	4 sec.	8 in.	1 pt.	<i>s</i> sec.
total	65 in.	<i>a</i> in.	<i>c</i> pt.	<i>e</i> oz.	<i>m</i> sec.	164 in.	25 pt.	3873 sec.

The metric system of measures

The most widely used system of measures in the world is the metric system. This system has the advantage that it has a base of ten like our system of numeration.

The Metric System

unit	approximate equivalent in our system
length	
centimeter (cm)	$\frac{2}{5}$ inch
meter (m) — 100 cm	39 inches
kilometer (km) — 1000 m	$\frac{5}{8}$ mile
volume (capacity)	
cubic centimeter (cc)	$\frac{1}{15}$ cubic inch
liter (l) — 1000 cc	1 quart
weight	
gram (gm)	$\frac{1}{25}$ ounce
kilogram (kgm) — 1000 gm	$2\frac{1}{5}$ pounds

EXERCISES

Complete the statements ($\leftrightarrow$ means "is approximately equivalent to").

- 1 centimeter $\leftrightarrow \frac{2}{5}$ inch, so 3 centimeters $\leftrightarrow$? inches.
- 1 meter $\leftrightarrow$ 39 inches, so 2 meters $\leftrightarrow$? inches.
- 1 liter $\leftrightarrow$ 1 quart, so 8 liters $\leftrightarrow$? quarts.
- 1 gram $\leftrightarrow \frac{1}{25}$ ounce, so 25 grams $\leftrightarrow$? ounces.
- 1 mile $\leftrightarrow$ 1600 m, so 5 miles $\leftrightarrow$? m.
- 1 km $\leftrightarrow$ 1000 m, so 3 km $\leftrightarrow$? m.
- *7. Bill's weight is 110 lb. or ? kgm.

A lot can happen in 10 years

A decade is 10 years. Read these facts about changes in the United States from 1950 to 1960.

	1950	1960
number of vehicles on U.S. roads	49,300,000	73,769,000
gallons of motor fuel sold	40,280,000 (thousands of gallons)	63,714,000
passenger miles on roads	458,246 (millions of miles)	718,845
miles of telephone wire	38,525,000	65,735,000

Discuss these facts and see if you can make a list of others like them.

EXERCISES

Write in words, numerals for each of the following numbers.

- | | | |
|----------------|----------------|------------------|
| 1. 246,000,000 | 3. 400,250,700 | 5. 500,000,000 |
| 2. 6,073,008 | 4. 13,200,000 | 6. 7,864,023,405 |

Write in digit form, a numeral for each of the numbers.

- seventy-two thousand, sixty-five
- three hundred thousand
- five million, two hundred forty-five thousand, six hundred
- eighty-four million, three hundred thousand
- two hundred thirty million, five hundred
- fifty-four billion, seven million, seventy-eight thousand
- three billion
- five hundred billion
- five hundred billion, forty-three thousand
- twenty-five billion, twenty-five million



List the objects in each of the sets.

1. {five whole numbers between 10 and 16}
2. {the northern-most and southern-most points in the world}

Name a superset for each of the sets.

3. {sparrows, eagles, parrots, hens}
4. {bread, butter, jam, fruit}

If $A = \{1, 2, 3, 4\}$ and $B = \{s, t, u\}$, name

5. $A \cup B$
6. $A \cap B$
7. $A \otimes B$
8. $B \otimes A$
9. $n(A)$
10. $n(B)$
11. $n(A \cup B)$
12. $n(A \cap B)$

Write the compact, decimal system numeral for each of the numbers.

13. $943,000,000 + 706,000 + 891$
14. $(8 \times 10^5) + (7 \times 10^4) + (3 \times 10^3) + (4 \times 10) + 6$
15. sixty-three million, four hundred five thousand, seventy-five

Write a base eight numeral for each of the numbers.

16. 13_{ten}
17. 23_{ten}
18. 33_{ten}
19. 54_{ten}
20. 73_{ten}

Write a decimal system numeral for each of the numbers.

21. 15_{eight}
22. 24_{eight}
23. 61_{eight}
24. 75_{eight}
25. 143_{eight}

Write a decimal system numeral for each of the numbers.

26. XXVI
27. LXIV
28. CVIII
29. MCLVI
30. MLXVI

Write a Roman numeral for each of the numbers.

31. 17
32. 39
33. 83
34. 146
35. 1492

Complete the sentences.

36. 20 pints are equivalent to ? gallons and ? quarts.
37. 93 ounces are equivalent to ? pounds and ? ounces.
38. 6 yards 7 feet are equivalent to ? inches.
39. 7240 pounds are equivalent to ? tons and ? pounds.
40. 2 hours 40 minutes are equivalent to ? seconds.



LOOKING BACK

Set A is a **subset** of set B if *all* the objects in A are in B . We write $A \subset B$. If A is a subset of B , B is a **superset** of A . We write $B \supset A$.

- For $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, name the subset S which contains all the odd numbers of A .

The **intersection** of sets A and B is {objects that are *both* in A and in B }. The **union** of sets A and B is {objects that are in A or in B }.

- For $A = \{1, a, m, e\}$ and $B = \{m, o, u, s, e\}$ name $A \cup B$ and $A \cap B$.

The **cross product** of sets A and B is the set of *all* pairs containing *first* a member of A , *then* a member of B .

- Name the cross product of $P = \{br, sl, fl\}$ and $Q = \{ing, ight, at\}$.

A **numeral** is a name for a number. Every number has many names. If $A = \{\bullet \bullet \bullet \blacktriangle \blacktriangle \blacktriangle \blacklozenge \blacklozenge\}$, names for $n(A)$ include eight, 8 , $7 + 1$, 4×2 , and $10 - 2$.

- List 10 different names for 20.

An **equation** is a statement that 2 numerals name the same number. An **inequality** is a statement that one number is greater than or less than another number.

Complete each sentence. Write $>$, $<$ or $=$ to replace each $\bigcirc$.

- $382 + 20 \bigcirc 401$
- $(5 \times 8^2) + (1 \times 8) + 3 \bigcirc 383$
- $(2 \times 10^2) + 36 \bigcirc 236$
- $(4 \times 8^2) + 5 \bigcirc 46_{\text{eight}}$
- $(3 \times 8^2) + (2 \times 8) + 7 \bigcirc (3 \times 10^2) + (1 \times 10) + 5$
- $3000 + 400 + 7 \bigcirc (3 \times 10^3) + (4 \times 10) + 7$



A billion is a very great number.

1. How long would it take to travel a billion miles in a jet?
The round trip by air from Los Angeles to New York and back again is about 5000 miles. To travel a million miles you would need to make this trip 200 times. To travel a billion miles you would need to make this trip 200,000 times. Making one round trip a day, it would take you nearly 550 years to travel a billion miles.
2. How much is a billion dollars?
A stack of a thousand one dollar bills is about 4 inches high. A stack of a million one dollar bills is about 4000 inches or 333 feet. This is about the length of a football field. A stack of a billion dollar bills is about 4,000,000 inches. This is about 63 miles.

Just For Fun

History number quizzes.

These dates may all be named with the same digits. Name the date.

3. 14 years before the Battle of Waterloo.
4. 15 years after William the Conqueror invaded Britain.
5. 34 years after the Declaration of Independence.
6. 474 years before Christopher Columbus sailed to America.

These dates may all be named with the same digits. Name the date.

7. 38 years after the beginning of the eighteenth century.
8. 30 years before the Wright Brothers flew their airplane.
9. 540 years before the end of World War I.

Solve the equations using Hebrew numerals (see p. 23).

10. $\aleph + \beth = a$

13. $\beth\aleph + \beth = d$

11. $\beth + \beth = b$

14. $\aleph\aleph - \beth = e$

12. $\beth + \beth = c$

15. $\aleph\aleph - \beth\aleph = f$

CHAPTER 2 • Addition, Subtraction, Multiplication, División

Looking back at addition

You have learned that

1. addition is used to name the cardinal number of the union of 2 <i>disjoint</i> sets.	$\{\star\} \cup \{\color{red}{\star}\color{blue}{\diamond}\} = \{\star\color{red}{\star}\color{blue}{\diamond}\}$ $n(S) + n(T) = n(S \cup T)$ $1 + 2 = 3$
2. addition is the renaming of 2 addends as a sum .	$\begin{array}{ccc} 6 & + & 8 & = & 14 \\ \swarrow & & \searrow & & \downarrow \\ \text{addends} & & & & \text{sum} \end{array}$
3. addition is commutative .	$4 + 6 = 6 + 4$
4. addition is associative .	$(2 + 3) + 5 = 2 + (3 + 5)$
5. when zero is one of the addends, the sum is equal to the other addend.	$8 + 0 = 8$
6. there are 45 basic facts to be memorized, with each addend between 0 and 10.	$6 + 9 = 15 \text{ is a basic fact.}$

EXERCISES

List 5 basic facts for each sum.

1. 10 2. 2 and 12 3. 4 and 14 4. 6 and 16 5. 8 and 18

List 4 basic facts for each sum.

6. 9 7. 11 8. 3 and 13 9. 5 and 15 10. 7 and 17
11. List the 14 basic facts which have the digit 4 in the addend or sum.



You have learned to

1. use the associative property when one addend is greater than 10 and the other addend is less than 10.	$ \begin{aligned} 26 + 3 &= (20 + 6) + 3 \\ &= 20 + (6 + 3) \\ &= 20 + 9 \\ &= 29 \end{aligned} $
2. use the associative property when one addend is a multiple of 10 and the other addend is greater than 10.	$ \begin{aligned} 20 + 18 &= 20 + (10 + 8) \\ &= (20 + 10) + 8 \\ &= 30 + 8 \\ &= 38 \end{aligned} $
3. use the associative and commutative properties when both addends are greater than 10.	$ \begin{aligned} 23 + 65 &= (20 + 3) + (60 + 5) \\ &= 20 + (3 + 60) + 5 \\ &= 20 + (60 + 3) + 5 \\ &= (20 + 60) + (3 + 5) \\ &= 80 + 8 \\ &= 88 \end{aligned} $

EXERCISES

Use the associative and commutative properties to name the sums.

- | | | | |
|-------------|---------------|---------------|---------------|
| 1. $14 + 5$ | 7. $40 + 16$ | 13. $64 + 15$ | 19. $46 + 48$ |
| 2. $34 + 3$ | 8. $30 + 48$ | 14. $43 + 35$ | 20. $57 + 62$ |
| 3. $46 + 9$ | 9. $50 + 29$ | 15. $34 + 23$ | 21. $73 + 59$ |
| 4. $78 + 5$ | 10. $70 + 84$ | 16. $52 + 47$ | 22. $88 + 94$ |
| 5. $69 + 7$ | 11. $80 + 38$ | 17. $51 + 39$ | 23. $76 + 77$ |
| 6. $83 + 8$ | 12. $90 + 17$ | 18. $27 + 35$ | 24. $98 + 93$ |

Use the associative and the commutative properties to name the sums.

- | | | |
|-------------------|-----------------------|---------------------------|
| 25. $18 + 12 + 6$ | 29. $63 + 18 + 7 + 2$ | 33. $15 + 8 + 19 + 4$ |
| 26. $18 + 6 + 12$ | 30. $23 + 25 + 27$ | 34. $37 + 6 + 8 + 15$ |
| 27. $63 + 7 + 18$ | 31. $39 + 40 + 41$ | 35. $8 + 7 + 6 + 32 + 17$ |
| 28. $7 + 63 + 18$ | 32. $49 + 30 + 41$ | 36. $37 + 8 + 12 + 6 + 7$ |

Addition function rules

If we begin with a set of numbers W and add 4 to each number in the set, we get another set of numbers Y . If we pair the corresponding numbers as shown, we form a set of pairs Z .

$$W = \{0, 1, 2, 3, \dots\}$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ Y = \{4, 5, 6, 7, \dots\} \end{array}$$

$$Z = \{(0, 4), (1, 5), (2, 6), (3, 7), \dots\}$$

We are careful to take the first number in each pair from W and the second number from Y .

A set of ordered pairs (like Z) with a rule (such as “add 4”) is a **function**. We may state the rule by means of an equation. If we write m for the **first number** in any pair, we write $f(m)$ for the **second number**. The rule to give us Z from W and Y is $m + 4 = f(m)$ where m is a member of W and $f(m)$ is a member of Y .

EXERCISES

Study the equations, then complete the ordered pairs.

1. $w + 2 = f(w)$: $\{(5, 7), (7, a), (0, b), (3, c), (9, d), (16, e)\}$
2. $x + 13 = f(x)$: $\{(5, 18), (7, f), (0, g), (9, h), (14, i), (28, j)\}$
3. $y + 38 = f(y)$: $\{(2, 40), (7, k), (10, l), (14, m), (23, n), (62, p)\}$
4. $z + 24 = f(z)$: $\{(0, 24), (16, q), (19, r), (81, s), (24, t), (98, u)\}$

If $M = \{2, 4, 6, 8, 10, 12\}$ and $N = \{12, 14, 16, 18, 20, 22\}$, then

5. write an equation which gives a rule for matching the members of M with the members of N .
6. write the set of ordered pairs for the rule in exercise 5 where the first number in the pair comes from M and the second number from N .

Write a function equation for each set of ordered pairs.

7. $\{(3, 9), (5, 11), (12, 18), (0, 6), (19, 25), (34, 40), \dots\}$
8. $\{(5, 22), (6, 23), (7, 24), (8, 25), (9, 26), \dots\}$
9. $\{(0, 29), (2, 31), (4, 33), (6, 35), (8, 37), \dots\}$
10. $\{(13, 13), (14, 14), (20, 20), (85, 85), (97, 97), \dots\}$

Study these methods for naming the sum $265 + 479$.

Number pattern	Four-step	Short												
<table><tr><td>200</td><td>60</td><td>5</td><td>265</td></tr><tr><td>400</td><td>70</td><td>9</td><td>479</td></tr><tr><td>600</td><td>130</td><td>14</td><td>744</td></tr></table>	200	60	5	265	400	70	9	479	600	130	14	744	$\begin{array}{r} 265 \\ + 479 \\ \hline 14 \\ 130 \\ 600 \\ \hline 744 \end{array}$	$\begin{array}{r} 11 \\ 265 \\ + 479 \\ \hline 744 \end{array}$
200	60	5	265											
400	70	9	479											
600	130	14	744											

EXERCISES

Complete the number patterns.

1.

400	50	3	<i>a</i>
100	40	9	<i>c</i>
<i>e</i>	<i>n</i>	<i>r</i>	<i>s</i>

2.

500	60	5	<i>u</i>
700	80	2	<i>v</i>
<i>s</i>	<i>v</i>	<i>x</i>	<i>z</i>

3.

800	90	8	<i>z</i>
300	90	7	<i>v</i>
<i>a</i>	<i>e</i>	<i>n</i>	<i>c</i>

Use the 4-step method to name the sums.

4.
$$\begin{array}{r} 526 \\ + 342 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 639 \\ + 248 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 751 \\ + 176 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 467 \\ + 848 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 936 \\ + 588 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 837 \\ + 786 \\ \hline \end{array}$$

Use the short method to name the sums.

10.
$$\begin{array}{r} 428 \\ + 365 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 652 \\ + 298 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 584 \\ + 519 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 397 \\ + 726 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 849 \\ + 784 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 956 \\ + 979 \\ \hline \end{array}$$

Name the sums.

16.
$$\begin{array}{r} 4728 \\ + 3655 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 526471 \\ + 391819 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 17948 \\ + 96235 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 26 \\ 87 \\ 65 \\ 43 \\ 32 \\ + 79 \\ \hline \end{array}$$

23.
$$\begin{array}{r} 438 \\ 297 \\ 645 \\ 338 \\ 704 \\ + 256 \\ \hline \end{array}$$

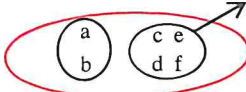
17.
$$\begin{array}{r} 34573 \\ + 28419 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 6249 \\ + 5393 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 807436 \\ + 795329 \\ \hline \end{array}$$

Looking back at subtraction

You have learned that

1. subtraction may be used to name the cardinal number of the subset that remains when a subset is removed.	 The arrow shows a subset being removed. $6 - 4 = 2$
2. subtraction is the renaming of a sum and an addend as an addend , commonly called the difference .	$\begin{array}{ccccc} 14 & - & 8 & = & 6 \\ / & & \backslash & & / \quad \backslash \\ \text{sum} & & \text{addends} & & \text{difference} \end{array}$
3. subtraction is the opposite of addition.	$\begin{array}{l} 6 + 8 = 14 \\ 14 - 8 = 6 \end{array}$
4. order is important in subtraction. Subtraction is not commutative .	$14 - 8 \neq 8 - 14$
5. we memorize addition – subtraction facts in families.	$\begin{array}{ll} 6 + 8 = 14 & 14 - 8 = 6 \\ 8 + 6 = 14 & 14 - 6 = 8 \end{array}$ A family of facts.

EXERCISES

Write the other facts in these families.

1. $7 + 9 = 16$ 2. $9 - 4 = 5$ 3. $8 + 8 = 16$ 4. $7 - 5 = 2$

Solve the equations.

5. $694 + 863 = 1557$, so $1557 - 694 = a$ and $1557 - 863 = b$

6. $3476 + 6295 = 9771$, so $9771 - 3476 = c$ and $9771 - 6295 = d$

Answer the questions.

*7. Is $(9 - 6) - 3$ equal to $9 - (6 - 3)$? Is subtraction associative?



You have learned how to subtract

1. a number less than 10 from a number greater than 10.

$ \begin{aligned} 17 - 6 &= (10 + 7) - 6 \\ &= 10 + (7 - 6) \\ &= 10 + 1 \\ &= 11 \end{aligned} $	$ \begin{aligned} 24 - 8 &= (10 + 14) - 8 \\ &= 10 + (14 - 8) \\ &= 10 + 6 \\ &= 16 \end{aligned} $
--	--

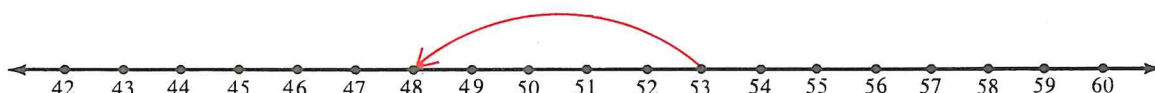
2. a number greater than 10 from another number greater than 10.

$ \begin{aligned} 48 - 27 &= 40 + 8 - 20 - 7 \\ &= 40 - 20 + 8 - 7 \\ &= (40 - 20) + (8 - 7) \\ &= 20 + 1 \\ &= 21 \end{aligned} $	$ \begin{aligned} 42 - 29 &= 30 + 12 - 20 - 9 \\ &= 30 - 20 + 12 - 9 \\ &= (30 - 20) + (12 - 9) \\ &= 10 + 3 \\ &= 13 \end{aligned} $
--	---

EXERCISES

Use the number line to name the differences.

The arrow shows $53 - 5 = 48$.



- | | | |
|-----------------|-----------------|-------------------|
| 1. $53 - 1 = a$ | 5. $60 - 4 = e$ | 9. $53 - 10 = i$ |
| 2. $53 - 2 = b$ | 6. $60 - 7 = f$ | 10. $56 - 14 = j$ |
| 3. $53 - 3 = c$ | 7. $60 - 9 = g$ | 11. $55 - 13 = k$ |
| 4. $53 - 4 = d$ | 8. $60 - 8 = h$ | 12. $60 - 14 = l$ |

Use the methods at the top of the page to name the differences.

- | | | |
|-------------------|-------------------|-------------------|
| 13. $29 - 6 = a$ | 17. $26 - 19 = e$ | 21. $43 - 17 = i$ |
| 14. $32 - 6 = b$ | 18. $85 - 68 = f$ | 22. $91 - 28 = j$ |
| 15. $85 - 8 = c$ | 19. $90 - 37 = g$ | 23. $78 - 64 = k$ |
| 16. $47 - 24 = d$ | 20. $71 - 17 = h$ | 24. $71 - 45 = l$ |



Practicing subtraction

Study these methods for naming the difference $71 - 39$.

Expanded numerals	Columns	Short
$\begin{array}{r} 71 = \\ - 39 = \\ \hline \end{array}$ $\begin{array}{r} 60 \quad 11 \\ \cancel{70} + \cancel{1} \\ - 30 - 9 \\ \hline 30 + 2 = 32 \end{array}$	$\begin{array}{r l} t & o \\ \hline 6 & 11 \\ \cancel{7} & \cancel{1} \\ - 3 & 9 \\ \hline 3 & 2 \end{array}$	$\begin{array}{r} 6 \quad 11 \\ \cancel{7} \quad \cancel{1} \\ - 3 \quad 9 \\ \hline 3 \quad 2 \end{array}$

EXERCISES

Use expanded numerals to name the differences.

1.
$$\begin{array}{r} 56 \\ - 23 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 56 \\ - 38 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 88 \\ - 29 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 648 \\ - 379 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 45 \\ - 33 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 61 \\ - 39 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 93 \\ - 58 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 534 \\ - 529 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 43 \\ - 15 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 72 \\ - 58 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 436 \\ - 119 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 766 \\ - 449 \\ \hline \end{array}$$

Use columns to name the differences.

13.
$$\begin{array}{r} 64 \\ - 27 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 83 \\ - 38 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 87 \\ - 69 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 869 \\ - 599 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 81 \\ - 47 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 75 \\ - 38 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 62 \\ - 37 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 822 \\ - 457 \\ \hline \end{array}$$

Use the short method to name the differences.

21.
$$\begin{array}{r} 77 \\ - 56 \\ \hline \end{array}$$

23.
$$\begin{array}{r} 74 \\ - 37 \\ \hline \end{array}$$

25.
$$\begin{array}{r} 146 \\ - 85 \\ \hline \end{array}$$

27.
$$\begin{array}{r} 456 \\ - 388 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 65 \\ - 45 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 42 \\ - 19 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 392 \\ - 79 \\ \hline \end{array}$$

28.
$$\begin{array}{r} 705 \\ - 238 \\ \hline \end{array}$$

Name the differences.

29. $495 - 299 = a$

31. $9006 - 4249 = c$

33. $16,257 - 8488 = e$

30. $511 - 399 = b$

32. $7307 - 3888 = d$

34. $29,356 - 7269 = f$



Use the function equation to help you complete the ordered pairs.

- $y - 5 = f(y)$: $\{(6, 1), (10, a), (12, e), (17, n), (23, u), (31, v)\}$
- $z - 13 = f(z)$: $\{(13, 0), (17, a), (33, e), (28, n), (41, u), (50, v)\}$
- $w - 28 = f(w)$: $\{(30, a), (40, e), (28, n), (51, u), (64, v), (76, x)\}$

Write a function equation for each set of ordered pairs.

- $\{(15, 8), (12, 5), (19, 12), (53, 46), (7, 0), (100, 93)\}$
- $\{(59, 30), (32, 3), (29, 0), (47, 18), (93, 64), (54, 25)\}$

Study the equations, then complete the patterns.

6. $x + 24 = f(x)$

x	12	16	37	49	63
$f(x)$	36	a	c	e	r

7. $81 - x = f(x)$

x	17	n	35	r	72
$f(x)$	a	23	c	46	e

8. $x - 48 = f(x)$

x	50	76	82	94	r
$f(x)$	a	c	e	n	22

9. $100 - x = f(x)$

x	22	38	47	n	r
$f(x)$	a	c	e	38	0

These patterns are called magic squares. In a magic square the sum of the numbers in each row, column, or diagonal is the same. Copy and complete the magic squares.

10.

a	17	16	6
14	c	7	11
10	12	e	5
15	3	4	n

11.

101	a	110	115
c	113	103	106
207	102	16	e
14	111	n	104

12.

34	48	2	16	a
46	10	14	c	32
e	12	26	40	44
20	n	38	42	6
22	36	r	4	18

13.

17	24	1	a	15
e	5	7	14	h
4	f	13	20	g
d	12	c	21	3
11	18	25	b	9

Using sets and numbers

To solve verbal problems we use a 5-step method, as in solving this problem:

How many people attended the school fair if 463 attended the first day and 649 attended the second day?

Step 1: Ask, "What are the sets in the problem?"

A set of 463 and a set of 649

Step 2: Ask, "What is happening to the sets?"

They are being joined.

Step 3: Write an equation.

$$463 + 649 = n$$

Step 4: Solve the equation.

$$463 + 649 = 1112$$

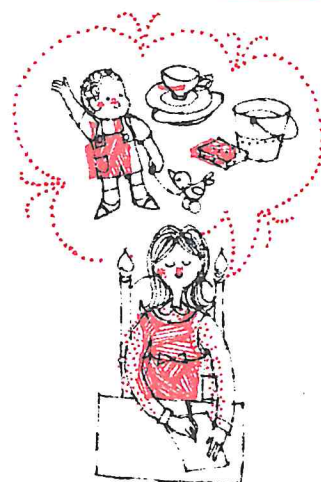
Step 5: Use the solution to answer the question in the problem.

There were 1112 people at the fair all together.

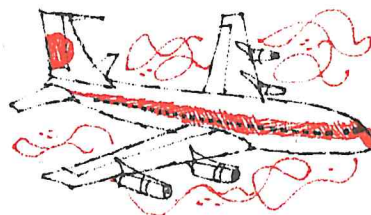
EXERCISES

Use the 5-step method to solve the problems.

1. Jim, Harry, and Willie are paper boys. Jim delivers 47 papers a day, Harry delivers 62, and Willie delivers 56. All together, how many papers do the three boys deliver each day?
2. In one week Nancy earned \$3.85 babysitting, \$1.40 washing dishes, and \$1.25 helping her Dad wash the car. How much did she earn all together? If she spent \$4.65 during the week, how much did she have left?
3. Steve's Soda Shop had 165 customers on Monday, 135 on Tuesday, 128 on Wednesday, 152 on Thursday, 193 on Friday, and 206 on Saturday. How many customers were there all week?
4. Phil's club sold candy. If Phil collected \$17.50, Bill collected \$18.75, Rick collected \$14.00, and Mark collected \$19.25, how much money did the boys collect for the candy?



1. Jim had saved \$5.78. He earned some money mowing lawns and now he has \$7.93. How much money did he earn?
2. Mary's family was driving from Cleveland to St. Louis. Mary knew that the total distance was 640 miles. When they reached Chicago, she saw a sign that said, "St. Louis 288 miles." How far had the family driven?
3. To fly from Los Angeles to Iowa City by commercial airline, you may fly east from Los Angeles to Chicago, a distance of 1745 miles. and then west from Chicago to Iowa City, a distance of 198 miles. What is the air distance between Los Angeles and Iowa City?
4. The weight of a car with its passengers is 4652 pounds. If the car weighs 3750 pounds without passengers, what is the weight of the passengers?
5. Jim's parents spent \$23,544 for their house, including the cost of the land, the building, and the furnishings. If the land cost \$6,500, and the building cost \$12,465, how much did they spend on furnishings?
6. The air distance from London to Berlin is 577 miles, from Berlin to Rome 738 miles, and from Rome to London 896 miles. What is the total air distance of a flight from London to Berlin to Rome and back to London?
- *7. A delivery truck weighs 5000 pounds, and it is carrying 12 barrels of oil each weighing 150 pounds. If the truck comes to a bridge that has a maximum capacity of 6500 pounds, can it safely go over the bridge?
- *8. Tom's class collected \$22.50 for the class trip. Each child went into the museum. If the admission price was 35¢ and there are 30 children in Tom's class, how much money was left?



Looking back at multiplication

You have learned that

1. multiplication may be used to name the cardinal number of the union of 2 or more disjoint equivalent sets.

$$\begin{aligned} A &= \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\} \\ B &= \{a, n, t, s\} \\ C &= \{6, 2, 9, 8\} \\ n(A \cup B \cup C) &= 4 + 4 + 4 \\ &= 3 \times 4 \\ &= 12 \end{aligned}$$

2. multiplication may be used to name the cardinal number of a cross product.

$$\begin{aligned} A &= \{a, i, o\} & B &= \{n, f, t, s\} \\ A \otimes B &= \{(a, n), (a, f), (a, t), (a, s), \\ &\quad (i, n), (i, f), (i, t), (i, s), \\ &\quad (o, n), (o, f), (o, t), (o, s)\} \\ n(A \otimes B) &= n(A) \times n(B) \\ &= 3 \times 4 = 12 \end{aligned}$$

3. there are 36 basic facts to be memorized with both factors between 1 and 10.

$$5 \times 7 = 35 \text{ is a \textbf{basic fact}.}$$

EXERCISES

Complete the chart by writing pairs of factors.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
product	21, 81	12, 32 42, 72	63	4, 14 24, 54 64	15, 25 35, 45	6, 16 36, 56	27	8, 18 28, 48	9 49	10, 20 30, 40
number of facts	2	5	1	6	4	6	1	5	2	4
factors	3×7 9×9	3×4 $a \times c$ $m \times n$ $w \times y$ $z \times s$	$a \times c$	2×2 $a \times c$ $m \times n$ $w \times y$ $z \times s$ $x \times r$	$a \times c$ $m \times n$ $w \times y$ $z \times s$	2×3 $a \times c$ $m \times n$ $w \times y$ $z \times s$ $x \times r$	$a \times b$	2×4 $a \times c$ $m \times n$ $w \times y$ $z \times s$	$a \times c$ $m \times n$	2×5 $a \times c$ $m \times n$ $w \times y$



You have also learned that

1. multiplication is the renaming of 2 factors as a product .	$\begin{array}{ccc} 3 \times 7 & = & 21 \\ \swarrow \quad \searrow & & \swarrow \\ \text{factors} & & \text{product} \end{array}$
2. multiplication is commutative .	$3 \times 7 = 7 \times 3$
3. multiplication is associative .	$\begin{array}{rcl} 3 \times (4 \times 5) & = & (3 \times 4) \times 5 \\ 3 \times 20 & = & 12 \times 5 \\ 60 & = & 60 \end{array}$
4. when 1 is one of the factors, the product is equal to the other factor.	$\begin{array}{rcl} 8 \times 1 & = & 8 \\ 1 \times 19 & = & 19 \end{array}$
5. multiplication is distributive over addition .	$\begin{array}{rcl} (3 \times 4) + (5 \times 4) & = & (3 + 5) \times 4 \\ 12 + 20 & = & 8 \times 4 \\ 32 & = & 32 \end{array}$

EXERCISES

Solve the equations.

- $7 \times 93 = 93 \times a$
- $8 \times 64 = 64 \times b$
- $2 \times 9846 = c \times 2$
- $2 \times (3 \times 4) = (2 \times d) \times 4$
- $2 \times 12 = e \times 2$
- $2 \times (3 \times 10) = (2 \times f) \times 10 = g$
- $8769 \times 1 = h$
- $483 \times 0 = i$
- $2 \times 4 \times 6 \times 0 = j$
- $6 \times (4 + 5) = 24 + k$
- $34 \times 6 = 180 + l$
- $14 + 21 = 7 \times (2 + m)$

Solve the equations.

- $6 \times 7 = a$
 $9 \times 7 = e$
 $15 \times 7 = c$
- $4 \times 9 = n$
 $9 \times 9 = r$
 $13 \times 9 = s$
- $5 \times 8 = u$
 $8 \times 8 = v$
 $13 \times 8 = x$
- $9 \times 6 = z$
 $8 \times 6 = a$
 $17 \times 6 = n$

More about multiplication

You have learned to

1. use the associative property when one of the factors is a multiple of 10, 100, and so on.	$ \begin{aligned} 4 \times 20 &= 4 \times (2 \times 10) \\ &= (4 \times 2) \times 10 \\ &= 8 \times 10 \\ &= 80 \end{aligned} $
2. use the distributive property when one of the factors is greater than 10.	$ \begin{aligned} 6 \times 43 &= 6 \times (40 + 3) \\ &= (6 \times 40) + (6 \times 3) \\ &= 240 + 18 \\ &= 258 \end{aligned} $
3. use the distributive property when both of the factors are greater than 10.	$ \begin{aligned} 26 \times 43 &= (20 + 6) \times 43 \\ &= (20 \times 43) + (6 \times 43) \\ &= 860 + 258 \\ &= 1118 \end{aligned} $

EXERCISES

Use the associative or distributive property to name the products.

1. 5×30 3. 4×86 5. 5×243 7. 28×43
 2. 5×50 4. 7×79 6. $7 \times 6 \times 8$ 8. 46×53

Complete the sentences. Write $<$, $>$, or $=$ to replace the $\bigcirc$.

9. $5 \times 40 \bigcirc 4 \times 50$ 13. $5 \times (20 + 7) \bigcirc (5 \times 20) + (5 \times 7)$
 10. $4 \times 13 \times 6 \bigcirc 24 \times 13$ 14. $46 \times 60 \bigcirc 2900$
 11. $99 \times 7 \bigcirc 700$ 15. $3 \times (80 + 80 + 80) \bigcirc 9 \times 80$
 12. $(100 - 2) \times 6 \bigcirc 6 \times 98$ 16. $4 \times 1,000,000 \bigcirc 100 \times 100 \times 4$

Complete the number charts.

17.

$\times$	5	3	8	4	2	6
27	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

18.

$\times$	2	6	8	4	5	9
47	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>

19.

$\times$	2	7	9	3	5	8
239	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>

20.

$\times$	4	2	6	3	5	8
419	<i>s</i>	<i>t</i>	<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>

Study these methods for naming the product 36×489 .

Number pattern	Three step (short)						
<table border="1"> <tr> <td>6×489</td><td>2934</td></tr> <tr> <td>30×489</td><td>14670</td></tr> <tr> <td>36×489</td><td>17604</td></tr> </table>	6×489	2934	30×489	14670	36×489	17604	$ \begin{array}{r} 489 \\ \times 36 \\ \hline 2934 \\ 14670 \\ \hline 17604 \end{array} $
6×489	2934						
30×489	14670						
36×489	17604						

EXERCISES

Complete the number patterns.

1.

4×32	a
50×32	c
54×32	e

2.

2×337	n
60×337	e
62×337	m

3.

8×649	r
70×649	s
78×649	i

Name the products using the 3-step method.

4.
$$\begin{array}{r} 54 \\ \times 37 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 153 \\ \times 26 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 2748 \\ \times 48 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 68 \\ \times 79 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 584 \\ \times 61 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 8057 \\ \times 57 \\ \hline \end{array}$$

Name the products.

10.
$$\begin{array}{r} 682 \\ \times 47 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 894 \\ \times 639 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 5285 \\ \times 275 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 406 \\ \times 352 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 518 \\ \times 703 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 8342 \\ \times 649 \\ \hline \end{array}$$

* Study the example, then use this factor method to name the products.

$$\begin{aligned}
 36 \times 28 &= (4 \times 9) \times 28 \\
 &= 4 \times (9 \times 28) \\
 &= 4 \times 252 \\
 &= 1008
 \end{aligned}$$

16. 12×24

20. 35×66

17. 24×24

21. 64×92

18. 32×44

22. 42×45

19. 48×37

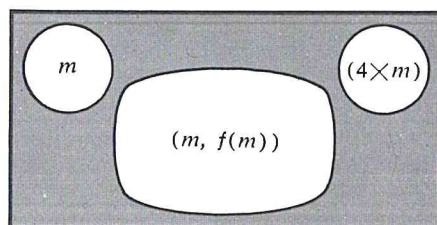
23. 73×98

Multiplication in function rules

We may show a function on a TV.

The number knob shows the first number of an ordered pair and the rule knob shows the rule. The screen shows the number pair.

The function equation is $4 \times m = f(m)$



EXERCISES

Study each function TV, then complete the ordered pairs.

1. $\{(6, a), (8, b), (11, c), (14, d), (19, e), (23, f)\}$

2. $\{(5, g), (0, h), (8, i), (1, j), (12, k), (17, l)\}$

3. $\{(10, m), (8, n), (3, s), (11, p), (15, q), (22, r)\}$

Study the equation, then complete the ordered pairs.

4. $4 \times s = f(s)$: $\{(2, a), (5, b), (9, c), (12, d), (18, e), (23, f)\}$
5. $15 \times t = f(t)$: $\{(3, g), (0, h), (9, i), (13, j), (17, k), (24, l)\}$
6. $44 \times u = f(u)$: $\{(1, m), (3, n), (7, s), (10, p), (14, q), (29, r)\}$

Write a function equation for each set of ordered pairs.

7. $\{(4, 8), (12, 24), (20, 40), (50, 100), (0, 0), (9, 18), \dots\}$
8. $\{(16, 256), (20, 320), (4, 64), (9, 144), (1, 16), (10, 160), \dots\}$
9. $\{(4, 4), (16, 16), (52, 52), (64, 64), (7, 7), (19, 19), \dots\}$

1, 2, 3, 4, 4, (5), 6, 7, (8), 8, 9, 10, 11, 12, 12, 13, ...

In this number pattern every multiple of 4 appears twice. The pattern would continue following this rule.

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ...

In this number pattern every number appears as many times as its name suggests. Three appears 3 times and so on.

Number patterns like these are called **sequences**. A member of a sequence is called a **term**. In the first sequence we have ringed the sixth and ninth terms.

EXERCISES

Copy and continue the sequences until each contains 12 terms.

1. 1, 1, 2, 3, 3, 4, 5, 5, ...

4. 11, 21, 31, 41, ...

2. 1, 2, 3, 6, 5, 4, 7, 8, 9, ...

5. 1, 5, 9, 13, ...

3. 2, 1, 4, 3, 6, 5, ...

6. 1, 2, 4, 8, 16, ...

Sometimes we can write an equation to name the term if we know its place in the sequence. For example, in the sequence: 5, 6, 7, 8, 9 ... we can use the rule $m + 4 = f(m)$, where m can be replaced by the number of the term and $f(m)$ by the term itself. The **sixth** term is $6 + 4 = 10$.

Complete this chart using the rule on the left to name the terms.

		number of term									
	rule	1	2	3	4	5	6	7	8	9	10
7.	$m + 5 = f(m)$	6	<i>c</i>	<i>e</i>	<i>x</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>	<i>v</i>	<i>w</i>
8.	$3 \times m = f(m)$	<i>a</i>	<i>c</i>	<i>e</i>	<i>x</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>	<i>v</i>	<i>w</i>
9.	$(3 \times m) + 1 = f(m)$	<i>a</i>	<i>c</i>	<i>e</i>	<i>x</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>	<i>v</i>	<i>w</i>
10.	$(5 \times m) - 3 = f(m)$	<i>a</i>	<i>c</i>	<i>e</i>	<i>x</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>	<i>v</i>	<i>w</i>

Two-stage problems

Larry was practicing his reading at home. On Tuesday night he read 218 words per minute for 35 minutes and on Wednesday night he read 196 words per minute for 46 minutes. How many words did he read all together?



June used the following 5-step method to solve this problem.

Step 1: Ask: What are the sets? 35 sets of 218 and 46 sets of 196.

Step 2: Ask: What is happening to the sets? The sets are being joined.

Step 3: Write an equation: $(35 \times 218) + (46 \times 196) = n$

Step 4: Solve the equation. $7630 + 9016 = 16,646$

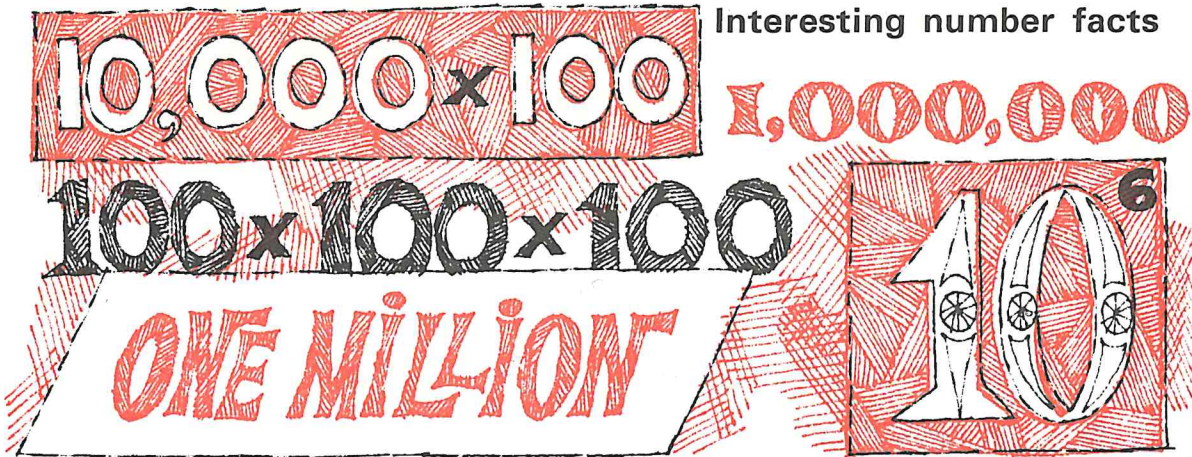
Step 5: Use your **solution** in the problem. "Larry read 16,646 words for the 2 nights."

In step 3, we must *first* name the products *and then* name the solution. This is why problems of this type are called *two-stage problems*.

EXERCISES

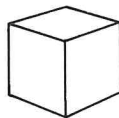
1. Sally read 234 words per minute for 18 minutes one night and 221 words per minute for 38 minutes another night. How many words did she read in all?
2. If a truck driver drives at the rate of 47 miles per hour for 58 hours and 51 miles per hour for 73 hours, how many miles does he drive all together?
3. A manufacturing company made parts at the rate of 1783 per hour for 12 hours. If 978 of the parts were defective, how many good parts did they make in 12 hours?
4. The supermarket ordered 957 cases of creamed corn and 1236 cases of whole corn. If each case contained 24 cans, how many cans of corn did they order all together?





A million is a very large number. Do you realize that millions “figure” in your life? Check these facts.

1. If you live to be 70 years of age, you will live
 $70 \times 365 \times 24 \times 60 = 36,792,000$ minutes. You are a minute multi-millionaire.
2. A calorie is a unit of energy. Your body uses about 2000 calories per day. During a life time of 70 years you will use up about $70 \times 365 \times 2000 = 51,100,000$ calories.
3. If you sleep 8 hours a day, you will sleep about
 $70 \times 365 \times 8 \times 60 = 12,264,000$ minutes in a lifetime of 70 years.
4. You take about 1000 breaths every hour. In a year you would take about $24 \times 365 \times 1000 = 8,760,000$ breaths.
5. You have about 5 billion red blood cells in the volume of blood shown in this picture:



Just For Fun

What's wrong with this?

6. There are 365 days in a year. You spend $\frac{1}{3}$ of your time asleep, that is 121 days. You spend $\frac{1}{3}$ of your time eating and playing, that is, another 121 days. If there are 104 Saturdays or Sundays and 18 other holidays in the year, you are left with $365 - 121 - 121 - 104 - 18 = 1$ day to go to school.

Rates and functions

Suppose football tickets sell at the **rate** of \$2 **per** ticket. We can show this with sets as in the diagram.



The number of dollars will depend on the number of tickets.
If m = number of tickets and $f(m)$ = number of dollars, then
 $2 \times m = f(m)$.

EXERCISES

Complete the sentences.

- If tickets sell at the rate of \$2 per ticket, 4 tickets will cost ? and 5 tickets will cost ? .
- If tickets sell at the rate of \$3 per ticket, 6 tickets will cost ? and 7 tickets will cost ? .
- If apples sell at the rate of 8¢ per apple, 9 apples will cost ? and 12 apples will cost ? .
- If milk sells at the rate of 20¢ per quart, 3 quarts will cost ? and 4 quarts will cost ? .

Complete the chart using the rate 5 cents per orange.
The rule is $5 \times m = f(m)$.

5.

number of oranges = m	1	2	4	8	9	7	16	27	43	82
number of cents = $f(m)$	5	a	c	e	n	m	r	s	u	v

Complete the chart using the rate 6 miles per hour.
The rule is $6 \times m = f(m)$.

6.

number of hours = m	1	2	3	5	8	9	12	14	17	23
number of miles = $f(m)$	6	a	c	e	n	m	r	s	u	v

Complete the number pairs. The first member of each pair is the number of hours, the second member is the number of miles. The rate is 8 miles per hour. The rule is $8 \times m = f(m)$.

7. $\{(1, 8), (2, 16), (3, a), (7, b), (9, c), (16, d), (18, e)\}$

We may use functions to rename quantities. We know, for example, that:

1 foot is equivalent to 12 inches or 12 inches *per* foot
 1 yard is equivalent to 3 feet or 3 feet *per* yard
 1 dime is equivalent to 10 cents or 10 cents *per* dime
 1 pound is equivalent to 16 ounces or 16 ounces *per* pound
 1 gallon is equivalent to 8 pints or 8 pints *per* gallon

EXERCISES

Complete the chart using the fact that 1 foot is equivalent to 12 inches. Rule: $m \times 12 = f(m)$.

1.

number of feet = m	1	2	3	4	5	6	7	8	9	10
number of inches = $f(m)$	12	24	a	c	e	n	m	r	s	u

Complete the chart using the fact that 1 yard is equivalent to 3 feet. Rule: $m \times 3 = f(m)$.

2.

number of yards = m	1	2	3	4	5	6	7	8	9	10
number of feet = $f(m)$	3	6	a	c	e	n	r	s	m	z

Complete the chart using the fact that 1 dime is equivalent to 10 cents. Rule: $m \times 10 = f(m)$.

3.

number of dimes = m	1	2	3	4	7	9	10	13	17	23
number of cents = $f(m)$	10	20	a	c	e	n	m	r	s	w

Complete the chart using the fact that 1 pound is equivalent to 16 ounces. Rule: $m \times 16 = f(m)$.

4.

number of pounds = m	3	10	2	7	8	13	1	19	20	47
number of ounces = $f(m)$	48	160	a	c	e	n	m	r	s	u

5. Make a rate chart for gallons and quarts.

Function rules again

If the function rule is given by the equation $(3 \times m) + 4 = f(m)$ and the set for m is A , the set for $f(m)$ is B and the set of ordered pairs is C .

$$A = \{0, 1, 2, 3, 4, \dots\}$$

$$B = \{4, 7, 10, 13, 16, \dots\}$$

$$C = \{(0, 4), (1, 7), (2, 10), (3, 13), \dots\}$$

EXERCISES

Complete the charts using the given rule.

1. Rule: $(2 \times m) + 3 = f(m)$

m	0	1	2	3	4	5	6	7	8	9	10
$f(m)$	3	5	7	a	c	e	n	r	s	u	v

2. Rule: $(2 \times m) - 1 = f(m)$

m	1	2	3	4	5	7	10	11	15	25	50
$f(m)$	1	3	5	i	x	z	w	m	n	r	s

3. Rule: $(3 \times m) + 5 = f(m)$

m	0	4	9	10	6	13	1	21	36	47	62
$f(m)$	5	17	32	m	r	s	u	z	v	w	x

Use the sets for m together with the given rule to complete the ordered pairs.

4. Rule: $(4 \times m) + 1 = f(m)$

Set for $m = \{0, 1, 2, 3, 4, 5, 6\}$

$$A = \{(0, 1), (1, a), (2, b), (3, c), (4, d), (5, e), (6, f)\}$$

5. Rule: $(3 \times m) - 2 = f(m)$

Set for $m = \{1, 4, 6, 8, 9, 10\}$

$$B = \{(1, 1), (4, g), (6, h), (8, i), (9, j), (10, k), (11, l)\}$$

Use the rule at the left to complete the ordered pairs.

6. $(2 \times m) + 7 = f(m)$: $D = \{(0, 7), (3, 13), (4, a), (8, b), (10, c), (14, d)\}$

7. $(4 \times m) - 4 = f(m)$: $E = \{(1, 0), (2, 4), (3, e), (6, f), (9, g), (11, h), (15, i)\}$

Complete the number sentences. Write $<$, $>$, or $=$ to replace each $\bigcirc$.

- | | | |
|------------------------------|------------------------------------|-------------------------------------|
| 1. $23 + 17 \bigcirc 40$ | 5. $376 + 281 \bigcirc 657$ | 9. $3964 + 2785 \bigcirc 6769$ |
| 2. $65 - 18 \bigcirc 37$ | 6. $67 \times 13 \bigcirc 875$ | 10. $8026 - 7399 \bigcirc 625$ |
| 3. $5 \times 14 \bigcirc 62$ | 7. $567 \times 23 \bigcirc 13,141$ | 11. $34 \times 710 \bigcirc 24,140$ |
| 4. $659 - 87 \bigcirc 582$ | 8. $9782 - 4316 \bigcirc 5366$ | 12. $2673 + 5842 \bigcirc 513$ |

Complete the equations. Write $+$, $-$, or $\times$ to replace each $\triangle$.

- | | |
|----------------------------------|------------------------------------|
| 13. $672 \triangle 300 = 372$ | 19. $384 \triangle 22 = 8448$ |
| 14. $310 \triangle 10 = 3100$ | 20. $3555 \triangle 2444 = 5999$ |
| 15. $48 \triangle 48 = 96$ | 21. $6781 \triangle 5493 = 12,274$ |
| 16. $3761 \triangle 5234 = 8995$ | 22. $9876 \triangle 9 = 9885$ |
| 17. $76 \triangle 32 = 2432$ | 23. $9876 \triangle 5 = 49,380$ |
| 18. $7635 \triangle 2796 = 4839$ | 24. $2926 \triangle 2854 = 72$ |

Complete the examples. Write a numeral to replace each letter.

- | | | | |
|--|--|---|---|
| 25. $\begin{array}{r} a68 \\ + 923 \\ \hline 169c \end{array}$ | 28. $\begin{array}{r} m8 \\ \times 6 \\ \hline 348 \end{array}$ | 31. $\begin{array}{r} 6v4 \\ \times 5 \\ \hline 3270 \end{array}$ | 34. $\begin{array}{r} 4x65 \\ \times 4 \\ \hline 1746z \end{array}$ |
| 26. $\begin{array}{r} 5e91 \\ - 2376 \\ \hline 311u \end{array}$ | 29. $\begin{array}{r} 95n6 \\ - 7777 \\ \hline r809 \end{array}$ | 32. $\begin{array}{r} 1761 \\ + 32w8 \\ \hline 5039 \end{array}$ | 35. $\begin{array}{r} 1r076 \\ + 14395 \\ \hline 274nx \end{array}$ |
| 27. $\begin{array}{r} 8e \\ \times 7 \\ \hline 623 \end{array}$ | 30. $\begin{array}{r} 3218 \\ + 4s57 \\ \hline 77u5 \end{array}$ | 33. $\begin{array}{r} 7612 \\ - 30n4 \\ \hline 455a \end{array}$ | 36. $\begin{array}{r} 8e9a \\ - 6999 \\ \hline n0c6 \end{array}$ |

Name the sums.

- | | | | | |
|---|--|---|--|---|
| 37. $\begin{array}{r} 37 \\ 56 \\ 42 \\ 83 \\ + 56 \\ \hline \end{array}$ | 38. $\begin{array}{r} 391 \\ 572 \\ 854 \\ 763 \\ + 206 \\ \hline \end{array}$ | 39. $\begin{array}{r} 2843 \\ 2617 \\ 5432 \\ 9876 \\ + 3002 \\ \hline \end{array}$ | 40. $\begin{array}{r} 59275 \\ 48634 \\ 65149 \\ 82738 \\ + 24921 \\ \hline \end{array}$ | 41. $\begin{array}{r} 976325 \\ 481720 \\ 610542 \\ 735961 \\ + 176542 \\ \hline \end{array}$ |
|---|--|---|--|---|

Name the products.

- | | | | | |
|--|---|--|--|---|
| 42. $\begin{array}{r} 37 \\ \times 49 \\ \hline \end{array}$ | 43. $\begin{array}{r} 368 \\ \times 47 \\ \hline \end{array}$ | 44. $\begin{array}{r} 7829 \\ \times 68 \\ \hline \end{array}$ | 45. $\begin{array}{r} 274 \\ \times 639 \\ \hline \end{array}$ | 46. $\begin{array}{r} 8234 \\ \times 297 \\ \hline \end{array}$ |
|--|---|--|--|---|

Looking back at division

You have learned that

1. division may be used to name the number of equivalent sets that can be formed from a set.	There are 4 sets of 3 in a set of 12. $12 \div (4 \times 3) = 0$ $12 \div 3 = 4$
2. division may be used to rename a product and one factor as a factor .	$ \begin{array}{c} \text{product} \swarrow \quad \searrow \text{factors} \\ 21 \div 7 = 3 \\ \swarrow \quad \searrow \\ \text{dividend} \quad \text{divisor} \quad \text{quotient} \end{array} $
3. division is the opposite of multiplication.	$3 \times 7 = 21 \quad 21 \div 7 = 3$ $3 \times (7 \div 7) = 3 \times 1 = 3$
4. order is important in division. Division is not commutative .	$21 \div 7 \neq 7 \div 21$
5. we memorize division facts in families with multiplication facts.	$3 \times 6 = 18 \quad 18 \div 6 = 3$ $6 \times 3 = 18 \quad 18 \div 3 = 6$ A family of facts.
6. some divisions have a remainder .	$55 \div 6 = n$ cannot be solved if n must be a whole number since $55 - (9 \times 6) = 1.$

EXERCISES

Name the family of facts for each set of factors and product.

1. {3, 8, 24} 2. {9, 6, 54} 3. {7, 6, 42} 4. {4, 9, 36}

Complete the chart.

5.

dividend	64	48	81	63	64	45	<i>g</i>	<i>h</i>	<i>i</i>
divisor	8	8	9	<i>d</i>	<i>e</i>	<i>f</i>	3	6	9
quotient	<i>a</i>	<i>b</i>	<i>c</i>	7	8	9	8	4	6

You have learned to

1. use the fact that division is the opposite of multiplication to name quotients.

$$\begin{array}{r} 3 \times 9 = 27, \\ \text{so } 27 \div 9 = 3 \\ 9 \overline{)27} \end{array}$$

2. use the fact that division is the opposite of multiplication to name quotients and remainders.

$$\begin{array}{r} (3 \times 9) + 2 = 27 + 2 \\ = 29 \\ 9 \overline{)29} \\ \underline{27} \\ 2 \end{array}$$

3. use the distributive property to name quotients.

$$\begin{aligned} 56 \div 4 &= (40 + 16) \div 4 \\ &= (40 \div 4) + (16 \div 4) \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

EXERCISES

Use the equations to name the quotients and remainders.

1. $(5 \times 8) + 3 = 43$

$$\begin{array}{r} a \\ 5 \overline{)43} \\ \underline{r} \\ e \end{array} \quad \text{or} \quad \begin{array}{r} c \\ 8 \overline{)43} \\ \underline{s} \\ n \end{array}$$

2. $(7 \times 9) + 4 = 67$

$$\begin{array}{r} e \\ 7 \overline{)67} \\ \underline{c} \\ r \end{array} \quad \text{or} \quad \begin{array}{r} n \\ 9 \overline{)67} \\ \underline{x} \\ s \end{array}$$

Use the divisions on the left to solve the equations on the right.

3. $\begin{array}{r} 7 \\ 4 \overline{)31} \\ \underline{28} \\ 3 \end{array} \quad (7 \times a) + 3 = b$

4. $\begin{array}{r} 6 \\ 9 \overline{)61} \\ \underline{54} \\ 7 \end{array} \quad (c \times 9) + 7 = d$

Use equations and the distributive property to name these quotients.

5. $36 \div 3$

7. $64 \div 4$

9. $72 \div 4$

11. $72 \div 6$

6. $48 \div 2$

8. $75 \div 5$

10. $96 \div 4$

12. $84 \div 3$

Division

Study the division and the equation.

$$30 - (7 \times 4) = 2$$

The dividend minus the product of the quotient and the divisor is equal to the remainder.

$$\begin{array}{r} 7 \\ 4 \overline{)30} \\ \underline{28} \\ 2 \end{array} \quad \begin{array}{l} 7 \times 4 = 28 \\ 30 - 28 = 2 \end{array}$$

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \\ \underline{\text{product}} \\ \text{remainder} \end{array}$$

EXERCISES

Name the quotients and remainders.

1. $(8 \times 4) + 3 = a$

$$\begin{array}{r} c \\ 4 \overline{)35} \\ \underline{r} \\ e \end{array} \quad \text{or} \quad \begin{array}{r} n \\ 8 \overline{)35} \\ \underline{s} \\ m \end{array}$$

2. $(9 \times 7) + 6 = r$

$$\begin{array}{r} s \\ 9 \overline{)69} \\ \underline{a} \\ u \end{array} \quad \text{or} \quad \begin{array}{r} v \\ 7 \overline{)69} \\ \underline{x} \\ w \end{array}$$

3. $(6 \times 8) + z = 53$

$$\begin{array}{r} a \\ 6 \overline{)53} \\ \underline{n} \\ w \end{array} \quad \text{or} \quad \begin{array}{r} c \\ 8 \overline{)53} \\ \underline{e} \\ r \end{array}$$

4. $61 - (7 \times 8) = n$

$$\begin{array}{r} u \\ 7 \overline{)61} \\ \underline{w} \\ v \end{array} \quad \text{or} \quad \begin{array}{r} m \\ 8 \overline{)61} \\ \underline{a} \\ x \end{array}$$

5. $75 - (9 \times 8) = r$

$$\begin{array}{r} s \\ 9 \overline{)75} \\ \underline{e} \\ u \end{array} \quad \text{or} \quad \begin{array}{r} a \\ 8 \overline{)75} \\ \underline{m} \\ n \end{array}$$

6. $57 - (9 \times 6) = r$

$$\begin{array}{r} a \\ 9 \overline{)57} \\ \underline{n} \\ e \end{array} \quad \text{or} \quad \begin{array}{r} c \\ 6 \overline{)57} \\ \underline{s} \\ x \end{array}$$

Complete the divisions.

7. $6 \overline{)53}$

11. $5 \overline{)49}$

15. $8 \overline{)66}$

19. $7 \overline{)38}$

23. $4 \overline{)27}$

8. $3 \overline{)22}$

12. $7 \overline{)68}$

16. $9 \overline{)50}$

20. $6 \overline{)50}$

24. $9 \overline{)38}$

9. $7 \overline{)27}$

13. $6 \overline{)43}$

17. $8 \overline{)52}$

21. $5 \overline{)27}$

25. $3 \overline{)26}$

10. $4 \overline{)33}$

14. $9 \overline{)80}$

18. $7 \overline{)65}$

22. $8 \overline{)17}$

26. $9 \overline{)59}$



Study these divisions.

A	B	C
$\begin{array}{r l} 4 \overline{)58} & \\ \underline{40} & 10 \\ 18 & \\ \underline{16} & 4 \\ 2 & 14 \end{array}$	$\begin{array}{r} 14 \\ 4 \overline{)58} \\ \underline{40} \\ 18 \\ \underline{16} \\ 2 \end{array}$	$58 - (4 \times 14) = 2$ quotient 14 remainder 2

EXERCISES

Name the quotients and remainders.

1.
$$\begin{array}{r|l} 2 \overline{)25} & \\ \underline{20} & a \\ 5 & \\ \underline{4} & m \\ c & n \end{array}$$

3.
$$\begin{array}{r|l} 7 \overline{)84} & \\ \underline{70} & i \\ 14 & \\ \underline{14} & x \\ & n \end{array}$$

5.
$$\begin{array}{r|l} 4 \overline{)85} & \\ \underline{40} & n \\ 45 & \\ \underline{40} & e \\ 5 & \\ \underline{4} & s \\ r & m \end{array}$$

7.
$$\begin{array}{r|l} 5 \overline{)96} & \\ \underline{50} & r \\ 46 & \\ \underline{45} & i \\ x & e \end{array}$$

2.
$$\begin{array}{r|l} 5 \overline{)63} & \\ \underline{50} & u \\ 13 & \\ \underline{10} & e \\ c & w \end{array}$$

4.
$$\begin{array}{r|l} 3 \overline{)59} & \\ \underline{30} & e \\ 29 & \\ \underline{27} & m \\ n & u \end{array}$$

6.
$$\begin{array}{r|l} 3 \overline{)89} & \\ \underline{30} & a \\ 59 & \\ \underline{30} & u \\ 29 & \\ \underline{27} & w \\ n & c \end{array}$$

8.
$$\begin{array}{r|l} 3 \overline{)97} & \\ \underline{30} & m \\ 67 & \\ \underline{60} & s \\ 7 & \\ \underline{6} & r \\ a & e \end{array}$$

Use method A to name the quotients and remainders.

9. $2 \overline{)37}$

12. $6 \overline{)84}$

15. $6 \overline{)95}$

18. $4 \overline{)87}$

10. $5 \overline{)64}$

13. $3 \overline{)41}$

16. $9 \overline{)89}$

19. $3 \overline{)73}$

11. $8 \overline{)77}$

14. $7 \overline{)92}$

17. $3 \overline{)97}$

20. $2 \overline{)75}$

21–33. Use method B to name the quotients and remainders in exercises 9–20.

Division

Study these methods for naming the quotient and remainder when 578 is divided by 4.

A	B	Check
$ \begin{array}{r l} 4 \overline{)578} & \\ \underline{400} & 100 \\ 178 & \\ \underline{160} & 40 \\ 18 & \\ \underline{16} & 4 \\ 2 & 144 \end{array} $	$ \begin{array}{r} 144 \\ 4 \overline{)578} \\ \underline{400} \\ 178 \\ \underline{160} \\ 18 \\ \underline{16} \\ 2 \end{array} $	$ \begin{array}{l} 100 \times 4 = 400 \\ 40 \times 4 = 160 \\ 4 \times 4 = 16 \\ 144 \times 4 = 576 \\ 576 + 2 = 578 \end{array} $
$578 - (144 \times 4) = 2.$ The quotient is 144; the remainder is 2.		

EXERCISES

Name the quotients and remainders.

$$\begin{array}{r|l}
 1. \quad 3 \overline{)485} & \\
 \underline{300} & a \\
 185 & \\
 \underline{180} & b \\
 5 & \\
 \underline{3} & c \\
 d & e
 \end{array}$$

$$\begin{array}{r|l}
 2. \quad 5 \overline{)792} & \\
 \underline{500} & f \\
 292 & \\
 \underline{250} & g \\
 42 & \\
 \underline{40} & n \\
 i & j
 \end{array}$$

$$\begin{array}{r|l}
 3. \quad 6 \overline{)849} & \\
 \underline{600} & k \\
 249 & \\
 \underline{240} & l \\
 9 & \\
 \underline{6} & m \\
 n & x
 \end{array}$$

$$\begin{array}{r|l}
 4. \quad 8 \overline{)987} & \\
 \underline{800} & p \\
 187 & \\
 \underline{160} & q \\
 27 & \\
 \underline{24} & r \\
 s & t
 \end{array}$$

5–8. Use the method above to check your work in exercises 1–4.

Use method A to name the quotients and remainders.

$$9. \quad 4 \overline{)586}$$

$$11. \quad 5 \overline{)758}$$

$$13. \quad 6 \overline{)376}$$

$$15. \quad 7 \overline{)298}$$

$$10. \quad 6 \overline{)823}$$

$$12. \quad 7 \overline{)954}$$

$$14. \quad 7 \overline{)625}$$

$$16. \quad 8 \overline{)586}$$

17–24. Use method B to name the quotients and remainders in exercises 9–16. Check your work.



Study these methods for naming the quotient and remainder when 578 is divided by 4. Method C is the short method. In this method we write only the remainder from each step to carry us on to the next step.

A $ \begin{array}{r} 144 \\ 4 \overline{)578} \\ \underline{400} \\ 178 \\ \underline{160} \\ 18 \\ \underline{16} \\ 2 \end{array} $	B $ \begin{array}{r} 144 \\ 4 \overline{)578} \\ \underline{4} \\ 17 \\ \underline{16} \\ 18 \\ \underline{16} \\ 2 \end{array} $	C $ \begin{array}{r} 144 \\ 4 \overline{)578} \\ \underline{11} \end{array} $ R. 2
--	--	---

EXERCISES

Use method A to name the quotients and remainders.

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 1. $2 \overline{)385}$ | 3. $6 \overline{)857}$ | 5. $3 \overline{)537}$ | 7. $7 \overline{)923}$ |
| 2. $4 \overline{)687}$ | 4. $5 \overline{)934}$ | 6. $8 \overline{)976}$ | 8. $9 \overline{)768}$ |

Use method B to name the quotients and remainders.

- | | | | |
|-------------------------|-------------------------|-------------------------|-------------------------|
| 9. $4 \overline{)596}$ | 11. $6 \overline{)507}$ | 13. $6 \overline{)754}$ | 15. $4 \overline{)623}$ |
| 10. $5 \overline{)839}$ | 12. $7 \overline{)897}$ | 14. $8 \overline{)976}$ | 16. $9 \overline{)975}$ |

Use method C to name the quotients and remainders.

- | | | | |
|-------------------------|-------------------------|-------------------------|-------------------------|
| 17. $6 \overline{)862}$ | 19. $5 \overline{)716}$ | 21. $8 \overline{)964}$ | 23. $4 \overline{)637}$ |
| 18. $2 \overline{)347}$ | 20. $7 \overline{)859}$ | 22. $9 \overline{)985}$ | 24. $3 \overline{)481}$ |

Name the greatest number of items you can buy and the amount of money you will have left

25. if you have \$7.44 and buy 9¢ rulers.
26. if you have \$7.50 and buy 8¢ erasers.
27. if you have \$7.60 and buy 7¢ pencils.

Estimating quotients

To **estimate** the quotient in the division $24 \overline{)57}$, we may use this method:

Step 1: $1 \times 24 = 24$ $10 \times 24 = 240$

Step 2: 57 is between 24 and 240,
so the quotient is between 1 and 10.

Step 3: 5 tens $\div$ 2 tens is about 2. This checks with step 2. Our estimated quotient is 2.

check

$$\begin{array}{r} 24 \overline{)57} \\ \underline{48} \\ 9 \end{array}$$

$9 < 24$, so
the estimate
is correct.

EXERCISES

Estimate the quotients.

1. $14 \overline{)57}$

3. $41 \overline{)86}$

5. $37 \overline{)82}$

7. $19 \overline{)85}$

2. $43 \overline{)86}$

4. $22 \overline{)69}$

6. $41 \overline{)97}$

8. $63 \overline{)92}$

9–16. Check your estimates in exercises 1–8. If your estimate is too large, try again.

To **estimate** the quotient in the division $37 \overline{)168}$ we may use this method.

Step 1: $1 \times 37 = 37$ $10 \times 37 = 370$

Step 2: 168 is between 37 and 370, so the quotient is between 1 and 10.

Step 3: Round the divisor up to 40 and the dividend down to 160.

Step 4: 16 tens $\div$ 4 tens = 4. This checks with step 2. Our estimated quotient is 4.

check

$$\begin{array}{r} 37 \overline{)168} \\ \underline{148} \\ 20 \end{array}$$

$20 < 37$, so
the estimate
is correct.

Estimate the quotients.

17. $21 \overline{)172}$

19. $25 \overline{)134}$

21. $28 \overline{)207}$

23. $93 \overline{)623}$

18. $42 \overline{)236}$

20. $52 \overline{)287}$

22. $64 \overline{)441}$

24. $97 \overline{)888}$

25–32. Check your estimates in exercises 17–24. If your estimate is too large, try again.

To **estimate** the quotient in the division $24 \overline{)576}$ we may use this method:

Step 1: $1 \times 24 = 24$, $10 \times 24 = 240$, $100 \times 24 = 2400$

Step 2: 576 is between 240 and 2400, so the quotient is between 10 and 100

Step 3: 50 tens $\div$ 2 tens is about 20. The estimated quotient is 20.

EXERCISES

Estimate the quotients.

1. $22 \overline{)485}$

5. $32 \overline{)974}$

9. $41 \overline{)856}$

13. $33 \overline{)698}$

2. $52 \overline{)748}$

6. $43 \overline{)916}$

10. $23 \overline{)723}$

14. $42 \overline{)923}$

3. $41 \overline{)778}$

7. $53 \overline{)876}$

11. $32 \overline{)712}$

15. $13 \overline{)653}$

4. $27 \overline{)378}$

8. $34 \overline{)592}$

12. $28 \overline{)992}$

16. $62 \overline{)876}$

To name the quotient in the division $24 \overline{)576}$ we may use these steps.

Step 1: **Estimate** quotient for $576 \div 24$ (top of this page).

Step 2: **Multiply:** $20 \times 24 = 480$.

Step 3: **Subtract:** $576 - 480 = 96$.

Step 4: **Estimate** quotient for $96 \div 24$. (page 60)

Step 5: **Multiply:** $4 \times 24 = 96$.

Step 6: **Subtract:** $96 - 96 = 0$.

The quotient is $20 + 4 = 24$ and the remainder is 0.

$$\begin{array}{r} 24 \overline{)576} \\ \underline{480} \quad 20 \\ 96 \\ \underline{96} \quad 4 \\ 0 \end{array}$$

Name the quotients.

17. $21 \overline{)861}$

21. $17 \overline{)884}$

25. $26 \overline{)858}$

29. $32 \overline{)192}$

18. $16 \overline{)730}$

22. $29 \overline{)667}$

26. $35 \overline{)735}$

30. $24 \overline{)792}$

19. $38 \overline{)836}$

23. $13 \overline{)975}$

27. $29 \overline{)957}$

31. $17 \overline{)918}$

20. $21 \overline{)693}$

24. $28 \overline{)896}$

28. $27 \overline{)864}$

32. $49 \overline{)588}$

Division

Study these methods for naming the quotient and remainder when 578 is divided by 24.

A	B	
$\begin{array}{r l} 24 \overline{)578} & \\ \underline{480} & 20 \\ 98 & \\ \underline{96} & 4 \\ 2 & 24 \end{array}$	$\begin{array}{r} 24 \\ 24 \overline{)578} \\ \underline{480} \\ 98 \\ \underline{96} \\ 2 \end{array}$	<i>Step 1: Estimate quotient:</i> 20 <i>Step 2: Multiply:</i> $20 \times 24 = 480$ <i>Step 3: Subtract:</i> $578 - 480 = 98$ <i>Step 4: Estimate quotient:</i> 4 <i>Step 5: Multiply:</i> $4 \times 24 = 96$ <i>Step 6: Subtract:</i> $98 - 96 = 2$
Check: $578 - (24 \times 24) = 2$. The quotient is 24 and the remainder is 2.		

EXERCISES

Use method A to name the quotients and remainders.

$$\begin{array}{r|l} 1. \quad 35 \overline{)476} & \\ \underline{350} & a \\ 126 & \\ \underline{105} & e \\ c & n \end{array}$$

$$\begin{array}{r|l} 2. \quad 27 \overline{)599} & \\ \underline{540} & v \\ 59 & \\ \underline{54} & u \\ r & s \end{array}$$

$$\begin{array}{r|l} 3. \quad 42 \overline{)895} & \\ \underline{840} & x \\ 45 & \\ \underline{42} & z \\ r & e \end{array}$$

$$\begin{array}{r|l} 4. \quad 23 \overline{)784} & \\ \underline{690} & m \\ 94 & \\ \underline{92} & n \\ a & c \end{array}$$

Use method A to name the quotients and remainders.

Check your work.

$$5. \quad 23 \overline{)498}$$

$$8. \quad 15 \overline{)327}$$

$$11. \quad 43 \overline{)486}$$

$$14. \quad 51 \overline{)705}$$

$$6. \quad 27 \overline{)569}$$

$$9. \quad 32 \overline{)793}$$

$$12. \quad 34 \overline{)753}$$

$$15. \quad 19 \overline{)613}$$

$$7. \quad 33 \overline{)678}$$

$$10. \quad 23 \overline{)875}$$

$$13. \quad 77 \overline{)999}$$

$$16. \quad 23 \overline{)932}$$

Use method B to name the quotients and remainders.

Check your work.

$$17. \quad 36 \overline{)796}$$

$$20. \quad 27 \overline{)579}$$

$$23. \quad 63 \overline{)798}$$

$$26. \quad 28 \overline{)632}$$

$$18. \quad 45 \overline{)624}$$

$$21. \quad 47 \overline{)985}$$

$$24. \quad 48 \overline{)993}$$

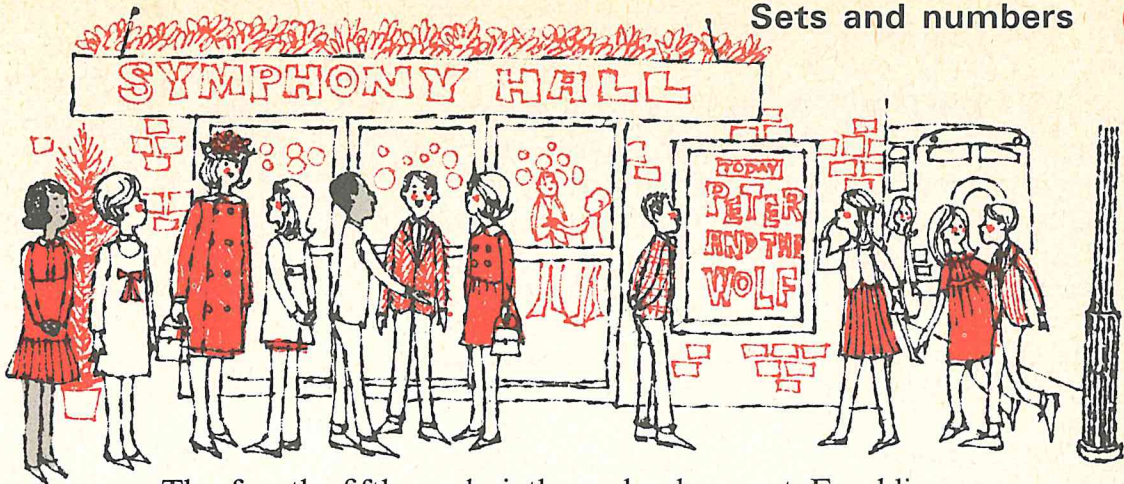
$$27. \quad 76 \overline{)961}$$

$$19. \quad 83 \overline{)974}$$

$$22. \quad 35 \overline{)813}$$

$$25. \quad 57 \overline{)743}$$

$$28. \quad 37 \overline{)728}$$



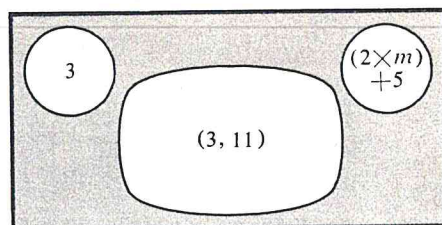
The fourth, fifth, and sixth grade classes at Franklin Elementary School visited a music concert. Solve these problems concerning their trip.

EXERCISES

1. There are 4 fourth grade classes with 124 children in all. If there are the same number of children in each class, how many children are there per class?
2. If the same number of fourth graders are to ride in 3 buses, how many is that per bus? How many children are left over?
3. There are 6 fifth grade classes with 174 children in all. If all the classes have the same number of children, how many children are there per class?
4. There are 183 sixth graders to ride in the buses. If each bus can hold 43 children, how many buses are needed for the sixth graders?
5. If there are 14 seats in each row in the concert auditorium, how many rows are needed for the 124 fourth graders?
6. If there are 14 seats in each row in the concert auditorium, how many rows are needed for the 183 sixth graders?
7. If the 174 fifth graders return home by cars that can each hold 6 children, how many cars are needed?
8. If the 183 sixth graders return home by small buses that can each hold 23 children, how many buses are needed?

Practice with functions

This is a new kind of function TV. The number knob shows the possible selections for m . The rule is in equation form and the screen shows the ordered pair $(3, 11)$ since $(2 \times 3) + 5 = 11$. A set of ordered pairs for this function is $\{(3, 11), (5, 15), (6, 17), (8, 21), (9, 23), (0, 5), (2, 9)\}$



EXERCISES

Use the function TV to complete the ordered pairs.

1. $\{(2, 3), (1, a), (3, b), (5, c), (7, d), (10, e)\}$

2. $\{(1, 8), (2, f), (3, g), (5, h), (6, i), (9, j), (11, k)\}$

3. $\{(3, 27), (1, l), (4, m), (6, n), (10, s), (2, p), (8, q)\}$

Use the set of selections for m on the left and the rule to name the set of ordered pairs.

- | | |
|-------------------------------------|-----------------------------------|
| 4. $A = \{0, 1, 3, 7, 9, 11\}$ | Rule: $(8 \times m) + 7 = f(m)$ |
| 5. $B = \{8, 10, 14, 19, 23, 27\}$ | Rule: $m - 6 = f(m)$ |
| 6. $C = \{1, 5, 6, 8, 12, 16\}$ | Rule: $(6 \times m) - 5 = f(m)$ |
| 7. $D = \{10, 20, 30, 40, 50, 60\}$ | Rule: $(2 \times m) + 5 = f(m)$ |
| 8. $E = \{1, 2, 3, 5, 6, 9\}$ | Rule: $(17 \times m) - 10 = f(m)$ |
| 9. $F = \{5, 6, 7, 8, 9, 10, 11\}$ | Rule: $(33 \times m) + 6 = f(m)$ |



Study the equations, then complete the ordered pairs.

1. $m + 3 = f(m)$: $\{(1, 4), (2, 5), (3, a), (5, b), (9, c), (10, d), (12, e)\}$
2. $x + 7 = f(x)$: $\{(2, 9), (4, 11), (6, f), (8, g), (10, h), (13, i), (22, j)\}$
3. $a - 2 = f(a)$: $\{(7, 5), (10, 8), (11, k), (23, l), (15, m), (19, n), (41, s)\}$
4. $(2 \times m) + 1 = f(m)$: $\{(0, 1), (2, 5), (4, p), (6, q), (9, r), (10, s), (14, t)\}$
5. $(4 \times a) - 2 = f(a)$: $\{(1, 2), (3, 10), (5, u), (11, v), (7, w), (17, x), (21, y)\}$

Name the sums or differences.

- | | | | | |
|--|--|---|---|---|
| 6. $\begin{array}{r} 23546 \\ + 78924 \\ \hline \end{array}$ | 8. $\begin{array}{r} 563417 \\ + 298502 \\ \hline \end{array}$ | 10. $\begin{array}{r} 4897652 \\ + 9876250 \\ \hline \end{array}$ | 12. $\begin{array}{r} 23 \\ 47 \\ 92 \\ 86 \\ 54 \\ 37 \\ 61 \\ + 42 \\ \hline \end{array}$ | 13. $\begin{array}{r} 5789 \\ 2346 \\ 5461 \\ 2092 \\ 3570 \\ 6854 \\ + 3167 \\ \hline \end{array}$ |
| 7. $\begin{array}{r} 78564 \\ - 39961 \\ \hline \end{array}$ | 9. $\begin{array}{r} 679031 \\ - 487630 \\ \hline \end{array}$ | 11. $\begin{array}{r} 3172652 \\ - 2983148 \\ \hline \end{array}$ | | |

Name the products.

- | | | | | |
|--|---|---|--|--|
| 14. $\begin{array}{r} 37 \\ \times 9 \\ \hline \end{array}$ | 16. $\begin{array}{r} 38765 \\ \times 8 \\ \hline \end{array}$ | 18. $\begin{array}{r} 89 \\ \times 17 \\ \hline \end{array}$ | 20. $\begin{array}{r} 654 \\ \times 62 \\ \hline \end{array}$ | 22. $\begin{array}{r} 60852 \\ \times 89 \\ \hline \end{array}$ |
| 15. $\begin{array}{r} 495 \\ \times 6 \\ \hline \end{array}$ | 17. $\begin{array}{r} 654321 \\ \times 5 \\ \hline \end{array}$ | 19. $\begin{array}{r} 728 \\ \times 19 \\ \hline \end{array}$ | 21. $\begin{array}{r} 3795 \\ \times 75 \\ \hline \end{array}$ | 23. $\begin{array}{r} 235468 \\ \times 42 \\ \hline \end{array}$ |

Name the quotients and remainders.

- | | | | |
|------------------------|------------------------|-------------------------|------------------------|
| 24. $7 \overline{)60}$ | 26. $5 \overline{)37}$ | 28. $8 \overline{)100}$ | 30. $7 \overline{)95}$ |
| 25. $6 \overline{)51}$ | 27. $4 \overline{)38}$ | 29. $9 \overline{)105}$ | 31. $6 \overline{)76}$ |

Estimate the quotients. Then name the quotients and remainders and check your estimates.

- | | | | |
|-------------------------|-------------------------|--------------------------|--------------------------|
| 32. $7 \overline{)967}$ | 35. $6 \overline{)354}$ | 38. $32 \overline{)673}$ | 41. $52 \overline{)634}$ |
| 33. $9 \overline{)486}$ | 36. $4 \overline{)589}$ | 39. $41 \overline{)896}$ | 42. $23 \overline{)725}$ |
| 34. $8 \overline{)567}$ | 37. $5 \overline{)533}$ | 40. $47 \overline{)923}$ | 43. $37 \overline{)855}$ |



LOOKING BACK

Addition is the renaming of 2 **addends** as a **sum**, subtraction is the renaming of a sum and an addend as the missing addend (the difference).

1. List the basic facts with 6 in the numeral for either an addend or the sum.

Multiplication is the renaming of 2 **factors** as a **product**. Division may be used to rename a product and one factor (the divisor) as the missing factor (quotient).

2. List the basic facts with 5 as one of the digits in the numeral for a factor or the product.

Addition and **subtraction** are **opposites**. Adding and then subtracting any number is the same as adding 0.

Multiplication and **division** are **opposites**. Multiplying and then dividing by the same number is the same as multiplying by 1.

Copy and complete the equations.

3. $(16 + 8) - 8 = a$

4. $(83 \times 7) \div 7 = a$

The **distributive property** of **multiplication** may be used to name products (or quotients) with one or both factors (or the dividend) greater than 10.

Copy and complete the equations.

5. $7 \times 43 = 7 \times (40 + 3)$
 $= (7 \times a) + (7 \times b)$
 $= c$

6. $84 \div 7 = (70 + 14) \div 7$
 $= (a \div 7) + (14 \div 7)$
 $= b$

Name the remainders for the divisions.

7.
$$\begin{array}{r} 8 \\ 6 \overline{)50} \end{array}$$

8.
$$\begin{array}{r} 8 \\ 7 \overline{)60} \end{array}$$

9.
$$\begin{array}{r} 3 \\ 4 \overline{)14} \end{array}$$

10.
$$\begin{array}{r} 5 \\ 5 \overline{)26} \end{array}$$



With addends between 10 and 100, try adding the tens and ones at the same time. The sums as we add from the top are shown in red. To add the first 2 addends, we think:

$$26 + 30 = 56, \quad 56 + 8 = 64$$

26	
38	64
47	111
63	174
29	203
+ 81	284
284	

Name the sums using this 2 column method for addition.

1. 48	2. 54	3. 84	4. 29	5. 96	6. 66
29	39	67	45	59	28
36	57	83	93	75	33
47	43	46	32	49	45
92	91	27	56	62	56
62	68	38	49	87	72
+ 38	+ 72	+ 55	+ 78	+ 66	+ 88

Complete the number patterns.

7.	8.	9.
$1 \times 9 = 10 - 1$	$1 \times 8 = 10 - 2$	$1 \times 7 = 10 - 3$
$2 \times 9 = 20 - 2$	$2 \times 8 = 20 - 4$	$2 \times 7 = 20 - 6$
$3 \times 9 = 30 - 3$	$3 \times 8 = 30 - 6$	$3 \times 7 = 30 - 9$
$4 \times 9 = a - b$	$4 \times 8 = g - h$	$4 \times 7 = m - n$
$5 \times 9 = c - d$	$5 \times 8 = i - j$	$5 \times 7 = o - p$
$6 \times 9 = e - f$	$6 \times 8 = k - l$	$6 \times 7 = q - r$

Just For Fun

Use this method for multiplying 2 numbers between 10 and 20.

$$15 \times 18$$

Step 1: Add, $15 + 8 = 23$.

Step 2: Multiply the sum by 10, $10 \times 23 = 230$.

Step 3: Multiply, $5 \times 8 = 40$.

Step 4: Add step 2 and step 3, $230 + 40 = 270$.

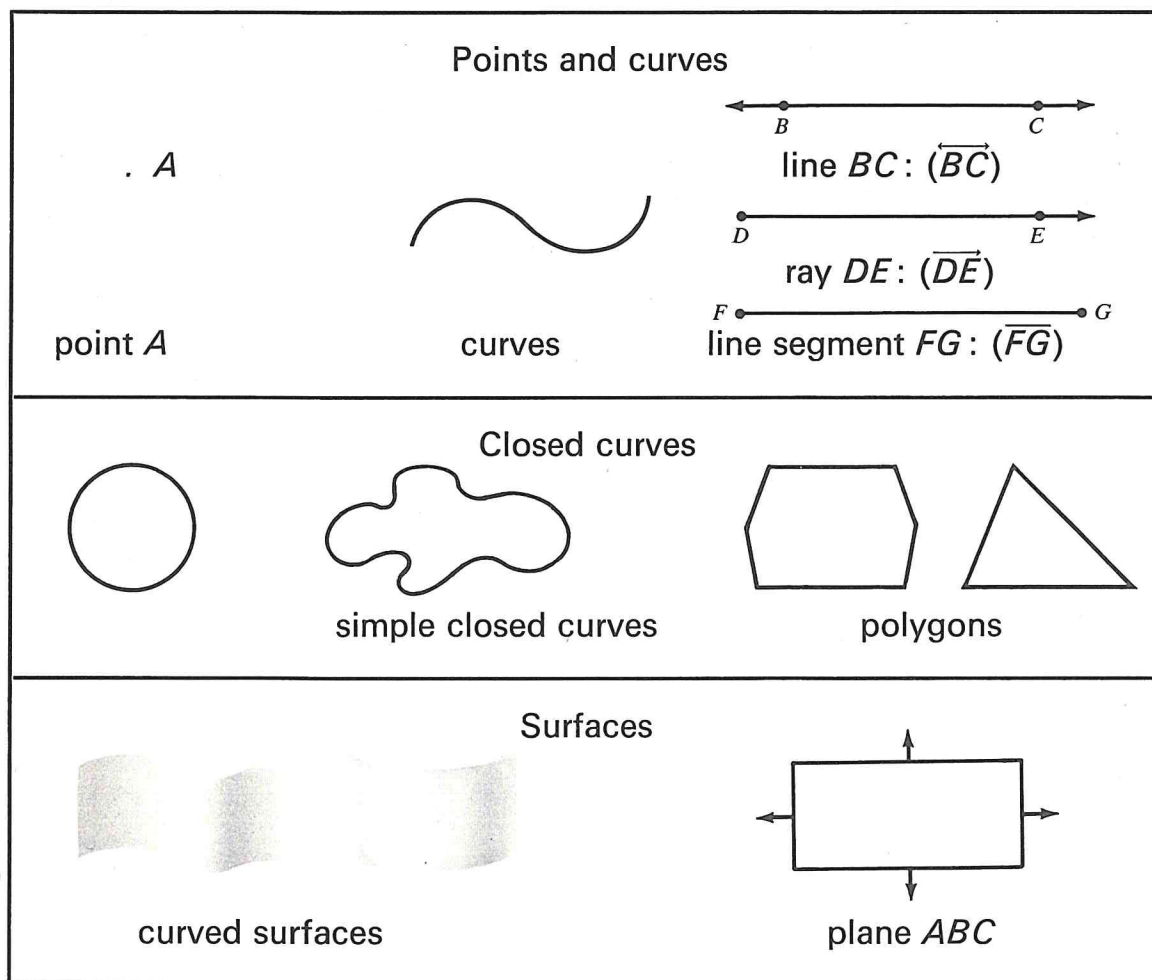
$$15 \times 18 = 270$$

Use this method to name the products.

10. 16	11. 14	12. 12	13. 19	14. 11	15. 13
$\times 13$	$\times 15$	$\times 17$	$\times 12$	$\times 18$	$\times 13$

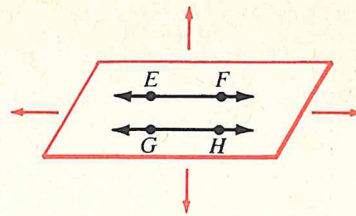
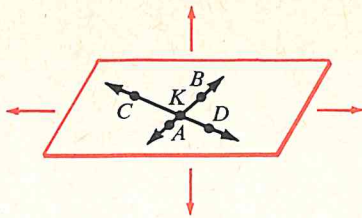
CHAPTER 3 • Geometry And Graphing

How much do you remember?



EXERCISES

1. What do the arrowheads mean when we show lines, rays, and planes?
2. How do you distinguish **a.** a closed curve from a simple closed curve, **b.** a simple closed curve from a polygon?
3. How many end points has a **a.** line **b.** ray **c.** line segment?
4. A simple closed curve separates the points in a plane into 3 sets. One of the sets is the points on the curve. Name the other two sets.



The point K is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Lines EF and GH do not intersect. We say that $\overleftrightarrow{EF}$ is parallel to $\overleftrightarrow{GH}$. If 2 lines are in the same plane and do not intersect, then they are **parallel**. Rays and line segments are parallel if they are parts of lines which are parallel.

EXERCISES

• P

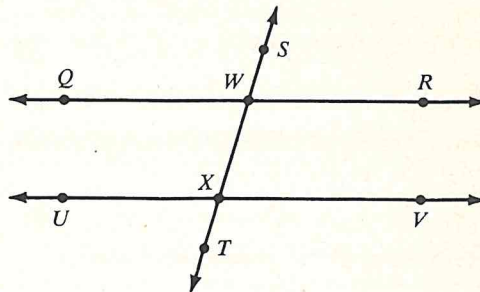
Study the figure on the right.

- How many lines can be drawn through P , parallel to $\overleftrightarrow{AB}$?



The lines QR and UV are parallel.

- Name the intersection of $\overleftrightarrow{QR}$ and $\overleftrightarrow{ST}$.
- Name the intersection of $\overleftrightarrow{ST}$ and $\overleftrightarrow{UV}$.
- Why is $\overleftrightarrow{QR}$ parallel to $\overleftrightarrow{UV}$?
- Name 4 rays with W as an endpoint.
- *6. Name the intersection of $\overleftrightarrow{UV}$ and $\overleftrightarrow{VT}$.



Draw pictures to show.

- two line segments in a plane which do not intersect and are not parallel.
- two rays in a plane which do not intersect and are not parallel.

You have learned that 2 points *determine* a line.

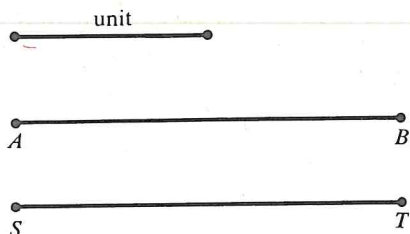
If no more than 2 points are on the same line, name the number of lines determined by

- | | | | |
|------------|--------------|--------------|--------------|
| 9. 1 point | 10. 2 points | 11. 3 points | 12. 4 points |
|------------|--------------|--------------|--------------|



Measuring line segments

The **measure** of a line segment is the **number** which tells how many units of length will match the line segment. $\overline{AB}$ and $\overline{ST}$ both match 2 units; $m(\overline{AB}) = m(\overline{ST}) = 2$. The length of $\overline{AB}$ and $\overline{ST}$ is 2 units.



Two line segments which have the same measure, in the same units, are **congruent**. $\overline{AB}$ and $\overline{ST}$ are congruent.

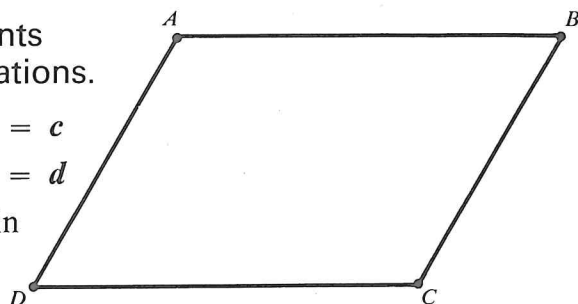
EXERCISES

Answer the following.

1. The **inch** is a unit of length. Name some other standard units for measuring length.
2. Why do we need standard units of measurement?

Use a ruler to measure the line segments in this polygon and complete the equations.

3. $m(\overline{AB}) = a$
4. $m(\overline{DC}) = b$
5. $m(\overline{BC}) = c$
6. $m(\overline{DA}) = d$
7. Name the congruent line segments in exercises 3–6.

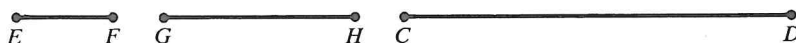


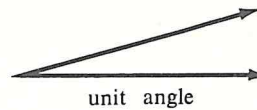
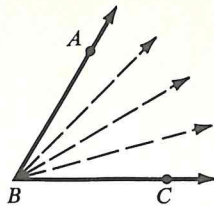
Draw line segments of length

- | | | |
|-----------------------|----------------------|----------------------|
| 8. 3 inches ($3''$) | 10. $2\frac{1}{4}''$ | 12. $\frac{3}{4}''$ |
| 9. $4\frac{1}{2}''$ | 11. $3\frac{3}{4}''$ | 13. $1\frac{3}{8}''$ |

Answer the questions.

14. Can you measure the length of a line? of a ray? Explain your answer.
15. Measure line segment $\overline{CD}$ with $\overline{EF}$ as the unit and then with $\overline{GH}$ as the unit. Does the unit used affect the length of $\overline{CD}$?



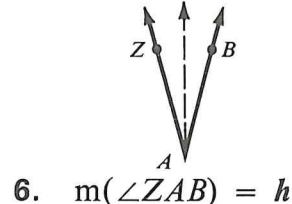
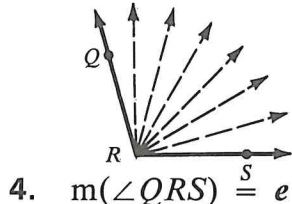
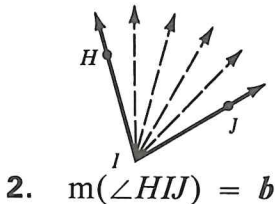
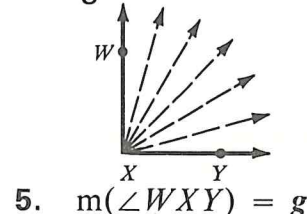
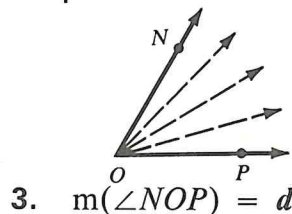
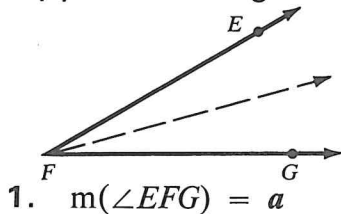


The union of 2 rays with a common endpoint is an **angle**. The rays are the **sides** of the angle and the common endpoint is the **vertex** of the angle. The angle above is named $\angle ABC$.

An angle is measured by comparing it with a **unit angle**. The measure of an angle tells *how many* unit angles it contains. The measure of angle ABC is 4. We write $m(\angle ABC) = 4$.

EXERCISES

Copy the unit angle and compare it to each of these angles.

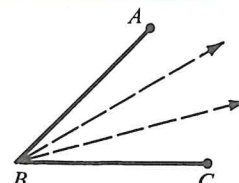
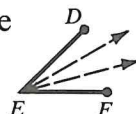


Angles which have the same measure with respect to the same unit are said to be **congruent**.

7. Name the congruent angles in exercises 1–6.

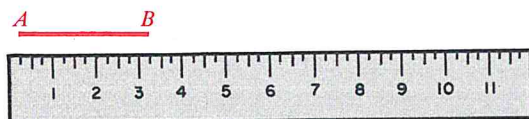
Often we refer to angles whose sides are line segments. In these cases, the line segments are parts of rays, but the rays are not drawn.

8. Are angles ABC and DEF congruent?
9. Will the lengths of the sides of an angle have any effect on the measure of the angle?



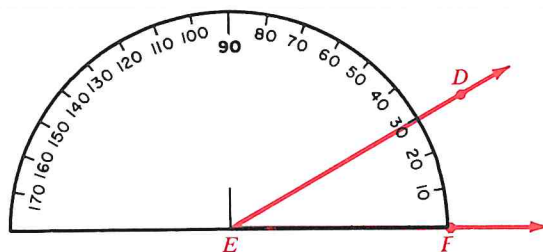


Measuring angles



A **ruler** is used to measure line segments. It is a kind of number line with the units numbered in order.

The *measure* of $\overline{AB}$ is 3. The *length* of $\overline{AB}$ is 3 inches, also written $3''$.

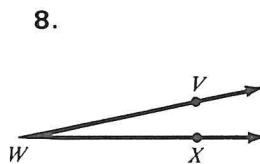
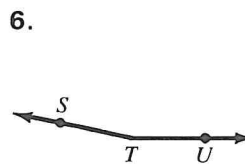
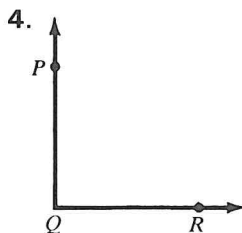
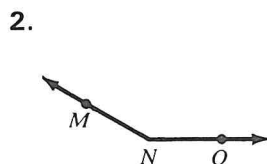
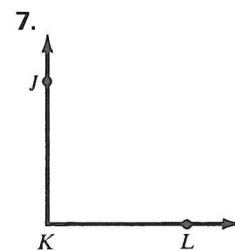
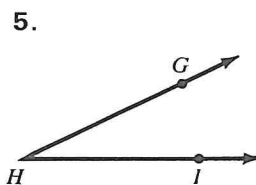
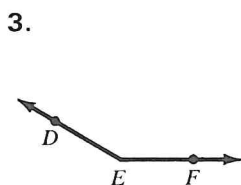
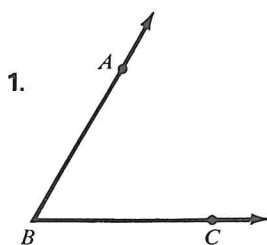


A **protractor** is used to measure angles. The units on a protractor are **degrees** and are shown on a kind of circular number line, numbered in order.

A **degree** is a small unit angle. The *measure* of $\angle DEF$ is 30. The *size* of $\angle DEF$ is 30 degrees, also written as 30° .

EXERCISES

Use your protractor to measure the angles.

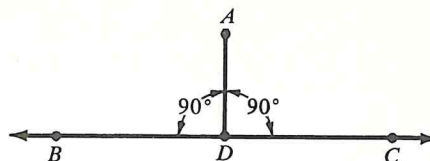


9. Name the angles in exercises 1 to 8 which are congruent.

An angle whose size is 90° is called a **right angle**.

10. Name the right angles in exercises 1–8.

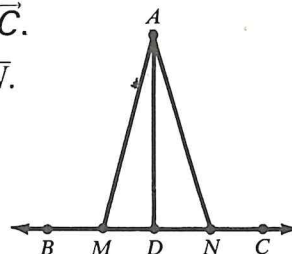
When 2 lines, line segments, or rays meet to form right angles, they are **perpendicular** to each other. $\angle ADB$ and $\angle ADC$ are both right angles, so $\overline{AD}$ is perpendicular to $\overline{BC}$.



EXERCISES

In the figure on the right, $\overline{AD}$ is perpendicular to $\overline{BC}$.

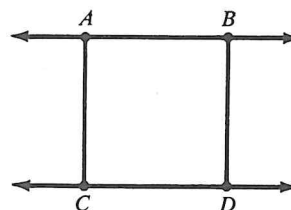
1. Name the shortest line segment— $\overline{AM}$, $\overline{AD}$, or $\overline{AN}$.
2. How would you find the **distance** from A to M ? from A to D ? from A to N ?
3. How would you find the **shortest distance** from point A to line BC ?



The perpendicular distance from a point to a line is referred to as **the distance** from a *point* to a *line*.

In the figure on the right, $\overline{AC}$ and $\overline{BD}$ are parallel. Measure each line segment and angle, and then answer the questions.

4. Are $\overline{AC}$ and $\overline{BD}$ perpendicular to $\overline{AB}$ and to $\overline{CD}$?
5. Are $\overline{AC}$ and $\overline{BD}$ congruent?
6. Will $\overline{AC}$ and $\overline{BD}$ always be the same distance apart?

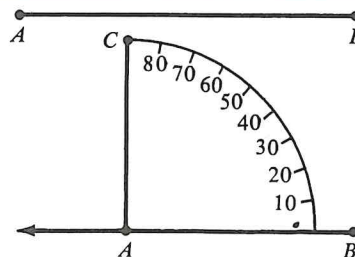


You may use a protractor to draw an angle.

Step 1: Draw a line segment $\overline{AB}$.

Step 2: Place the edge of the protractor along the line segment so that the center is on A .

Step 3: Measure the angle and mark the point C . Join C and A .

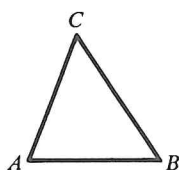


Follow these steps to draw angles whose sizes are

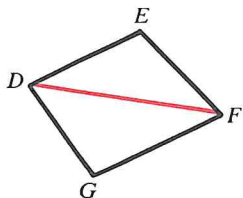
7. 90°
8. 45°
9. 135°
10. 30°
11. 53°
12. 15°

Polygons

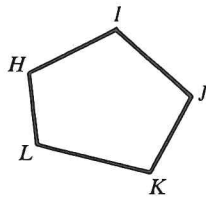
A **polygon** is a simple closed curve formed by the union of line segments. Here are some polygons:



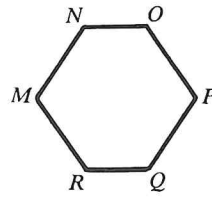
Triangle
ABC



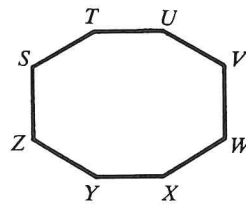
Quadrilateral
DEFG



Pentagon
HIJKL



Hexagon
MNOPQR



Octagon
STUVWXYZ

The line segments are **sides**, and the common endpoints are **vertices**; a line segment which joins 2 vertices and is not a side is a **diagonal**. $\overline{DF}$ is a diagonal of quadrilateral *DEFG*.

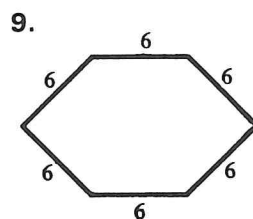
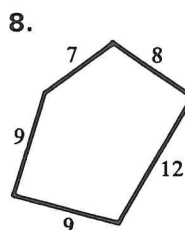
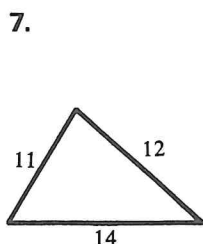
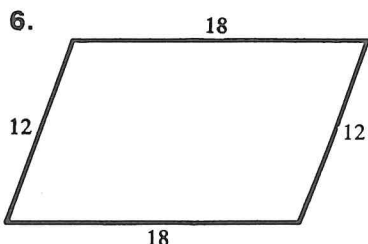
EXERCISES

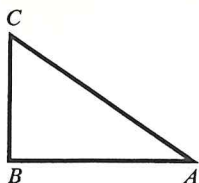
Use the figures above to help you complete this table.

	polygon	no. of sides	no. of vertices	no. of diagonals
1.	triangle	<i>a</i>	<i>b</i>	<i>c</i>
2.	quadrilateral	<i>d</i>	<i>e</i>	2
3.	pentagon	<i>f</i>	<i>g</i>	<i>h</i>
4.	hexagon	<i>i</i>	<i>j</i>	9
5.	octagon	<i>k</i>	<i>l</i>	<i>m</i>

The sum of the measures (in the same units) of the sides of a polygon names the measure of the **perimeter** of the polygon.

Name the perimeters of each polygon, using the measures given.





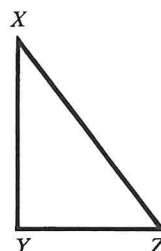
A triangle which has one angle of 90° is a **right triangle**.



A triangle which has 2 congruent sides is an **isosceles triangle**.

EXERCISES

Name the measures in inches or degrees for the right triangle XYZ .

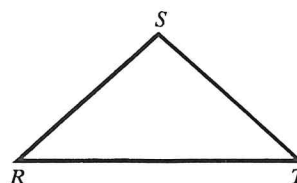


- | | | |
|--------------------|-----------------------|-----------------------|
| 1. $m(\angle XYZ)$ | 3. $m(\angle YXZ)$ | 5. $m(\overline{XZ})$ |
| 2. $m(\angle YZX)$ | 4. $m(\overline{XY})$ | 6. $m(\overline{YZ})$ |

Draw other right triangles and measure their angles and sides, then complete the sentences.

- The sum of the measures of the other angles of a right triangle is (less than, equal to, greater than) the measure of the right angle.
- The side opposite the right angle is the (longest, shortest) side of a right triangle.

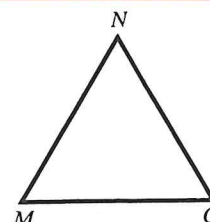
Name the measures in inches or degrees for the isosceles triangle RST .



- | | | |
|---------------------|------------------------|------------------------|
| 9. $m(\angle RST)$ | 11. $m(\angle SRT)$ | 13. $m(\overline{RT})$ |
| 10. $m(\angle RTS)$ | 12. $m(\overline{RS})$ | 14. $m(\overline{ST})$ |
15. Name the congruent sides and the congruent angles.

A triangle which has 3 sides congruent is an **equilateral triangle**. (Equilateral is from 2 Latin words meaning "equal sides.")

Name the measures, in inches or degrees, for triangle MNO .

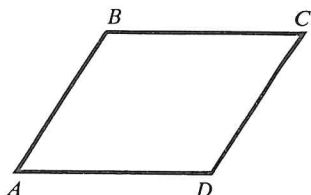


- | | | |
|---------------------|------------------------|------------------------|
| 16. $m(\angle NMO)$ | 18. $m(\angle MON)$ | 20. $m(\overline{NO})$ |
| 17. $m(\angle MNO)$ | 19. $m(\overline{MN})$ | 21. $m(\overline{MO})$ |

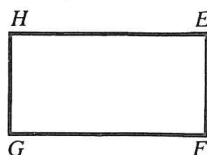


Quadrilaterals

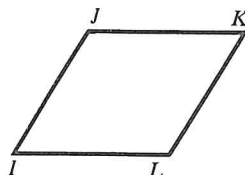
Special Quadrilaterals



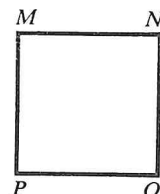
Parallelogram
ABCD



Rectangle
EFGH



Rhombus
IJKL

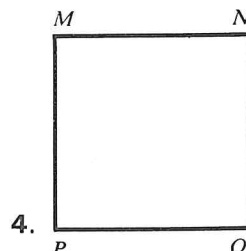
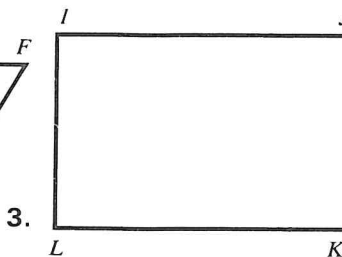
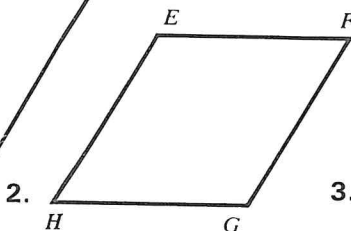
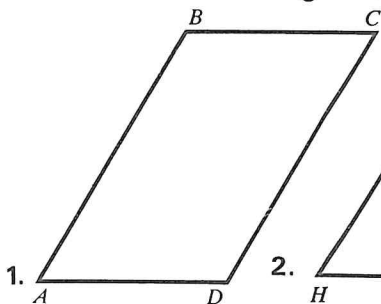


Square
MNOP

In any quadrilateral $ABCD$, $\overline{AB}$ and $\overline{CD}$ are called **opposite sides**.

EXERCISES

Use a protractor and a ruler to name the measures of each side and angle for these quadrilaterals.



For each quadrilateral in exercises 1–4, name

- | | |
|------------------------|-------------------------|
| 5. the congruent sides | 7. the perimeter |
| 6. the parallel sides | 8. the congruent angles |

Write *parallelogram*, *rectangle*, *rhombus*, and *square* across the top of your paper. Under each one name the numbers of the properties which describe the figure.

- | | |
|-------------------------------|-----------------------------|
| 9. opposite sides parallel | 12. all sides congruent |
| 10. opposite sides congruent | 13. all angles right angles |
| 11. opposite angles congruent | |

Complete the sentence.

14. If a quadrilateral is a rectangle and a rhombus, then it must be a ____.



Answer the questions in A as quickly as you can.
Then see if you can answer the questions in B even more quickly.

A

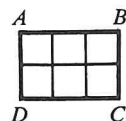
1. Name the sum of 222, 444, and 666.
2. Name the product of 321 and 9.
3. Name the perimeter of a square with sides of length 18".
4. Is $1332 \div 3$ greater or less than 49×9 ?
5. Name the union and intersection of {whole numbers less than 10} and {even numbers less than 10}.
- *6. Give an example of a set of whole numbers that is empty.
7. There are 5 bears, 5 lions, 10 tigers, and 19 monkeys at the zoo. How many animals are there in all?
8. Is 6 feet 9 inches longer or shorter than 80 inches?
9. Can 1001 be divided evenly by 11?
10. $531 + 315 + 153 = a$
11. Name the cost of 6 books at 99¢ each.

B

1. Name the difference between 10,000 and 3478.
2. Which statement is true, $675 \div 25 < 540 \div 20$, $675 \div 25 = 540 \div 20$, or $675 \div 25 > 540 \div 20$?
3. Name the perimeter of a rectangle with length 19" and width 14".
4. Name the property which says that $24 + 36 = 12 \times (2 + 3)$.
5. Which costs more, 5 dresses at \$21 each or 6 dresses at \$19 each?
6. Name the cardinal number of $A \times B$ if $n(A) = 15$ and $n(B) = 20$.
7. If a rhombus has a perimeter of 52', name the length of each side.
8. Name the function equation for this set of ordered pairs: $\{(10, 5), (20, 10), (100, 50), \dots\}$.
9. Name the number of angles in an octagon.
10. $56 \times 8 = 448$, so $5600 \times 80 = a$.
11. Name the cost of 12 books at \$1.09 each.

Measuring area

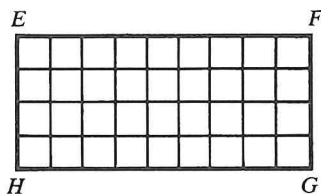
We name the area of a region by naming the number of square units that match the region. This number is the **measure**. Some standard units of area are the square inch, the square foot, and the square yard.



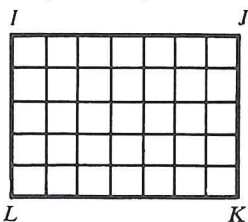
The area of $ABCD$ is 6 square units.

EXERCISES

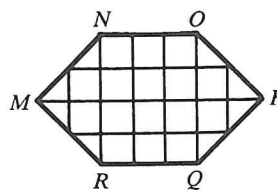
Name the number of square units in each of the polygonal regions.



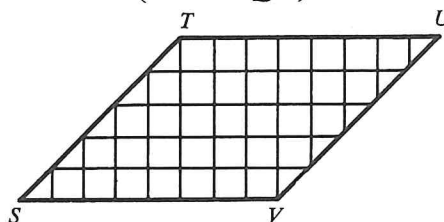
1. $m(EFGH) = a$



2. $m(IJKL) = b$

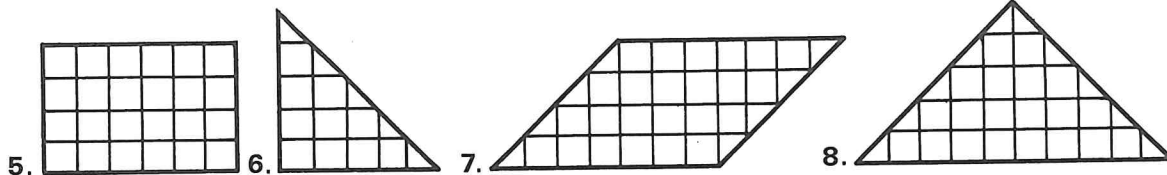


3. $m(MNOPQR) = c$



4. $m(STUV) = d$

Name the area of each of the polygonal regions (in square units).

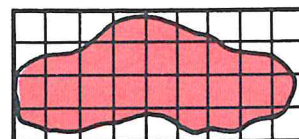


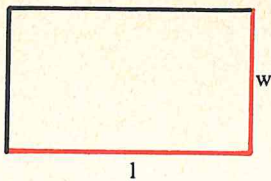
*Follow these steps to **estimate** the area of the simple closed curve.

Step 1: Count the total number of square units.

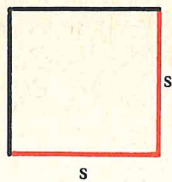
Step 2: Count the number of whole square units inside the curve (shaded red).

Step 3: Name the *average* of the numbers in steps 1 and 2 by dividing their sum by 2.

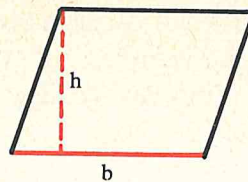




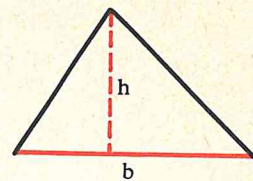
$$A = l \times w$$



$$A = s^2$$



$$A = b \times h$$

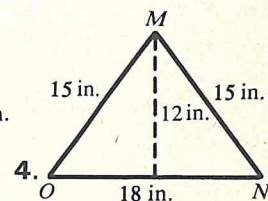
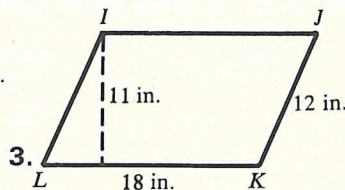
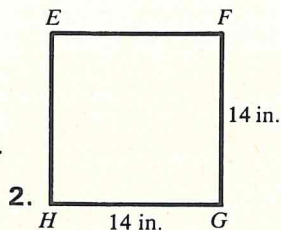
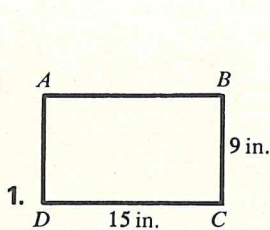


$$A = \frac{1}{2} \times b \times h$$

In each case A names the total number of units of area. The letters l , w , s , b , and h name the measures of the length, width, side, base, and height.

EXERCISES

Name the area and perimeter of each of the polygonal regions.



Name the area and perimeter of the regions formed by

5. a square with sides 8"
6. a square with sides 26"
7. a right triangle with sides of 5', 12', and 13'
8. a rectangle with length 15" and width 9"

Name the area of the regions formed by

9. a parallelogram with base 14" and height 11"
10. a triangle with base 12" and height 9"
11. a triangle with base 9" and height 12"

A rectangle has sides of 4' and 6'. Name the perimeter and area in

12. feet and square feet
13. inches and square inches

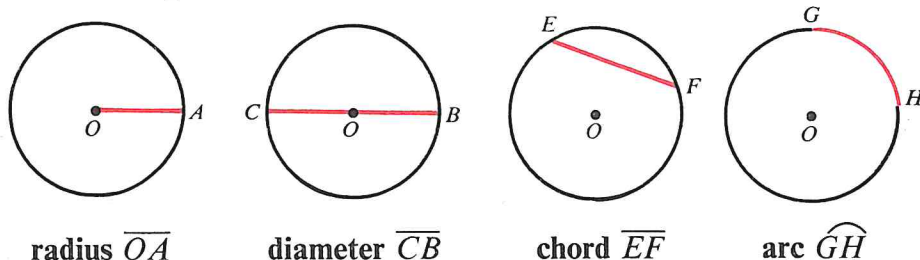
*A right triangle has sides of 27m, 36m, and 45m. Name

14. the perimeter
15. the area



Circles

A **circle** is the set of all points in a plane a fixed distance from a given point called **the center**. The distance from a point on the circle to the center is called **the radius** of the circle. A *radius* is a line segment.

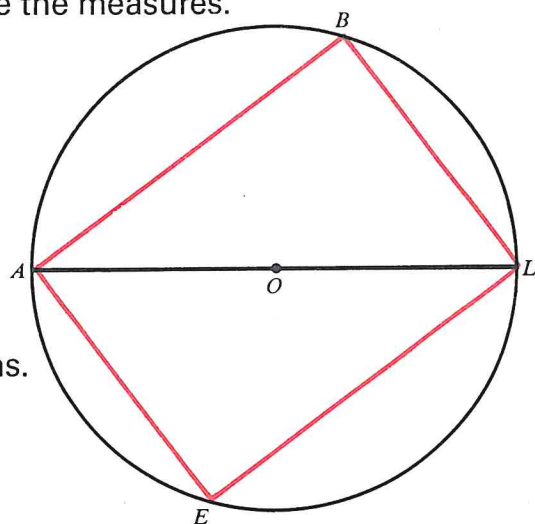


EXERCISES

1. How is a circle different from a polygon?
2. Use a compass to draw a circle with a radius of 2".
3. Draw a diameter of your circle. How long is it?
4. Draw a chord 4" long. What will be the longest chord of a circle?
5. Choose any 2 points on your circle. How many arcs are formed? How many chords can you draw with the 2 points as endpoints?

Use a protractor and a ruler to help you name the measures.

- | | |
|-------------------------|---------------------------|
| 6. $m(\angle ABL) = a$ | 11. $m(\angle BLA) = f$ |
| 7. $m(\angle LEA) = b$ | 12. $m(\widehat{AE}) = g$ |
| 8. $m(\angle BAL) = c$ | 13. $m(\widehat{BL}) = h$ |
| 9. $m(\angle ALE) = d$ | 14. $m(\widehat{AB}) = i$ |
| 10. $m(\angle LAE) = e$ | 15. $m(\widehat{EL}) = j$ |



Use exercises 6–15 to complete the questions.

16. Name the congruent angles.
17. Name the congruent sides.
18. Is $ABLE$ a rectangle? Why?
19. $\overline{AL}$ is a diagonal of $ABLE$. $\overline{AL}$ is also a ____ of the circle.



Circles and arcs can help us draw many other figures.

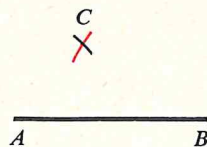
1. Draw a line segment $\overline{AB}$ with length 2". Draw all of the points in the plane 1" from point A .
2. Follow these steps to draw a triangle with sides of 2", 1", and $1\frac{1}{2}$ ".



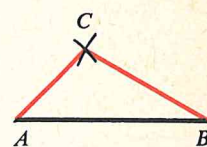
step 1



step 2



step 3



step 4

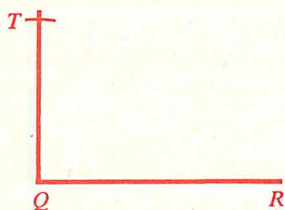
Step 1: Using your ruler, draw $\overline{AB}$ of length 2".

Step 2: With A as center draw an arc of a circle with radius 1".

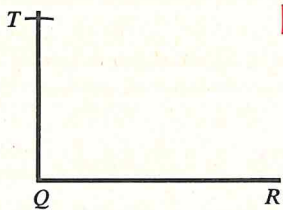
Step 3: With B as center draw an arc of a circle with radius $1\frac{1}{2}$ " so that it intersects the first arc in point C .

Step 4: Join points, A , B , and C . C is 1" from A and $1\frac{1}{2}$ " from B .

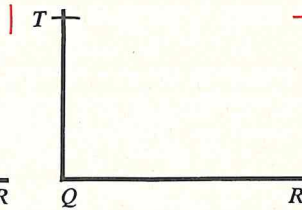
3. Follow these steps to draw a rectangle with sides of 3" and 2".



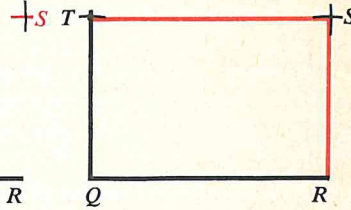
step 1



step 2



step 3



step 4

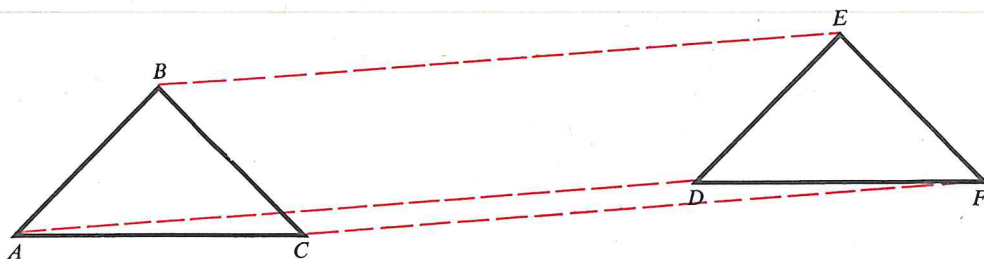
Step 1: Draw a line segment $\overline{QR}$ of length 3". At Q draw a right angle. On the line perpendicular to QR measure line segment $\overline{QT}$ of length 2".

Step 2: With T as center draw an arc of a circle with radius 3".

Step 3: With R as center draw an arc of a circle with radius 2", intersecting the first arc in S .

Step 4: Join S to points T and R .

● Congruent triangles

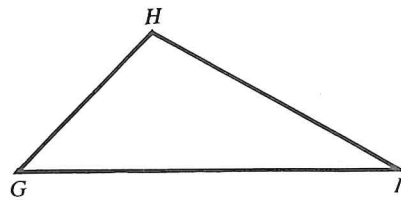


Triangles ABC and DEF are the same size and shape. Matching sides and angles of these triangles are congruent. $\overline{AB}$ is congruent to $\overline{DE}$, $\overline{BC}$ to $\overline{EF}$, $\overline{CA}$ to $\overline{FD}$. $\angle ABC$ is congruent to $\angle DEF$, $\angle BCA$ to $\angle EFD$, $\angle CAB$ to $\angle FDE$. We say that triangle ABC is **congruent** to triangle DEF .

EXERCISES

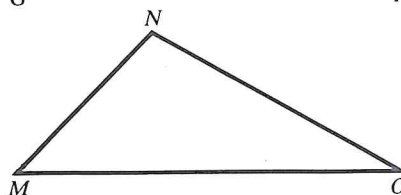
For triangles GHI and MNO name

1. the measures of the sides, in inches.
2. the measures of the angles, in degrees.

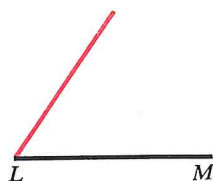
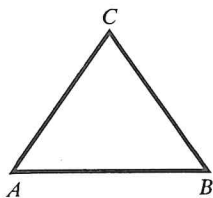


For the triangles GHI and MNO name

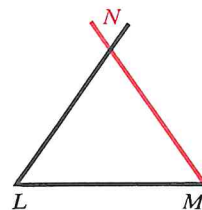
3. the pairs of congruent sides.
4. the pairs of congruent angles.
5. Are the triangles GHI and MNO congruent?



Follow these steps to draw a triangle congruent to ABC .



step 1



step 2

6. *Step 1:* Draw $\overline{LM}$ congruent to $\overline{AB}$. At L draw an angle congruent to $\angle CAB$.
Step 2: At M draw an angle congruent to $\angle ABC$. The sides of each angle should intersect at N .

Answer the following question.

7. Two triangles of the same size and shape are ____?

Consider the triangle in figure 1 where a , b , and c are the measures of the sides, c being the longest side. The squares of the measures of these sides are a^2 , b^2 , and c^2 .

The measure of the square region on each side of the triangle can also be named a^2 , b^2 and c^2 .

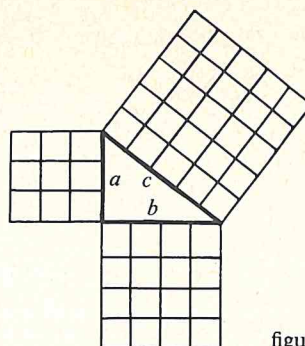


figure 1

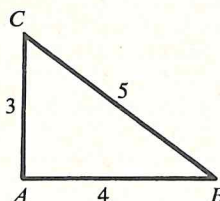
If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

This property was discovered by a famous Greek mathematician named Pythagoras and is called the **Theorem of Pythagoras**.

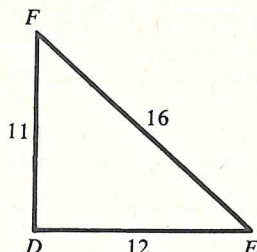
EXERCISES

Name the square of each measure of the triangles, then name the right triangles.

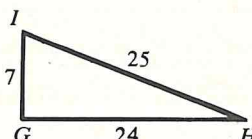
1.



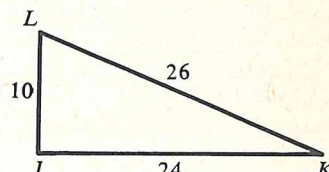
2.



3.



4.



Can a right triangle have sides of

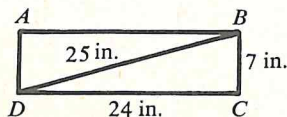
5. 5, 12, 13?

6. 8, 9, 12?

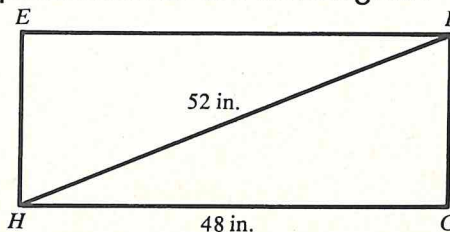
7. 13, 25, 28?

Which of these quadrilaterals are rectangles? Why?

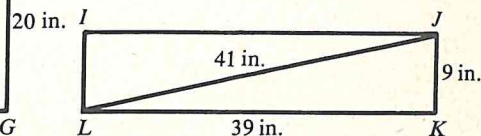
8.



9.



10.

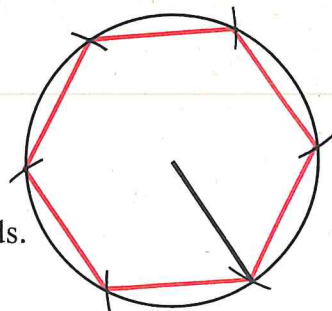


11-12. Name the perimeter and area of each figure in exercises 8 and 9.

Working with figures

Follow these steps for drawing a hexagon.

1. *Step 1:* Draw a circle with radius $1''$.
- Step 2:* Mark 6 arcs around the circle, using the radius of the circle as the length of the chords.
- Step 3:* Draw the 6 chords.



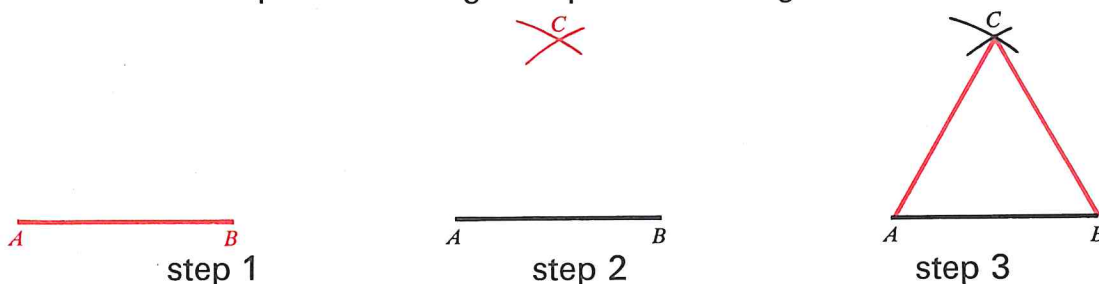
Use the hexagon in exercise 1 to answer the questions.

2. Name the measure of each side of the hexagon. Are all the sides congruent?
3. Name the measure of each angle of the hexagon. Are all the angles congruent?
4. Join each vertex to the center of the circle. What are these line segments called? What is their measure?

A polygon which has all of its sides and angles congruent is said to be **regular**.

5. Another name for a regular quadrilateral is ____?
6. A regular triangle is called an ____? triangle.
7. Is the hexagon in exercise 1 regular?

Follow these steps for drawing an equilateral triangle with sides of $2''$.



8. *Step 1:* Draw line segment $\overline{AB}$ of length $2''$.
- Step 2:* With radius of $2''$, draw an arc with A as center and another arc with B as center, so that the arcs intersect in point C .
- Step 3:* Draw $\overline{AC}$ and $\overline{BC}$.
9. Draw 6 equilateral triangles like those in exercise 8. Cut them out and arrange them to form a regular hexagon.

Draw a right triangle with sides of $1\frac{1}{2}$ ", 2" and $2\frac{1}{2}$ ".

1. Name the perimeter.

*2. Name the area.

Draw a triangle AFT with sides of 5", 5", and 6".

3. What kind of triangle is this?

4. Name the measure of each angle.

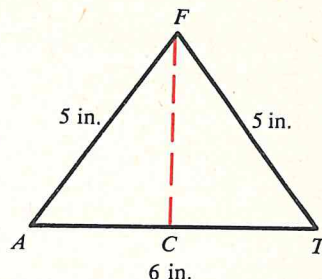
5. Name the congruent angles.

6. Locate the point C on $\overline{AT}$, 3" from A and 3" from T and join C to F .

7. Are angles ACF and FCT right angles?

8. Name the measure of $\overline{FC}$.

9. Use the Theorem of Pythagoras to see if triangles FAC and FTC are right triangles.



10. Are triangles AFC and TFC congruent?

11. Name the areas of triangles FCA , FTC , and FTA .

12. Name the perimeter of triangle FTA .

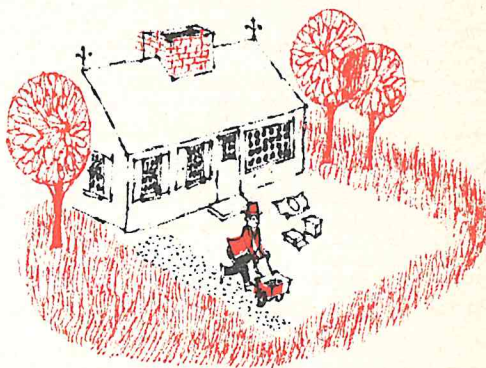
Solve the problems.

13. Mr. Jordan's backyard forms a rectangular shape 45' wide and 60' long. If he builds a wire fence around the yard, how long would the fencing have to be?

14. Mr. Jordan bought some grass seed for his backyard. If 1 box of grass seed covers 90 square feet, how many boxes would he need?

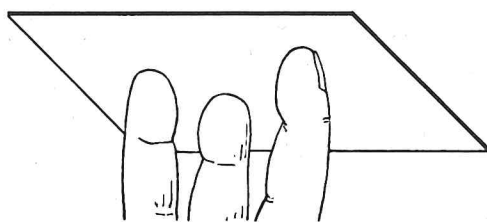
*15. Wendy made a triangular picture 78 inches long and 52 inches high. How many square inches of wall would the picture cover?

*16. If 1 quart of paint covers 80 square feet, would 42 quarts be enough to cover a wall that is 72' long and 48' high?

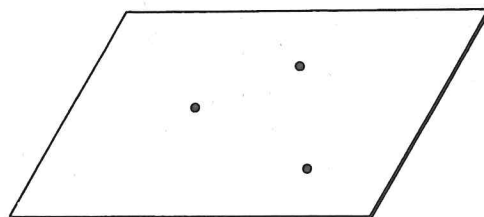




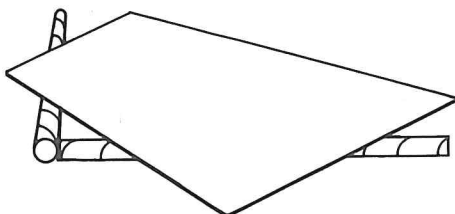
Facts about planes



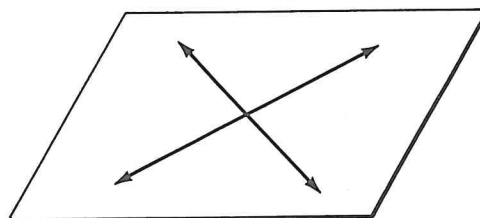
The card can rest on the 3 fingers without being bent.



There is one plane that can contain 3 points not in one line.



The card can rest on the 2 straws.

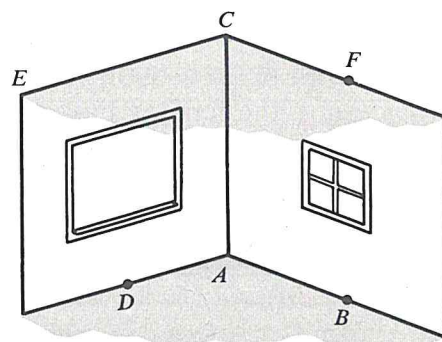


There is one plane that can contain 2 intersecting lines.

EXERCISES

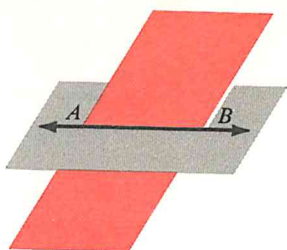
Complete each sentence with *floor*, *ceiling*, *window wall* or *chalkboard wall*.

1. The surface that contains points A , D , and C is the ____?
2. The surface which contains $\overrightarrow{AC}$ and $\overrightarrow{CE}$ is the ____?
3. The surface which contains $\overrightarrow{AB}$ and point F is the ____?
4. The surface which contains points E , C , and F is the ____?
5. Two surfaces which contain $\overrightarrow{AD}$ are the ____? and the ____?.

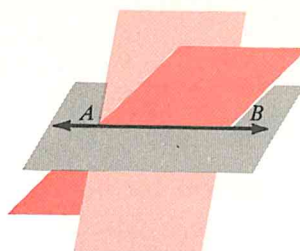


Use the figure for exercises 1–5 to

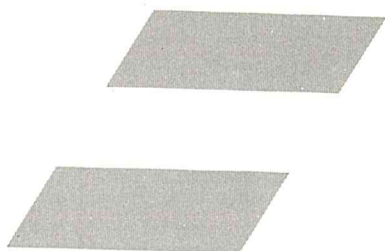
6. name four points that are *not* in the same plane.
7. name four points that *are* in the same plane.



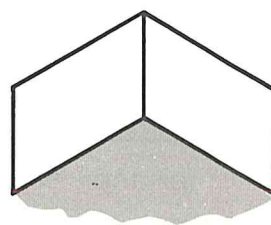
The intersection of 2 planes is a line.



The intersection of 3 planes may be a line.



Two planes that do not intersect are parallel.



The intersection of 3 planes may be a point.

EXERCISES

Study this surface, then complete the exercises.

1. Name the intersection of planes ABC and AEH .
2. Name 3 planes which intersect in line FG .
3. Name a plane parallel to AEH .
4. Name 3 planes which intersect in point F .

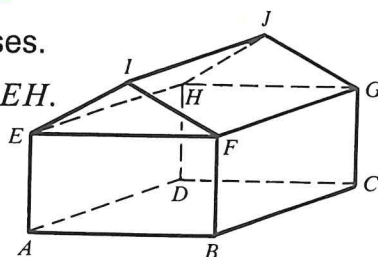


figure 1

The trails of the 2 jets do not intersect and are not parallel.

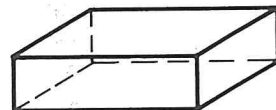
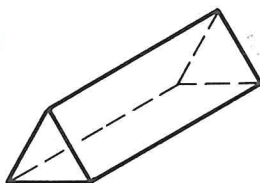
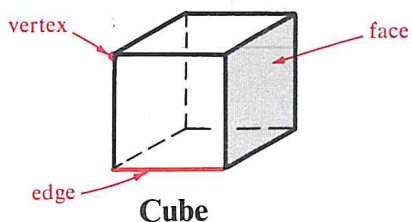
Lines that do not intersect and are not parallel are skew.



5. In figure 1, $\overline{IJ}$ and $\overline{BF}$ are skew. Name other line segments in figure 1 that are skew.



Closed surfaces



These are examples of **simple closed surfaces** called **prisms**. They are made up of 5 or more polygons and their regions. The regions are called **faces**, the sides of the polygons which form the regions are called **edges**, and the intersections of the edges are called **vertices**.

EXERCISES

Study the closed surfaces above and then complete the table.

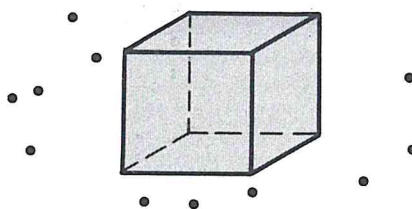
	surface	no. of vertices	no. of faces	no. of edges	shape of faces
1.	cube	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
2.	rectangular prism	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
3.	triangular prism	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i> and <i>m</i>

Name the number to answer the questions.

4. A prism has ____?____ or more rectangular faces.
5. How many edges of a cube are congruent?
6. How many faces of a cube are congruent?
7. How many pairs of faces of a rectangular prism are congruent?
8. How many faces of a triangular prism are congruent?

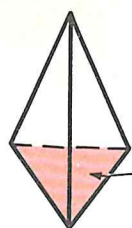
A **simple closed surface** separates space into 3 sets of points:

- a. the points **outside** the surface
- b. the points **inside** the surface
- c. the **surface** itself.

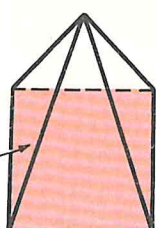


9. Is it possible to join a point inside the closed surface and a point outside the closed surface without intersecting the closed surface itself?

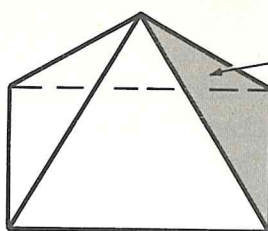
Other closed surfaces



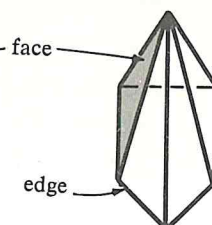
Triangular
Pyramid



Square
Pyramid



Rectangular
Pyramid



Pentagonal
Pyramid

These simple closed surfaces are called **pyramids**. They are made up of 4 or more regions. The face of the pyramid that is shaded red is also called the **base**. A pyramid is named according to the shape of the base.

EXERCISES

Study the surfaces above, then complete the table.

	shape of base	no. of vertices	no. of faces	no. of edges	shape of faces other than the base
1.	triangle	a	b	c	d
2.	square	e	f	g	h
3.	rectangle	i	j	k	l
4.	pentagon	m	n	p	q
5.	hexagon	r	s	t	u

Complete the sentences.

- All the faces of a pyramid except the base are ____?
- All the faces of a pyramid except the base intersect in a ____?

Use figure 1 to help you answer the exercises.

- Name the number of faces of the rectangular pyramid that contain point A .
- Name the edges of the rectangular pyramid that are congruent to $\overline{AD}$.
- Name the face of the rectangular pyramid that does not contain point A .

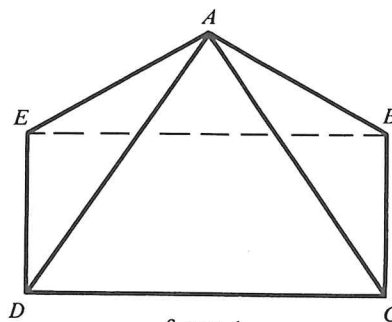
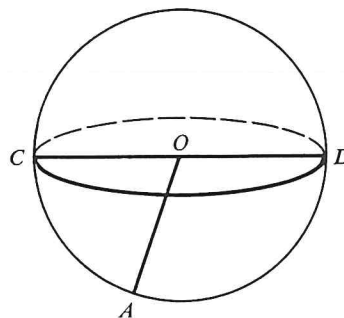


figure 1

Spheres

The set of all points in space which are a fixed distance from a point called the center is called a **sphere**. The distance from the center to a point on the sphere is called the **radius** of the sphere. $\overline{OA}$ is a radius. $\overline{CD}$ is a diameter.



EXERCISES

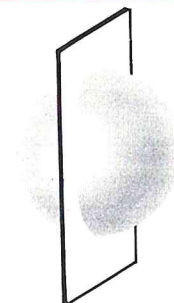
Answer the questions:

1. Are all radii of a sphere congruent?
2. Are all diameters of a sphere congruent?
3. If the length of a radius of a sphere is 5'', name the diameter.

If a plane intersects a sphere in more than one point, then the intersection will be a **circle**.

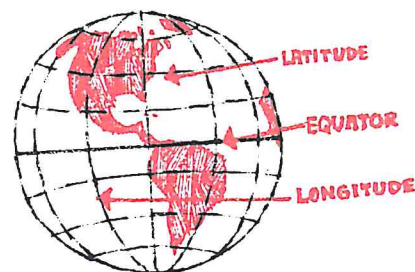
If the center of the sphere is a point in the intersecting plane, then the intersection is called a **great circle**.

4. Is the center of a great circle the same as the center of the sphere?
5. Is the radius of a great circle the same as the radius of the sphere?
6. How many great circles can be drawn through the endpoints of a diameter of a sphere?
7. How many great circles will a sphere have?



*The earth is like a sphere. To complete each sentence use one of these words: *equator*, *latitude* or *longitude*.

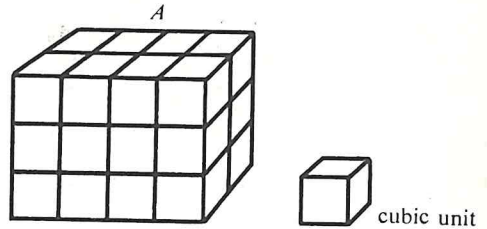
8. A plane through the center of the earth at right angles to the axis cuts the earth in a great circle called the ____?
9. Planes parallel to this plane cut the earth in circles called parallels of ____?
10. Planes through the axis cut the earth in great circles of ____?



Measuring volume

To measure the volume of the *space inside* a closed surface we compare a standard unit of volume, such as a **cubic inch**, with the space.

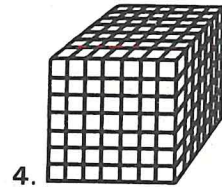
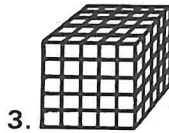
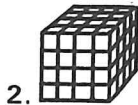
In the surface on the right,
 $m(A) = 4 \times 3 \times 2 = 24$ and the volume of A is 24 cubic units.



EXERCISES

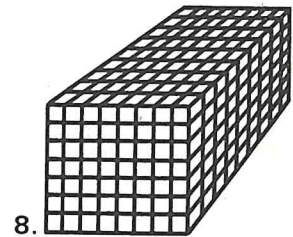
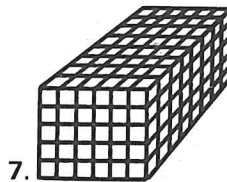
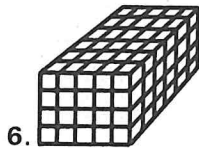
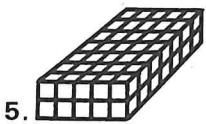
If s is the measure of each edge of a cube, then the total number of units of volume (V) in the cube is $s \times s \times s = s^3$.
 We write $V = s^3$.

Name the volume in each of the cubes.

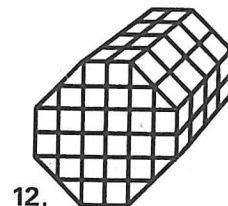
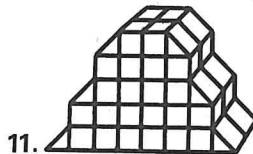
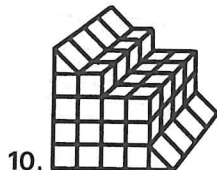
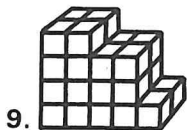


If l , w , and h are the measures of each edge of a rectangular prism, the number of units of volume (V) will be $l \times w \times h$.
 We write $V = l \times w \times h$.

Name the volumes in each of the prisms.



Name the volume in each of the closed surfaces.



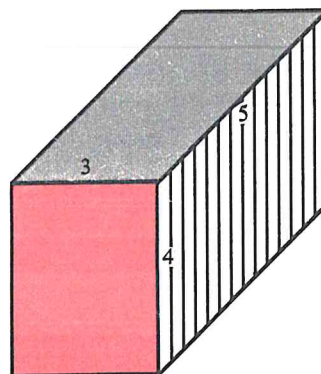
Area and closed surfaces

The sum of the areas of *all* the faces of a closed surface is called the **surface area**.
For the rectangular prism in figure 1, the measure of area of the

red face is $3 \times 4 = 12$,
striped face is $4 \times 5 = 20$,
shaded face is $3 \times 5 = 15$.

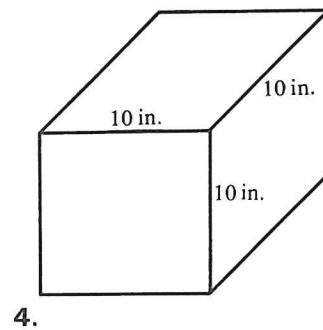
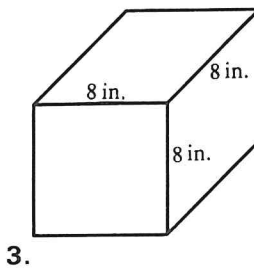
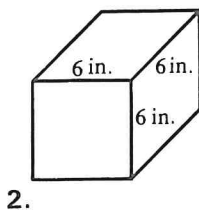
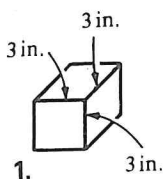
Since there are 2 red faces, 2 striped faces, and 2 shaded faces, the surface area measure is

$$L = (2 \times 12) + (2 \times 20) + (2 \times 15) = 94.$$

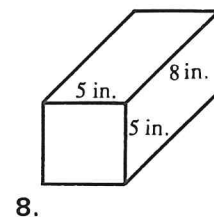
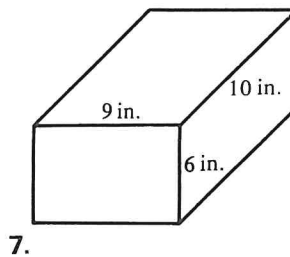
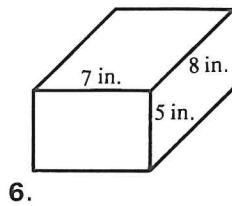
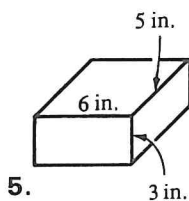


EXERCISES

Name the surface areas of the cubes.



Name the surface areas of the rectangular prisms.



Answer the questions.

9–16. Name the volume in each surface in exercises 1–8.

- *17. If a rectangular prism has edges of measure l , w , and h , write an equation for the number of units of area (L) in the surface.

Working with area and volume

Use the figures to help you work the exercises.

1. Name the volume and surface area of the rectangular prism (fig. 1).

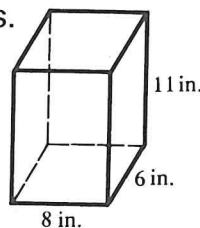


figure 1

2. Name the volume and surface area of the triangular prism (fig. 2).

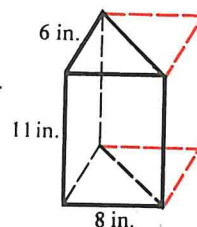


figure 2

3. Name the volume of the pool (fig. 3) by naming the sum of the volumes in the rectangular prism and the triangular prism.

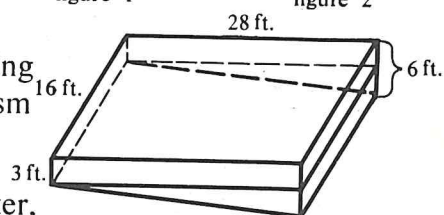


figure 3

4. If one cubic foot holds about 7 gallons of water, how many gallons of water will the pool hold?

5. Name the area of the triangular region (fig. 4).

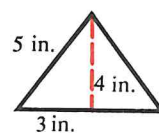


figure 4

6. Name the surface area of the square pyramid (fig. 5).

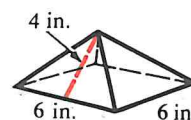


figure 5

7. Name the area of the larger square region in figure 6.
8. Name the area of the smaller square region in figure 6.
9. Estimate the area of the circular region in figure 6.

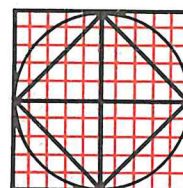


figure 6

** In figures 7 and 8, the area of each circular base is 616 square units and the perimeter of each base is 88 units. The height is 24 units.

10. Name the volume.

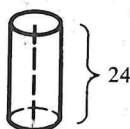


figure 7

11. Name the surface area.

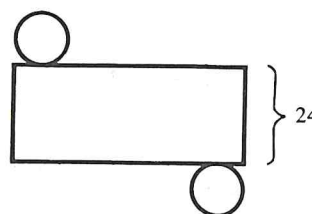
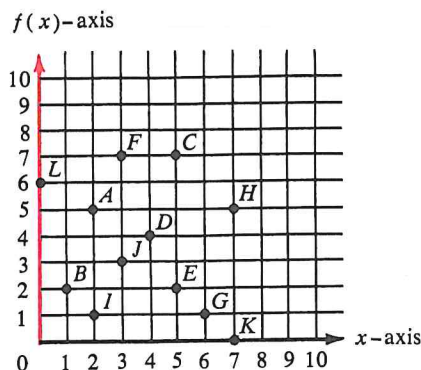


figure 8



Number planes

A number plane may be formed by drawing 2 number lines perpendicular to each other. The red number line is called the $f(x)$ -axis; the black number line is called the x -axis. The intersection of the x and $f(x)$ -axes is called the **origin**.



A point is located in a number plane by an **ordered pair** of numbers. The first number in a pair tells how many units the point is along the x -axis; the second number tells how many units it is along the $f(x)$ -axis. The ordered pair for the point A is (2, 5).

EXERCISES

Use the number plane above to name the ordered pairs for the points.

1. D 2. H 3. F 4. G 5. K 6. I 7. B 8. L

Draw a number plane with 12 units on each axis.

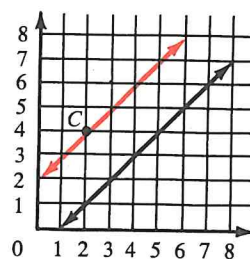
9. Locate and label the following points on your plane:
 $A: (3, 6)$ $B: (4, 8)$ $C: (5, 10)$ $D: (6, 12)$
10. Join points A , B , C , and D . Do they lie on the same line?
11. Would the origin be a point on this line?
12. Write a function equation for the set of ordered pairs on your line.
13. Complete the set of ordered pairs for the function equation $x + 2 = f(x)$: $\{(0, a), (1, b), (3, c), (5, d), (8, e), (10, f)\}$
14. Locate and join this set of points on your number plane.
15. Name the point common to graphs of the functions in exercises 12 and 13.
- *16. Does the ordered pair you named in exercise 15 make each function equation in exercises 12 and 13 true? Are there any other ordered pairs which will make both sentences true?

This number plane shows the **graphs** of the function equations

$$x + 2 = f(x) \text{ (in red)}$$

$$\text{and } x - 1 = f(x) \text{ (in black)}$$

The ordered pair $(2, 4)$ locates the point C on the graph of $x + 2 = f(x)$. If we replace x by 2 and $f(x)$ by 4 we have a true equation, $2 + 2 = 4$. The pair $(2, 4)$ is a **solution** of the equation, $x + 2 = f(x)$.



EXERCISES

Study the number plane above.

1. Name 4 other solution pairs for $x + 2 = f(x)$.
2. Name 7 solution pairs for $x - 1 = f(x)$.
3. Are the graphs of these equations parallel?

Draw a number plane with 12 units on each axis.

4. Name 4 solution pairs for $3 \times x = f(x)$. Locate and join these points on your number plane.
5. Name 4 solution pairs for $x + 4 = f(x)$. Locate and join these points on your number plane.
6. Do the graphs of these equations have any points in common?

Draw a number plane with 12 units on each axis.

7. Use the equation $x + 2 = f(x)$ to complete the ordered pairs: $(0, a)$, $(3, c)$, $(6, e)$, $(9, i)$. Locate and join these points in your number plane.
8. Use the equation $2 \times x = f(x)$ to complete the ordered pairs: $(0, a)$, $(1, c)$, $(4, e)$, $(6, i)$. Locate and join these points in your number plane.
9. Name the point common to the graphs of these equations.

*Draw a number plane with 25 units on each axis.

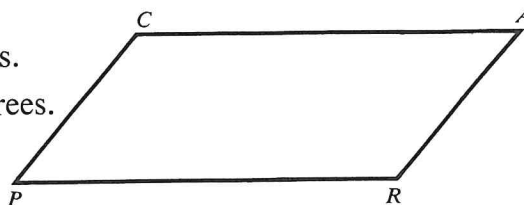
10. For the equation $x^2 = f(x)$ let x be a member of $\{0, 1, 2, 3, 4, 5\}$, then name the solution pairs.
11. Locate and join the 6 solution pairs on your number plane.
12. Do the 6 solution pairs lie on a line?



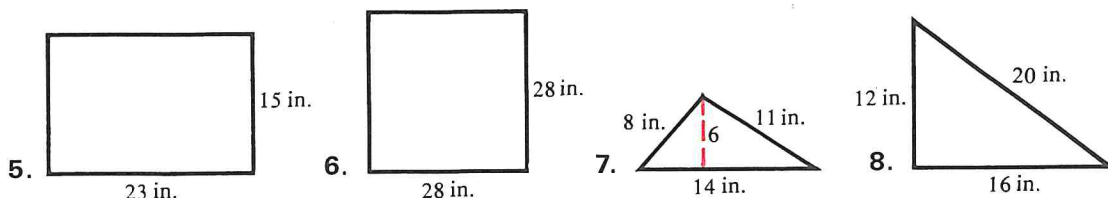
CHECKING UP

Study parallelogram *CARP*.

1. Name the measure of each side in inches.
2. Name the measure of each angle in degrees.
3. Name the congruent sides.
4. Name the congruent angles.



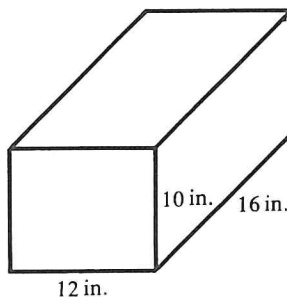
Name the perimeter and area of each plane figure.



9. Use the Theorem of Pythagoras to show that the triangle in exercise 8 is a right triangle.

Study the rectangular prism.

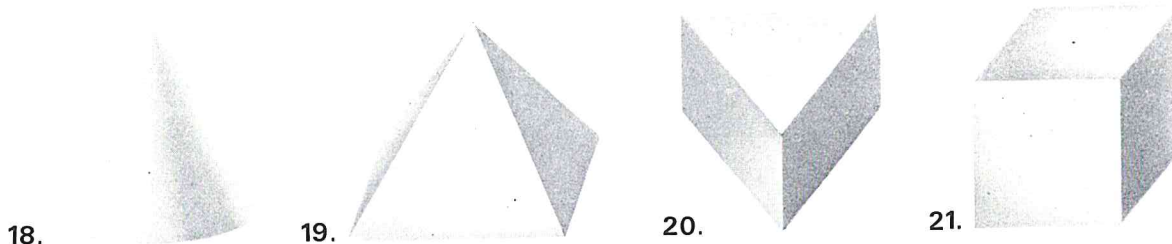
10. Name the number of faces.
11. Name the number of edges.
12. Name the number of vertices.
13. Name the volume.
14. Name the surface area.



Draw a number plane with 10 units on each axis.

15. Locate and join the following points: $\{(4, 1), (6, 3), (3, 0), (7, 4), (9, 6), (5, 2)\}$.
16. Write a function equation for the graph in exercise 15.
17. Draw a triangle with sides of length $4''$, $2\frac{1}{2}''$ and $3\frac{1}{2}''$.

Name the closed surfaces.

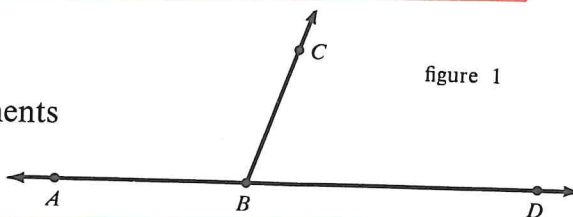




Geometry is concerned with different sets of points.

In figure 1, name

1. a line
2. 3 rays
3. 4 line segments
4. 2 angles

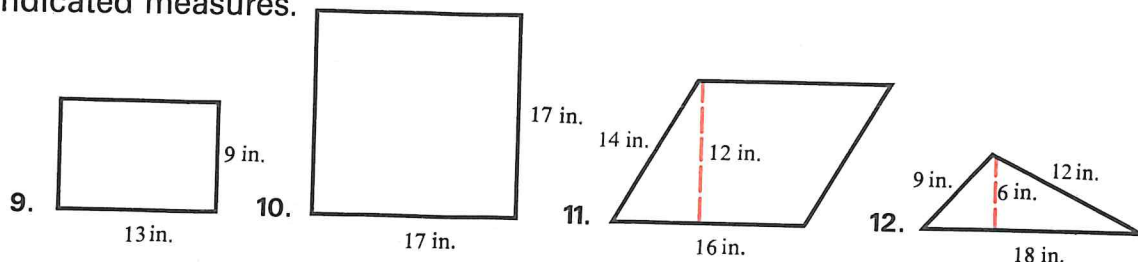


Measurement involves the comparison of a standard unit with the object being measured.

In figure 1, name the measures in inches or degrees.

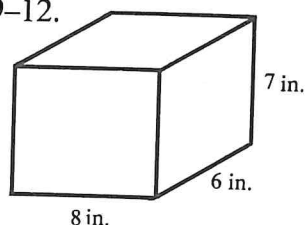
5. $m(\overline{AB})$
6. $m(\overline{BD})$
7. $m(\angle ABC)$
8. $m(\angle CBD)$

Name the areas of the polygonal regions using the indicated measures.



13–16. Name the perimeter of each polygon in exercises 9–12.

17. Name the volume in the rectangular prism in figure 2.
18. Name the surface area of the rectangular prism in figure 2.



19. Name the volume and surface area of a cube with sides of 5". figure 2

A number plane may be formed by 2 perpendicular number lines. Each point in the plane is located by an ordered number pair.

Draw a 10 by 10 number plane, then locate and label these points on your number plane.

20. A: (4, 3)
21. B: (7, 1)
22. C: (5, 5)
23. D: (2, 8)
24. E: (0, 6)



LOOKING WAY BACK

For the sets $A = \{0, 3, 6, 9, 12, 15, 18\}$ and $B = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$, name

1. $A \cup B$
2. $A \cap B$
3. $n(A)$
4. $n(B)$
5. $n(A \cup B)$
6. $n(A \cap B)$
7. a universal set for A and B .
8. Draw a diagram showing sets A and B .

Complete the sentences. Write $<$, $>$ or $=$ to replace each $\bigcirc$.

9. $300 + 40 + 6 \bigcirc 346$
10. $6^2 + 7^2 \bigcirc 13^2$
11. $64 \times 48 \bigcirc 32 \times 96$
12. $3056 - 887 \bigcirc 1134 + 876$
13. $90 - 7^2 \bigcirc 4^2 + 5^2$
14. $11^2 + 60^2 \bigcirc 61^2$
15. $306 \div 9 \bigcirc 918 \div 17$
16. $42 \times (30 + 8) \bigcirc (42 \times 30) + (42 \times 8)$
17. $3^3 + 4^3 \bigcirc 5^3$
18. $4^2 + 5^2 \bigcirc 20^2$

Name the sums or differences.

- | | | | | |
|---|---|---|---|---|
| 19. $\begin{array}{r} 4357 \\ + 5296 \\ \hline \end{array}$ | 21. $\begin{array}{r} 5346 \\ - 2698 \\ \hline \end{array}$ | 23. $\begin{array}{r} 9031 \\ - 5478 \\ \hline \end{array}$ | 25. $\begin{array}{r} 76304 \\ + 63918 \\ \hline \end{array}$ | 27. $\begin{array}{r} 41372 \\ - 34185 \\ \hline \end{array}$ |
| 20. $\begin{array}{r} 7322 \\ - 6866 \\ \hline \end{array}$ | 22. $\begin{array}{r} 6395 \\ + 7847 \\ \hline \end{array}$ | 24. $\begin{array}{r} 84372 \\ + 13848 \\ \hline \end{array}$ | 26. $\begin{array}{r} 73468 \\ + 41439 \\ \hline \end{array}$ | 28. $\begin{array}{r} 63204 \\ - 58739 \\ \hline \end{array}$ |

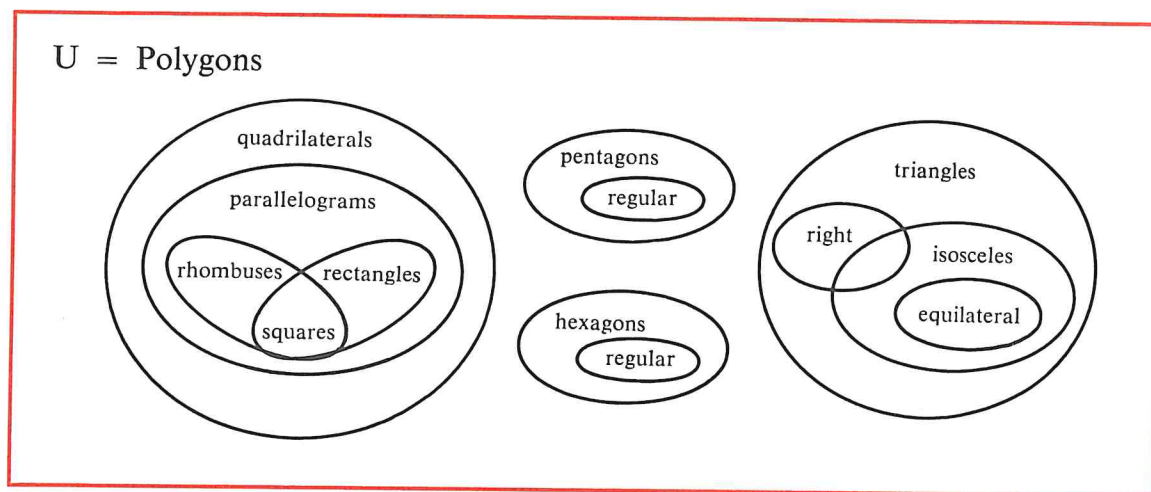
Name the products.

- | | | | | |
|---|---|--|--|---|
| 29. $\begin{array}{r} 576 \\ \times 8 \\ \hline \end{array}$ | 31. $\begin{array}{r} 782 \\ \times 46 \\ \hline \end{array}$ | 33. $\begin{array}{r} 8437 \\ \times 73 \\ \hline \end{array}$ | 35. $\begin{array}{r} 5983 \\ \times 76 \\ \hline \end{array}$ | 37. $\begin{array}{r} 84395 \\ \times 24 \\ \hline \end{array}$ |
| 30. $\begin{array}{r} 5495 \\ \times 9 \\ \hline \end{array}$ | 32. $\begin{array}{r} 478 \\ \times 69 \\ \hline \end{array}$ | 34. $\begin{array}{r} 4096 \\ \times 37 \\ \hline \end{array}$ | 36. $\begin{array}{r} 9388 \\ \times 95 \\ \hline \end{array}$ | 38. $\begin{array}{r} 90417 \\ \times 57 \\ \hline \end{array}$ |

Name the quotients and remainders.

- | | | | |
|-------------------------|--------------------------|--------------------------|--------------------------|
| 39. $8 \overline{)536}$ | 41. $6 \overline{)439}$ | 43. $68 \overline{)538}$ | 45. $19 \overline{)635}$ |
| 40. $7 \overline{)609}$ | 42. $23 \overline{)322}$ | 44. $76 \overline{)984}$ | 46. $15 \overline{)570}$ |

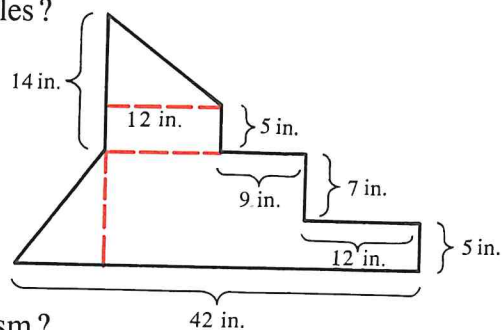
Study this set diagram, then answer the questions.



1. What is the intersection of the set of rectangles and the set of rhombuses? Why?
2. Are all pentagons regular?
3. Can a triangle be right and isosceles?
4. Are all parallelograms quadrilaterals?
5. Are all equilateral triangles isosceles?
6. Are some equilateral triangles right triangles?

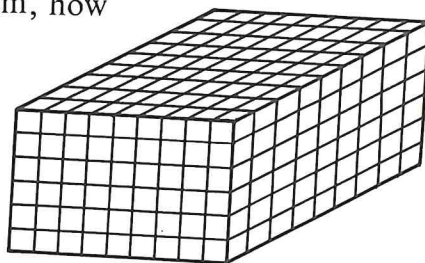
Study the figure, then

7. name the perimeter.
8. name the area.



Answer the questions.

9. What is the volume of this rectangular prism?
10. If you painted the surface of this prism, how many of the small cubes would have
 - a. 4 faces painted?
 - b. 3 faces painted?
 - c. 2 faces painted?
 - d. 1 face painted?
 - e. 0 faces painted?



CHAPTER 4 • Multiplication, División

One factor very great

Study these two methods for naming the product, $8 \times 57,469,326$.

A

$$\begin{array}{r}
 57469326 \\
 \times 8 \\
 \hline
 40 \quad 48 \\
 56 \quad 16 \\
 32 \quad 24 \\
 48 \\
 72 \\
 \hline
 459,754,608
 \end{array}$$

B

$$\begin{array}{r}
 5357224 \\
 57469326 \\
 \times 8 \\
 \hline
 459,754,608
 \end{array}$$

EXERCISES

Name the products using method A.

1. 39765423
 $\times 7$

2. 45672935
 $\times 4$

3. 84910272
 $\times 6$

Name the products using method B.

4. 59234587
 $\times 5$

7. 61782947
 $\times 9$

10. 87652946
 $\times 6$

5. 40578119
 $\times 5$

8. 76389741
 $\times 3$

11. 98246547
 $\times 8$

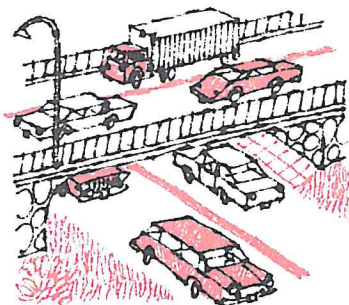
6. 96573274
 $\times 9$

9. 55795181
 $\times 8$

12. 62340987
 $\times 6$

Solve the problems.

13. If 23,197,642 cars travel on the Expressway each week, how many cars will travel on it in 4 weeks?
14. If 2,396,853 cars travel in and out of the city every day, how many cars would this be in 9 days?
15. Which is greater, $6,438,962 \times 8$ or $5,723,521 \times 9$?



In method **B** at the top of the last page we multiplied and *then* added. The first instance was $(8 \times 2) + 4 = 16 + 4 = 20$.

We may practice this kind of multiplication and addition with the function rule $(8 \times m) + 4 = f(m)$. This would give us the set of pairs $\{(1, 12), (2, 20), (3, 28), \dots\}$.

EXERCISES

Study the function equations, then complete the patterns.

1. $(5 \times m) + 3 = f(m)$

first number: m	1	2	3	4	5	6	7	8	9	10
second number: $f(m)$	8	13	a	c	e	n	r	s	u	v

2. $(9 \times m) + 6 = f(m)$

first number: m	2	10	7	11	12	3	9	6	5	15
second number: $f(m)$	24	96	69	a	c	e	n	r	s	u

3. $(10 \times m) + 10 = f(m)$

m	3	5	7	9	12	15	10	8	13	19	20	30	17
$f(m)$	40	60	80	a	c	e	n	r	s	u	v	x	z

Name the set of ordered pairs if the first number in each pair is from $M = \{1, 2, 4, 7, 8, 12, 13, 17\}$ and the rule is

4. $(2 \times m) + 3 = f(m)$ 5. $(4 \times m) + 9 = f(m)$ 6. $(6 \times m) - 6 = f(m)$

Study each set of ordered pairs, then write a function equation and complete the ordered pairs.

7. $\{(3, 27), (7, 63), (0, 0), (5, 45), (11, a), (16, c), (12, e)\}$

8. $\{(4, 1), (8, 5), (19, 16), (10, a), (11, c), (13, e), (18, n)\}$

*9. $\{(1, 6), (2, 11), (3, 16), (4, 21), (5, a), (12, c), (18, e)\}$

*10. $\{(0, 2), (1, 5), (2, 8), (3, 11), (4, a), (6, c), (8, e), (9, n)\}$

*11. $\{(0, 2), (1, 6), (2, 10), (3, 14), (4, a), (5, c), (7, e), (10, n)\}$



Another way to multiply



Lattice Method

		4	8	6	×
		1 2	2 4	1 8	3
		2 8	5 6	4 2	7
1	7	9	8	2	

A very famous Scottish mathematician, Napier, invented rods for multiplication. Napier's rods used the multiplication called the Lattice Method which was common in the 15th century. Our example shows the multiplication $37 \times 486 = 17,982$.

EXERCISES

Copy and complete the multiplications using the lattice method.

1.

		3	9	7	×
		2 4	7 2	5 6	8
		1 5	4 5	3 5	5
3	3	7	4	5	

3.

		6	3	8	×
		1 2	0 6	1 6	2
		5 4	2 1	7 2	9
		3 6	1 5	4 8	5
1	8	8	2	1	0

2.

		6	9	4	×
		2 4	3 6	1 6	4
		4 8	7 2	3 2	8
3	3	3	1	2	

Use the lattice method to name the products.

4. 42×369

8. 476×652

5. 37×981

9. 564×381

6. 58×764

10. 473×519

7. 27×465

11. 358×623

Compare the lattice method with the 5-step method.

	5	2	4	6	×
	3	1	2	4	7
3	6	7	2	2	

$$\begin{array}{r}
 5246 \\
 \times 7 \\
 \hline
 42 \\
 280 \\
 1400 \\
 35000 \\
 \hline
 36,722
 \end{array}$$

EXERCISES

Copy and complete the multiplications using the lattice method.

1.

	6	4	3	8	×
	3	2	1	4	6
3	8	6	2	8	

2.

		4	6	2	9	×
		3	5	1	8	9
	4	1	6	6	1	

Use the lattice method to name the products.

3. 297×28

4. 567×89

5. 7963×75

6. 8724×396

7. 3692×854

8. 4876×972

Name the product 365×478 using

9. the lattice method.

10. the short method.

11. Can you see any advantages that the lattice method has over the short method?

Name the factors and the product for each of the lattices.

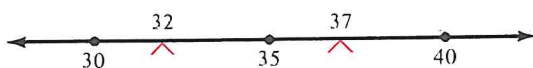
12.

		a3	c6	9	×
		1	2	3	4
		2	4	6	
		2	4	6	7n
		1	2	3	
r1	s7	u3	v4	z3	

13.

		a4	c5	e7	×
		0	1	1	2n
		8	0	4	
		1	2	2	4r
		6	0	8	
w1	v0	s9	u6	8	

Rounding—estimates



To **round** a number to the **nearest 10** means to name the multiple of 10 to which it is closest. Using the number lines, we see that 32 may be rounded to 30, 274 to 270, 37 to 40 and 279 to 280. In these instances, 32 and 274 were **rounded down** to the lower multiple of 10; 37 and 279 were **rounded up** to the higher multiple of 10. To round a number which is halfway between 2 multiples of 10, we usually **round up**. For example, 35 is rounded to 40 and 275 is rounded to 280.

EXERCISES

Round each of the numbers to the nearest 10.

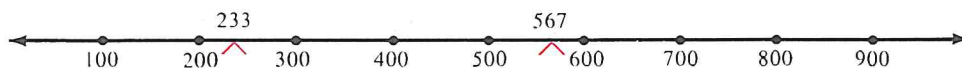
1. 23 2. 47 3. 78 4. 59 5. 65 6. 136 7. 652 8. 516

Round each factor to the nearest 10, then estimate the product.

For example, 12×42 is estimated to be 10×40 or 400.

9. 18×36 11. 25×37 13. 254×59 15. 295×268
10. 24×83 12. 137×21 14. 156×238 16. 569×143

17-24. Name each product in exercises 9 to 16. Then name the difference between your estimate and the exact product.



To **round** a number to the nearest 100 means to name the multiple of 100 to which it is closest. On the number line we see that 567 may be rounded to 600, 233 to 200.

Round each of the numbers to the nearest 100.

25. 239 26. 561 27. 750 28. 852 29. 674 30. 988 31. 799

Round each factor to the nearest 100, then estimate the product.

For example, 287×343 is estimated to be $300 \times 300 = 90,000$.

32. 347×786 33. 217×476 34. 538×627 35. 167×982

Study these methods for estimating the product 26×487 .

1. Round 26 up to next 10. Round 487 up to next 100. $30 \times 500 = 15,000$	3. Round 26 down to next 10. Round 487 up to next 100. $20 \times 500 = 10,000$
2. Round 26 up to next 10. Round 487 down to next 100. $30 \times 400 = 12,000$	4. Round 26 down to next 10. Round 487 down to next 100. $20 \times 400 = 8000$

Since $26 \times 487 = 12,662$, our estimate was best when we rounded the greater factor down and the other factor up (method 2).

EXERCISES

Make 4 estimates for each of the products, then compare each estimate with the exact product to see which estimate is closest.

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 1. 34×273 | 3. 56×778 | 5. 85×355 | 7. 276×53 |
| 2. 52×778 | 4. 56×728 | 6. 87×329 | 8. 853×56 |

Answer the questions for exercises 1–8.

9. In which exercises did method 3 give the closest estimate?
10. In which exercises did method 2 give the closest estimate?
11. Did methods 1 or 4 give the closest estimate in any exercises?

Usually when we estimate products we follow these rules:

- a. 2 digit factors are rounded to the nearest 10.
- b. 3 digit factors are rounded to the nearest 100.

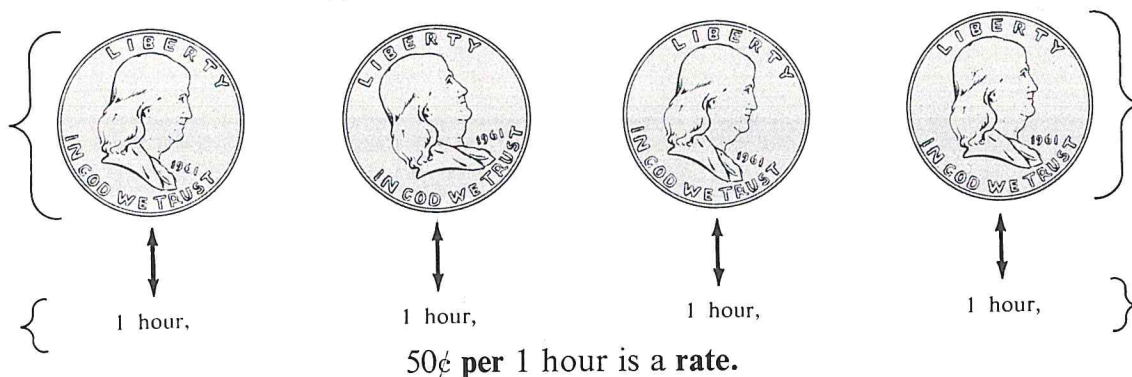
For example, 53×382 is estimated as 50×400 .

Use the rules above to estimate the products.

- | | | | |
|--------------------|---------------------|----------------------|----------------------|
| 12. 54×86 | 14. 46×245 | 16. 256×789 | 18. 195×432 |
| 13. 76×37 | 15. 76×383 | 17. 324×961 | 19. 843×527 |

Rates

Rita earns 50¢ an hour baby sitting.
She thinks, “For every hour I baby sit
I shall earn 50¢. So for 4 hours
I shall earn 4 sets of 50¢, that is \$2.”



EXERCISES

Complete the short sentences.

1. 75¢ per hour: a ¢ for 4 hours; b ¢ for 7 hours
2. 25 pennies per quarter: c pennies per 7 quarters; d pennies per 16 quarters
3. 25 miles per hour: e miles in 6 hours; f miles in 14 hours
4. 24 hours per day: g hours in 4 days; h hours in 7 days
5. 16 ounces per pound: i ounces per 9 pounds; j ounces per 23 pounds
6. If a doughnut costs 15 cents, what is the cost of 6 doughnuts?
7. The Walker family traveled 300 miles in 6 hours. About how many miles per hour did they travel?
8. There are 5280 feet per mile. How many feet above the ocean is a plane if it is 6 miles up?
9. The admission price for the movie is 65¢ per person. If 267 people attend, what is the total amount of money collected?
10. There are 16 ounces per pound and 2000 pounds per ton. How many ounces per ton?



If you earn 50¢ per hour, you will earn $(50 \times m)$ ¢ in m hours. We can write this as a function rule.

$$\begin{aligned}\text{Number of hours} &= m \\ \text{Total number of cents} &= f(m) \\ 50 \times m &= f(m)\end{aligned}$$

EXERCISES

Given the rate and the function rule, name the number pairs.

1. **The rate:** 40¢ per hour. **The rule:** $40 \times m = f(m)$

number of hours: m	1	2	3	4	5	6	7	8
number of cents: $f(m)$	40	80	a	c	e	n	r	s

2. **The rate:** \$6 per day. **The rule:** $6 \times m = f(m)$

number of days: m	1	2	3	4	6	7	12	25
number of dollars: $f(m)$	6	a	c	e	n	r	s	u

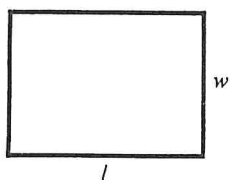
3. **The rate:** 5 boys per car. **The rule:** $5 \times m = f(m)$

number of cars: m	2	4	7	9	15	23	36	49
number of boys: $f(m)$	a	c	e	n	r	s	u	v

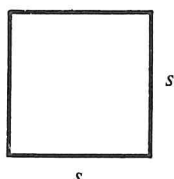
Write the function rule, then name the quantities.

- 6 horses per truck: a horses in 7 trucks; b horses in 29 trucks
- 12 balls per box: c balls in 15 boxes; d balls in 37 boxes
- 3 feet per yard: e feet in 5 yards; f feet in 7 yards
- 8 pints per gallon: g pints in 19 gallons; h pints in 37 gallons
- 30 children per room: i children in 6 rooms; j children in 16 rooms
- 12 cents per candy bar: k cents for 8 candy bars; l cents for 24 candy bars
- 14 children per table: m children at 7 tables; n children at 13 tables

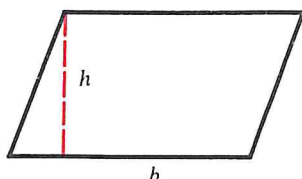
Area functions



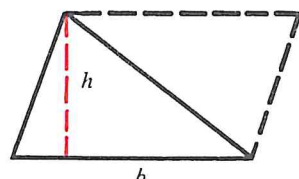
$$l \times w = A$$



$$s^2 = A$$



$$b \times h = A$$



$$\frac{1}{2} \times b \times h = A$$

l , w , s , b , and h name the measures of the sides and the height. A is the total number of units of area.

EXERCISES

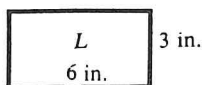


figure 1

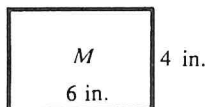


figure 2

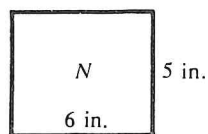


figure 3

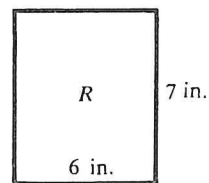
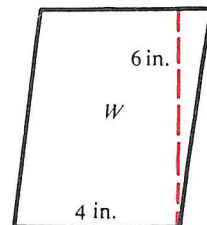
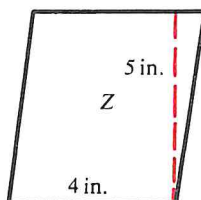
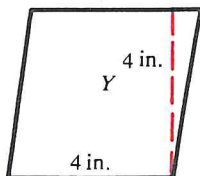
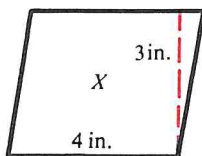


figure 4

In each of these rectangles, $l = 6$. For their areas we may write the function rule: $6 \times w = A$. Use the rule to name the answers.

- Complete: $\{(3, a), (4, c), (5, e), (7, n)\}$.
- Name the areas of L, M, N, and R (figures 1, 2, 3, and 4).

Study these parallelograms, then use the function rule $b \times h = A$ to answer the questions.



- Using $b = 4$, write a function rule for the number of units of area in the above parallelograms.
- Use the function rule in exercise 3 to complete the ordered pairs: $\{(3, a), (4, c), (5, e), (6, n)\}$.
- Name the areas in X, Y, Z, and W.

Use the function rule for a square ($s^2 = A$) to complete the ordered pairs

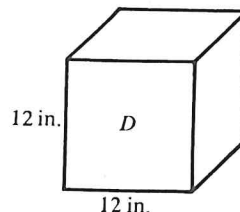
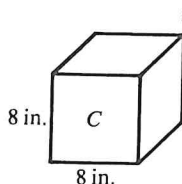
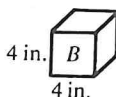
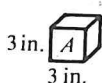
- $\{(2, 4), (3, 9), (4, a), (5, c), (6, e), (7, n), (8, r), (9, s)\}$.

Volume and surface area functions

The equation for the number of units of volume in a cube is $s^3 = V$. This is a function equation where s names the measure of each edge of the cube.

The total area of all the faces of a cube is called the **surface area** of the cube.

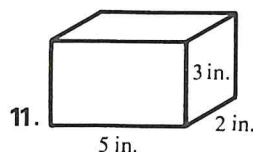
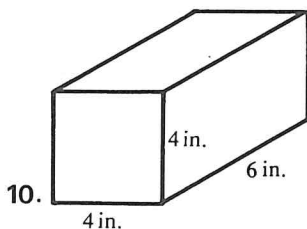
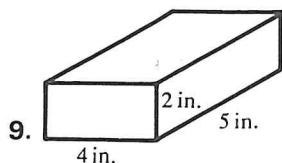
EXERCISES



Use each of the cubes to answer the questions.

1. For each cube write the set of ordered pairs where the first member of the pair is a replacement for s , and the second member is the replacement for V .
2. Name the number of faces in a cube.
3. Name the figure formed by each face. Are all the faces of a cube congruent?
4. Name the area for a face of each cube.
5. Do congruent squares have the same area?
6. Name the surface area of each cube above.
7. Write a function equation for the surface area of a cube if T is the number of units in the surface area and A is the number of units of area of one face.
- **8. Write a function equation for the surface area of a cube if T is the number of units in the surface area and s is the measure of one edge.

Name the volume and surface area for each rectangular prism.



Division

Study these divisions.

A

$$\begin{array}{r|l}
 4 \overline{)5349} & 1000 \\
 \underline{4000} & \\
 1349 & \\
 \underline{1200} & 300 \\
 149 & \\
 \underline{120} & 30 \\
 29 & \\
 \underline{28} & 7 \\
 1 & 1337
 \end{array}$$

$$Q = 1337, R = 1$$

B

$$\begin{array}{r}
 1337 \\
 4 \overline{)5349} \\
 \underline{4000} \\
 1349 \\
 \underline{1200} \\
 149 \\
 \underline{120} \\
 29 \\
 \underline{28} \\
 1
 \end{array}$$

$$Q = 1337, R = 1$$

$$\begin{array}{r}
 9337 \\
 4 \overline{)37349} \\
 \underline{36000} \\
 1349 \\
 \underline{1200} \\
 149 \\
 \underline{120} \\
 29 \\
 \underline{28} \\
 1
 \end{array}$$

$$Q = 9337, R = 1$$

EXERCISES

Use method A to name the quotients and remainders.

1. $3 \overline{)4156}$

4. $4 \overline{)5382}$

7. $3 \overline{)5630}$

10. $4 \overline{)6521}$

2. $5 \overline{)6403}$

5. $6 \overline{)6785}$

8. $9 \overline{)9876}$

11. $5 \overline{)7826}$

3. $7 \overline{)9654}$

6. $8 \overline{)9951}$

9. $2 \overline{)3047}$

12. $6 \overline{)8376}$

13–25. Use method B to name the quotients and remainders in exercises 1–12.

Name the quotients and remainders.

26. $2 \overline{)14396}$

29. $5 \overline{)17986}$

32. $7 \overline{)45165}$

35. $6 \overline{)49739}$

27. $3 \overline{)21592}$

30. $4 \overline{)38275}$

33. $5 \overline{)28014}$

36. $9 \overline{)78392}$

28. $4 \overline{)16507}$

31. $3 \overline{)29318}$

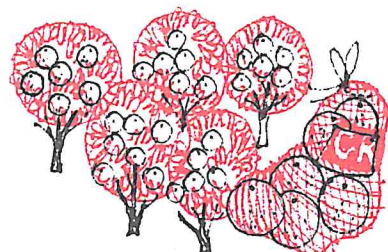
34. $8 \overline{)59161}$

37. $7 \overline{)50985}$

Solve the problems.

38. How many bags of 6 grapefruit can be made from 3827 grapefruit?

39. How many boxes of 8 candy bars can be made from 53,205 candy bars?





To estimate the tens' digit in the quotient in the division $34 \overline{)1663}$, we may use this method.

Step 1: $10 \times 34 = 340$, $100 \times 34 = 3400$

Step 2: 1663 is between 340 and 3400 so the quotient is between 10 and 100.

Step 3: Round the divisor 34 up to 40 and the dividend 1663 down to 1600.

Step 4: $1600 \div 40 = 40$ (This checks with step 2, 40 is between 10 and 100.)

Our estimated quotient is 40.

To check our estimate, we see that

$40 \times 34 = 1360$, $1663 - 1360 = 303$ and $303 < 34$ tens.

We cannot subtract another 10×34 , so our estimate of 4 tens in the quotient is correct.

EXERCISES

Estimate the quotients.

1. $63 \overline{)4892}$

5. $42 \overline{)2962}$

9. $31 \overline{)1754}$

13. $42 \overline{)2436}$

2. $51 \overline{)3316}$

6. $52 \overline{)4902}$

10. $33 \overline{)7348}$

14. $73 \overline{)8157}$

3. $43 \overline{)3867}$

7. $54 \overline{)2823}$

11. $64 \overline{)5162}$

15. $41 \overline{)2536}$

4. $63 \overline{)2827}$

8. $84 \overline{)7032}$

12. $62 \overline{)4124}$

16. $32 \overline{)2516}$

17–32. Name the quotients in exercises 1–16 using your estimates to help you.

Name the quotients and remainders.

33. $47 \overline{)3477}$

37. $58 \overline{)4461}$

41. $35 \overline{)1609}$

45. $28 \overline{)1086}$

34. $27 \overline{)1069}$

38. $38 \overline{)2786}$

42. $49 \overline{)3081}$

46. $68 \overline{)5128}$

35. $36 \overline{)1264}$

39. $53 \overline{)1892}$

43. $38 \overline{)1312}$

47. $49 \overline{)1612}$

36. $84 \overline{)2334}$

40. $37 \overline{)1979}$

44. $73 \overline{)6265}$

48. $69 \overline{)3338}$

Division

Study the division $49 \overline{)6489}$.

Step 1: Estimate: $49 \times 100 = 4900$, $49 \times 1000 = 49,000$
 6489 is between 4900 and $49,000$
 The quotient is between 100 and 1000 .
 $6000 \div 50$ is about 100 . We estimate the first quotient as **100**.

$$\begin{array}{r} 49 \overline{)6489} \\ \underline{4900} \text{100} \\ 1589 \end{array}$$

Multiply: $100 \times 49 = 4900$

Subtract: $6489 - 4900 = 1589$

Check: $1589 < 49$ hundreds (This means we cannot subtract another 100×49 and we are ready to work with tens.)

Step 2: Estimate: $49 \times 10 = 490$, $49 \times 100 = 4900$
 1589 is between 490 and 4900
 The quotient is between 10 and 100 .
 $1500 \div 50 = 30$. We estimate the second quotient as **30**.

$$\begin{array}{r} 49 \overline{)6489} \\ \underline{4900} 100 \\ 1589 \\ \underline{1470} \text{30} \\ 119 \end{array}$$

Multiply: $30 \times 49 = 1470$

Subtract: $1589 - 1470 = 119$

Check: $119 < 49$ tens

Step 3: Estimate: $2 \times 50 = 100$. We estimate the third quotient as **2**

Multiply: $2 \times 49 = 98$

Subtract: $119 - 98 = 21$

Check: $21 < 49$, so 21 is the remainder.

$$\begin{array}{r} 49 \overline{)6489} \\ \underline{4900} 100 \\ 1589 \\ \underline{1470} 30 \\ 119 \\ \underline{98} \text{2} \\ 21 132 \end{array}$$

The quotient is 132 and the remainder is 21 .

EXERCISES

Name the quotients and remainders.

1. $58 \overline{)6726}$

6. $77 \overline{)8986}$

11. $73 \overline{)9651}$

16. $28 \overline{)2195}$

2. $58 \overline{)7452}$

7. $39 \overline{)4825}$

12. $38 \overline{)8346}$

17. $87 \overline{)5916}$

3. $29 \overline{)3632}$

8. $56 \overline{)6709}$

13. $46 \overline{)2275}$

18. $63 \overline{)4996}$

4. $45 \overline{)5785}$

9. $84 \overline{)9055}$

14. $35 \overline{)1968}$

19. $57 \overline{)5583}$

5. $62 \overline{)9264}$

10. $33 \overline{)7416}$

15. $92 \overline{)9875}$

20. $79 \overline{)4572}$

Compare the examples.

A	B	check
$\begin{array}{r l} 49 \overline{)6489} & \\ \hline 4900 & 100 \\ \hline 1589 & \\ 1470 & 30 \\ \hline 119 & \\ 98 & 2 \\ \hline 21 & 132 \end{array}$	$\begin{array}{r} 132 \\ 49 \overline{)6489} \\ \hline 4900 \\ \hline 1589 \\ \hline 1470 \\ \hline 119 \\ \hline 98 \\ \hline 21 \end{array}$	$\begin{array}{l} 100 \times 49 = 4900 \\ 30 \times 49 = 1470 \\ 2 \times 49 = 98 \\ \hline 132 \times 49 = 6468 \\ 6489 - 6468 = 21 \end{array}$

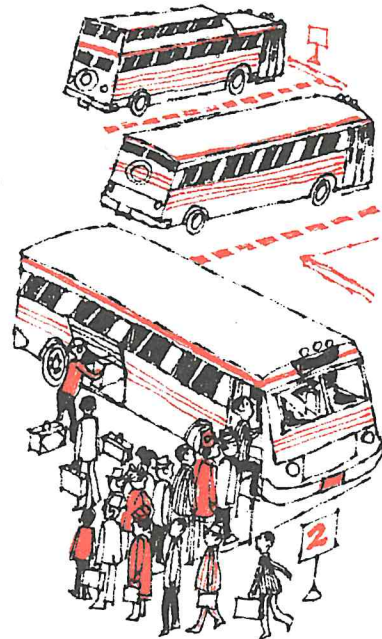
EXERCISES

Name the quotients and remainders using method B, then check each example.

- | | | | |
|--------------------------|--------------------------|--------------------------|---------------------------|
| 1. $27 \overline{)4642}$ | 4. $65 \overline{)2861}$ | 7. $78 \overline{)6634}$ | 10. $94 \overline{)2499}$ |
| 2. $16 \overline{)1448}$ | 5. $54 \overline{)4265}$ | 8. $36 \overline{)3428}$ | 11. $89 \overline{)5488}$ |
| 3. $42 \overline{)6537}$ | 6. $19 \overline{)2528}$ | 9. $83 \overline{)6027}$ | 12. $98 \overline{)8973}$ |

Solve the problems.

13. A bus makes six 70 mile round trips between Midville and Center Town every day.
 - a. How many miles does the bus travel per day?
 - b. How many miles does the bus travel per week?
 - c. If the bus must be inspected every 3000 miles, about how many trips can it make between inspections?
14. In 6 days a bus carried 3096 passengers. If the bus carried the same number of passengers each trip and makes 6 trips a day, how many people made each trip?
15. A bus travels 2359 miles in 7 days. How far does it travel each day if it travels the same distance each day?



Sets and numbers

Compare these two problems :

1. If 23 overcoats together cost \$3749, what is the cost of one overcoat if each overcoat costs the same?
2. If one overcoat costs \$23, how many overcoats of the same price can be bought for \$3749?

Our equation in either problem is $3749 \div 23 = n$.
The solution is 163. In problem 1, our answer is \$163;
in problem 2, it is 163 overcoats.

We must be careful when we fit the solution of our equation into the sets in our problem.

EXERCISES

It costs Mr. Roberts \$85.44 a week for milk.

1. If he buys 96 gallons a week, what is the cost of 1 gallon?
2. If the cost of 1 gallon is 96¢, how many gallons does he buy?

It cost the students \$83.20 to go to the fair.

3. If 65 students went to the fair, how much did it cost each student?
4. If it cost 65¢ for each student to go to the fair, how many students went to the fair?

Name the quotient $6608 \div 28$, then answer the questions.

5. If 28 model airplanes cost \$66.08, what is the cost of 1 airplane?
6. If 28 machines produce 6608 parts in a day, then what does one machine produce in a day?
7. How many record players, at \$28 each, can be bought for \$6608?
8. How many records, on sale for 28¢ each, can be bought for \$66.08?
- **9. How many albums, on sale for \$2.80 each, can be bought for \$660.80?

To name the quotient for the division $\$55.30 \div 35$, we notice that $55 \div 35$ is a little greater than 1, so our quotient will be a little greater than \$1. The division shows that $5530 \div 35 = 158$, so our answer is \$1.58.

$$\begin{array}{r} 158 \\ 35 \overline{) 5530} \\ \underline{3500} \\ 2030 \\ \underline{1750} \\ 280 \\ \underline{280} \\ 0 \end{array}$$

EXERCISES

Name the quotients.

- | | | |
|--------------------------|---------------------------|---------------------------|
| 1. $\$65.32 \div 46 = a$ | 6. $\$97.35 \div 59 = f$ | 11. $\$26.25 \div 25 = k$ |
| 2. $\$70.68 \div 62 = b$ | 7. $\$25.84 \div 34 = g$ | 12. $\$79.38 \div 42 = l$ |
| 3. $\$66.15 \div 49 = c$ | 8. $\$81.22 \div 31 = h$ | 13. $\$85.02 \div 78 = m$ |
| 4. $\$41.36 \div 22 = d$ | 9. $\$96.60 \div 92 = i$ | 14. $\$78.44 \div 74 = n$ |
| 5. $\$66.24 \div 46 = e$ | 10. $\$90.71 \div 47 = j$ | 15. $\$93.15 \div 27 = p$ |

Solve the problems.

- On Saturday night Mr. Smiley collected \$45.12 from 96 customers. How much would this be per customer?
- On Friday afternoon Mr. Smiley had a special sale of Super Duper Triple Scooper sundaes for 29¢. If 68 people bought these sundaes, how much did he collect from the special sale?
- If Mr. Smiley collected \$7.25 from the sale of his 29¢ Super Duper Triple Scooper sundaes, how many sundaes did he sell?
- Mr. Smiley paid Mary, his helper, \$60.80 for 38 hours of work. How much does Mary earn per hour?
- Fifty-eight people each gave \$2.50 for the music fund. How much was given in all? If \$200 was needed for the fund before these gifts, how much more was needed after the gifts?



Extending division

Study this example.

$$\begin{array}{r}
 \text{A} \\
 57 \overline{)35482} \\
 \underline{34200} \quad 600 \\
 1282 \\
 \underline{1140} \quad 20 \\
 142 \\
 \underline{114} \quad 2 \\
 28 \quad 622
 \end{array}$$

$$\begin{array}{r}
 \text{B} \\
 622 \\
 57 \overline{)35482} \\
 \underline{34200} \\
 1282 \\
 \underline{1140} \\
 142 \\
 \underline{114} \\
 28
 \end{array}$$

$$\begin{array}{l}
 \text{check} \\
 600 \times 57 = 34200 \\
 20 \times 57 = 1140 \\
 2 \times 57 = 114 \\
 622 \times 57 = 35454 \\
 35,482 - 35,454 = 28
 \end{array}$$

EXERCISES

Name the quotients and remainders.

$$\begin{array}{r}
 1. \quad 37 \overline{)21264} \\
 \underline{18500} \quad a \\
 2764 \\
 \underline{2590} \quad m \\
 174 \\
 \underline{148} \quad n \\
 e \quad c
 \end{array}$$

$$\begin{array}{r}
 2. \quad 48 \overline{)98372} \\
 \underline{96000} \quad s \\
 2372 \\
 \underline{1920} \quad r \\
 452 \\
 \underline{432} \quad u \\
 v \quad x
 \end{array}$$

$$\begin{array}{r}
 3. \quad 27 \overline{)89273} \\
 \underline{81000} \quad m \\
 8273 \\
 \underline{8100} \quad a \\
 173 \\
 \underline{162} \quad r \\
 n \quad s
 \end{array}$$

Name the quotients and remainders using method A.

$$4. \quad 39 \overline{)23264}$$

$$7. \quad 49 \overline{)33498}$$

$$10. \quad 68 \overline{)23678}$$

$$13. \quad 78 \overline{)49965}$$

$$5. \quad 59 \overline{)32782}$$

$$8. \quad 27 \overline{)25685}$$

$$11. \quad 83 \overline{)19354}$$

$$14. \quad 26 \overline{)37463}$$

$$6. \quad 43 \overline{)26256}$$

$$9. \quad 65 \overline{)50419}$$

$$12. \quad 96 \overline{)78708}$$

$$15. \quad 25 \overline{)20358}$$

Name the quotients and remainders using method B.

$$16. \quad 27 \overline{)17392}$$

$$19. \quad 38 \overline{)13195}$$

$$22. \quad 42 \overline{)21356}$$

$$25. \quad 97 \overline{)42516}$$

$$17. \quad 59 \overline{)26981}$$

$$20. \quad 67 \overline{)42398}$$

$$23. \quad 79 \overline{)41618}$$

$$26. \quad 23 \overline{)18265}$$

$$18. \quad 48 \overline{)37921}$$

$$21. \quad 88 \overline{)27189}$$

$$24. \quad 45 \overline{)80190}$$

$$27. \quad 27 \overline{)96448}$$

28–39. Check your answers to exercises 16–27.

Name the products.

1.
$$\begin{array}{r} 56 \\ \times 9 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 97 \\ \times 13 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 5643 \\ \times 29 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 147 \\ \times 304 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 3167 \\ \times 105 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 387 \\ \times 8 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 82 \\ \times 37 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 4187 \\ \times 38 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 698 \\ \times 574 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 2894 \\ \times 267 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 5679 \\ \times 5 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 98 \\ \times 67 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 6905 \\ \times 74 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 332 \\ \times 111 \\ \hline \end{array}$$

23.
$$\begin{array}{r} 5837 \\ \times 391 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 48231 \\ \times 7 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 354 \\ \times 55 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 56321 \\ \times 56 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 584 \\ \times 623 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 64903 \\ \times 467 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 690837 \\ \times 6 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 897 \\ \times 48 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 49376 \\ \times 43 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 790 \\ \times 216 \\ \hline \end{array}$$

25.
$$\begin{array}{r} 510200 \\ \times 518 \\ \hline \end{array}$$

Name the quotients and remainders.

26. $9 \overline{)584}$

30. $3 \overline{)5672}$

34. $46 \overline{)5752}$

38. $95 \overline{)34855}$

27. $25 \overline{)467}$

31. $6 \overline{)53201}$

35. $36 \overline{)8928}$

39. $73 \overline{)56049}$

28. $17 \overline{)977}$

32. $4 \overline{)35723}$

36. $25 \overline{)8437}$

40. $64 \overline{)22208}$

29. $95 \overline{)760}$

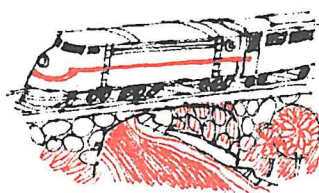
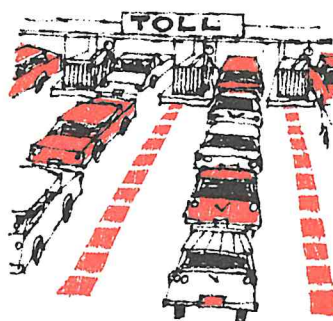
33. $8 \overline{)59672}$

37. $43 \overline{)9872}$

41. $26 \overline{)61048}$

Solve the problems.

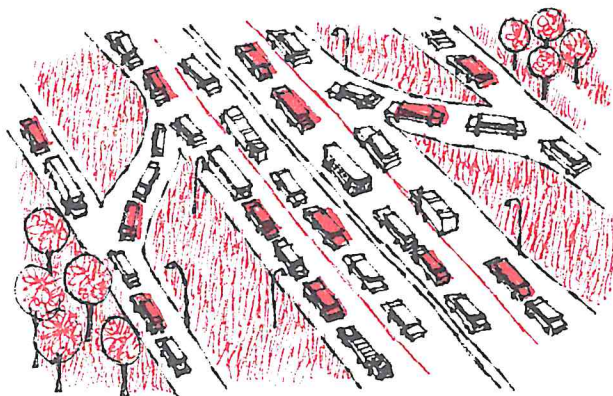
42. If 70 cars pass through the highway toll booth every hour, how long will it take for 1750 cars to pass through? How long will it take for 3640 cars to pass through the toll booth?
43. If each car pays 25¢ at the toll booth and \$18.75 is collected in 10 minutes, how many cars pass through the toll booth during those 10 minutes?
44. An express train travels 73 miles an hour. Estimate how long it will take the train to travel 1685 miles at that speed.
45. If the express train travels 65 miles an hour, estimate how long it will take to travel 1650 miles?



Data-range

year	vehicles (nearest million)
1930	26,000,000
1940	32,000,000
1950	49,000,000
1960	73,000,000

The number of vehicles registered for use on United States highways is shown as **data** in the above table.



The greatest number is 73,000,000 and the least number is 26,000,000. The **range** is $73 - 26 = 47$ millions.

EXERCISES

Solve the problems.

- From Monday through Friday, the following number of students were present in Mrs. Jackson's sixth grade class: 27, 29, 30, 28, 31. Show this data in a table and name the range.
- In 9 innings, a baseball pitcher threw the following number of pitches: 10, 9, 12, 6, 15, 8, 8, 4, 7. Show this data in a table and name the range.
- From 7:00 a.m. to 2:00 p.m., the following number of cars crossed over the Washington Bridge each hour: 3427, 3976, 2998, 2708, 4162, 4002, 3856, 1691. Show this data in a table and name the range.



Name the range for the data on the charts.

Central School Absences

4.

day (November)	16	17	18	19	20
number of pupils absent	26	37	43	39	25

Library Visitors

5.

day	20	21	22	23	24	25	26
number of customers	195	217	166	173	228	254	202

The measures of the height of the students in Pat's row are numbers. The **average** of these numbers tells us what the heights would be if they were all the same and the total of the heights didn't change.

$$305 \div 5 = 61$$

Average height is 61 inches.

student	height (in.)
Pat	60
Don	63
Jim	66
Sue	59
Eve	57
total	305

EXERCISES

Name the range and average for each set of data.

- 6', 8', 12', 5', 15', 8'.
- 140 lbs., 160 lbs., 180 lbs.
- 68 yds., 43 yds., 28 yds., 25 yds.
- 75 points, 85 points, 95 points.

Study the table, then

- name the *total* number of customers for the week.
- name the *total* amount of money collected for the week.
- name the *average* number of customers per day.
- name the *average* amount of money collected per day.
- name the *range* for the number of customers.
- name the *range* for the amount of money collected.

Smiley's Sundae Shop
November 14–20.

date	number of customers	amount collected
14	184	\$64.40
15	173	\$55.36
16	158	\$50.95
17	179	\$65.18
18	204	\$89.87
19	245	\$108.79
20	215	\$98.50

Name the range and average for the sets of data.

- 68, 25, 36, 42, 603, 16, 8, 18
- \$5500, \$3950, \$6250, \$4775, \$126,400
- *13. Why is the average not a good number to describe each of these sets of data?



Median and mode

An average is a number which may be used to describe a set of data. Two other such numbers are the **mode** and the **median**. The **mode** is the number which occurs most often in a set of data. The **median** is the number which comes in the middle of the data when the members are arranged in order. If the number of numbers in the data is even, then the median is the average of the 2 middle numbers.

15, 17, 17, 17, 18, 21, 21, 26, 28

The *average* is 20.

The *mode* is 17.

The *median* is 18

13, 18, 22, 28, 35, 46

The *average* is 27.

There is no *mode*.

The *median* is $\frac{22 + 28}{2} = 25$

EXERCISES

Name the average, median, mode, and range for each set of data.

1. {42, 42, 45, 52, 64}

5. {205, 222, 226, 230, 243, 243, 262}

2. {95, 87, 87, 87, 63, 49}

6. {30, 20, 25}

3. {29, 24, 24, 24, 19, 19, 17, 17, 16, 11}

7. {55, 43, 27, 86, 43, 38, 76, 19, 88,

4. {15, 15, 15}

65, 43}

For each set of data decide if the average, median, or mode best describes the data.

8. 5196

9. 1512

10. 400

11. 5829

4485

1512

400

345

3679

1512

325

262

225

1512

325

169

225

1512

325

110

1512

250

9072

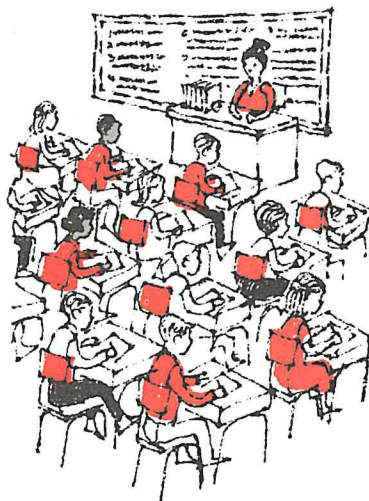
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Solve the problems.

12. If the average number of people that attended 8 football games is 3872, how many people attended all together?

13. If the range of a set of test scores is 38 and the lowest score is 57, name the highest score.

1. Washington, D. C. has an area of 61 square miles. In 1960 the population of the city was 763,964. What was the average number of people per square mile?
2. The high temperatures for 7 days were 63, 72, 82, 76, 70, 62, 58. Name the
 - a. average,
 - b. median,
 - c. range.
3. George is 5 feet tall. His father told him that the average depth of the river is 4 feet. Should George wade across the river?
4. The weights of the people in Ken's row are 136, 153, 168, 147, and 126. What is the
 - a. average weight?
 - b. median weight?
5. The average class size for 18 classes at Central School is 28. The average class size for 23 classes at North School is 26. How many students attend
 - a. Central School?
 - b. North School?
6. Mary scored 90, 87, 85, 73, 87, and 94 on her math tests. What was her
 - a. average score?
 - b. mode score?
 - c. range?
 - d. median score?
7. Miss Anderson's sixth grade mathematics class took a test. There were 2 persons who scored 100, 3 who scored 92, 4 who scored 84, 7 who scored 80, 6 who scored 76, 7 who scored 72, and 1 who scored 68. What was
 - a. the range?
 - *b. the median score?
 - c. the average score?



Squaring

In chapter one we saw that 2^2 means 2×2 . We call 2^2 , the **second power of 2**. We also call it the square of 2, or **2 squared**. If a square has sides of length 2 inches, the measure of its area, in square inches, is $2 \times 2 = 2^2 = 2 \text{ squared} = 4$.

EXERCISES

Name the products (squares).

- | | | | |
|--------------|---------------|-----------|----------------|
| 1. 3^2 | 4. 1^2 | 7. 14^2 | 10. 26 squared |
| 2. 4 squared | 5. 9^2 | 8. 16^2 | 11. 40^2 |
| 3. 7^2 | 6. 10 squared | 9. 19^2 | 12. 500^2 |

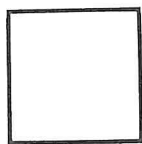
Name the areas of the square regions using the measures given.

13.



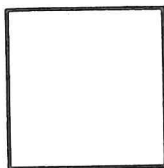
4 in.

14.



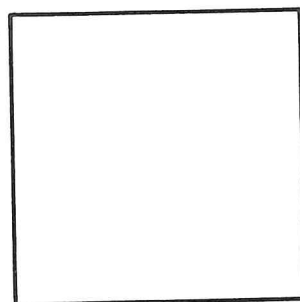
11 in.

15.



13 in.

16.



24 in.

Do you remember that the **Theorem of Pythagoras** says that if a , b , and c are the measures of the 3 sides of a right triangle and c is the side opposite the right angle, then $c^2 = a^2 + b^2$?

Use the Theorem of Pythagoras to check whether triangles with the following sides are right triangles.

- | | | |
|---------------|----------------|----------------|
| 17. 6, 8, 10 | 19. 11, 14, 18 | 21. 26, 27, 39 |
| 18. 5, 12, 13 | 20. 7, 24, 25 | 22. 11, 60, 61 |

If a square has sides of length 9' name the area

- 23. in square feet,
- 24. in square yards,
- 25. in square inches.

Naming the square root

Naming the square root is the opposite of squaring.

The sign $\sqrt{\quad}$ means “the square root of.”

$$6^2 = 36$$

$$\sqrt{36} = 6$$

EXERCISES

Solve the equations.

1. $16^2 = 256$, so $\sqrt{256} = a$

4. $69^2 = 4761$, so $\sqrt{4761} = d$

2. $20^2 = 400$, so $\sqrt{400} = b$

5. $85^2 = 7225$, so $\sqrt{7225} = e$

3. $34^2 = 1156$, so $\sqrt{1156} = c$

6. $127^2 = 16129$, so $\sqrt{16129} = f$

Name the square roots.

7. $\sqrt{4}$

10. $\sqrt{36}$

13. $\sqrt{64}$

16. $\sqrt{121}$

*19. $\sqrt{10000}$

8. $\sqrt{9}$

11. $\sqrt{49}$

14. $\sqrt{81}$

17. $\sqrt{144}$

*20. $\sqrt{4900}$

9. $\sqrt{25}$

12. $\sqrt{16}$

15. $\sqrt{100}$

18. $\sqrt{900}$

*21. $\sqrt{12100}$

Name the length of the sides of a square if the area is

22. 64 square feet

23. 100 square yards

24. 225 square inches

Solve the equations.

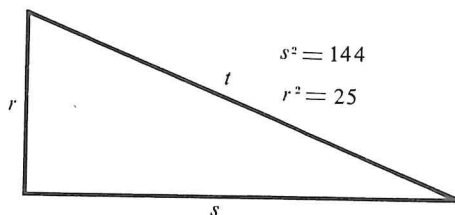
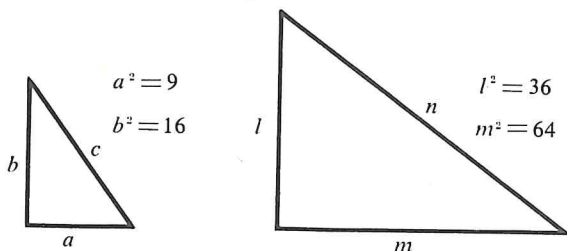
25. $13 \times a = 169$

27. $32 \times c = 1024$

26. $24 \times b = 576$

28. $50 \times d = 2500$

Use the Theorem of Pythagoras to help you name the measures of the sides of the right triangles. The first one is done for you.



29. $c^2 = 9 + 16 = 25$ 30. $n^2 = ?$ 32. $m = ?$ 34. $t^2 = ?$ 36. $r = ?$
 $a = 3, b = 4, c = 5$ 31. $l = ?$ 33. $n = ?$ 35. $s = ?$ 37. $t = ?$

Naming square roots

If m is the square root of 529, we know that when 529 is divided by m , the quotient is m .

$$m \times m = 529 \qquad m \overline{)529}^m$$

To name the square root of 529, we can use the idea that the divisor and the quotient must be equal as here:

- Step 1:* **Estimate** the square root. Let us estimate 20.
Step 2: **Divide** 529 by 20. The quotient is 26. (We shall ignore the remainder).
Step 3: **Average** 20 and 26. $(20 + 26) \div 2 = 23$.
Step 4: **Divide** 529 by 23. $529 \div 23 = 23$.
 23 is the square root of 529.

In some problems, the division and the quotient in step 4 will not be equal the *first time* this method is used. If this happens, keep repeating the method until two equal factors are obtained.

EXERCISES

Use the averaging method to name the square roots. The first estimated quotient is named in each case.

- | | | |
|----------------------|----------------------|------------------------|
| 1. $\sqrt{576}$ (20) | 5. $\sqrt{625}$ (30) | 9. $\sqrt{841}$ (30) |
| 2. $\sqrt{324}$ (16) | 6. $\sqrt{529}$ (20) | 10. $\sqrt{2601}$ (50) |
| 3. $\sqrt{81}$ (8) | 7. $\sqrt{441}$ (20) | 11. $\sqrt{961}$ (30) |
| 4. $\sqrt{169}$ (11) | 8. $\sqrt{784}$ (30) | 12. $\sqrt{1681}$ (40) |

Use the averaging method to name the square roots.

- | | | |
|------------------|------------------|-------------------|
| 13. $\sqrt{361}$ | 17. $\sqrt{225}$ | 21. $\sqrt{1296}$ |
| 14. $\sqrt{484}$ | 18. $\sqrt{676}$ | 22. $\sqrt{1024}$ |
| 15. $\sqrt{729}$ | 19. $\sqrt{289}$ | 23. $\sqrt{1369}$ |
| 16. $\sqrt{196}$ | 20. $\sqrt{841}$ | 24. $\sqrt{2809}$ |



Answer the questions in A as quickly as you can.
Then see if you can answer the questions in B even more quickly.

A

1. Name the difference between 10,000 and 3456.
2. Name the cardinal number of the cross product of a set of 15 and a set of 25.
3. $9^2 + 11^2 = a$
4. Name the average of 25, 30, and 35.
5. Name the area in a triangle with base of 8'' and height of 13''.
6. How many inches will match 12'?
7. Name the perimeter of a parallelogram with sides of 24'' and 31''.
8. $18 \times 22 = a$
9. $96 \times 24 = 2304$, so $9600 \times 240 = a$
10. How much greater is the area of a square with sides 12'' than a square with sides 6''?

B

1. Which is greater, $6^2 + 8^2$ or $5^2 + 9^2$?
2. How many inches will match 3 yards?
3. Name the sum of 1234, 2341, 2412, and 4123.
4. If Jane earns 75¢ per hour, how much will she earn in 6 hours?
5. Name the quotient $5000 \div 25$.
6. If it takes 7 minutes to bake 7 cookies, how many minutes will it take to bake 35 cookies?
7. Mary is 56'' tall. John is 8'' taller. Is John over 5' tall?
8. $4321 - 1234 = a$.
9. $(3 \times 17) + (3 \times 13) = a$.
10. Name the product:
 $5 \times 9 \times 64 \times 0 \times 18 = a$

CHECKING UP

Study the equations, then complete the ordered pairs.

- $x + 15 = f(x)$: $\{(2, 17), (13, a), (24, b), (31, c), (46, d), (59, e)\}$
- $x - 19 = f(x)$: $\{25, f), (19, g), (37, h), (33, i), (56, j), (65, k)\}$

For a cube with sides 12" long, name

- the volume.
- the surface area.

Name the products.

- | | | | | | |
|---|--|--|--|---|--|
| 5. $\begin{array}{r} 57 \\ \times 84 \\ \hline \end{array}$ | 6. $\begin{array}{r} 438 \\ \times 16 \\ \hline \end{array}$ | 7. $\begin{array}{r} 784 \\ \times 38 \\ \hline \end{array}$ | 8. $\begin{array}{r} 436 \\ \times 95 \\ \hline \end{array}$ | 9. $\begin{array}{r} 3529 \\ \times 53 \\ \hline \end{array}$ | 10. $\begin{array}{r} 8908 \\ \times 67 \\ \hline \end{array}$ |
|---|--|--|--|---|--|

Name the quotients and remainders.

- | | | | |
|---------------------------|---------------------------|---------------------------|----------------------------|
| 11. $7 \overline{)994}$ | 13. $47 \overline{)3218}$ | 15. $56 \overline{)9327}$ | 17. $73 \overline{)26061}$ |
| 12. $23 \overline{)1104}$ | 14. $65 \overline{)928}$ | 16. $43 \overline{)8772}$ | 18. $89 \overline{)35262}$ |

Complete the rates.

- 50 pennies per half dollar; a pennies per 9 half dollars.
- 12 inches per foot; e inches per 28 feet.
- \$1.65 per hour; n per 40 hours; r per 160 hours.

For these data, $\{39, 56, 43, 28, 39, 43, 52, 39, 48\}$, name the

- average
- median
- mode
- range

Solve the equations.

- $13^2 = a$
- $16^2 = b$
- $28^2 = c$
- $47^2 = d$
- $64^2 = e$
- $15^2 = 225$, so $\sqrt{225} = f$;
- $49^2 = 2401$, so $\sqrt{2401} = g$

Name the square roots. If necessary, use the averaging method.

- $\sqrt{64}$
- $\sqrt{100}$
- $\sqrt{900}$
- $\sqrt{169}$
- $\sqrt{4900}$



Multiplication: We may extend multiplication through the use of the distributive property and the idea of place value in our numeration system.

Name these products.

$$\begin{array}{r} 1. \quad 26847936 \\ \times \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 74862954 \\ \times \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 3864 \\ \times \quad 63 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 5942 \\ \times \quad 89 \\ \hline \end{array}$$

Division: We may extend division through the use of the distributive property. For each dividend we **estimate** the quotient, **multiply**, **subtract** the product, and **check** that the remainder is less than the divisor.

Name the quotients and remainders.

$$5. \quad 43 \overline{)2408}$$

$$7. \quad 74 \overline{)6254}$$

$$9. \quad 82 \overline{)13366}$$

$$11. \quad 38 \overline{)22156}$$

$$6. \quad 65 \overline{)1885}$$

$$8. \quad 58 \overline{)4369}$$

$$10. \quad 46 \overline{)13110}$$

$$12. \quad 94 \overline{)39452}$$

Rates: A matching such as 5 pennies per nickel is called a rate. If m is the number of nickels and $f(m)$ is the number of pennies, then this rate can be written as the function equation $5 \times m = f(m)$.

Write a function equation, then complete each rate.

13. 12 in. per foot; a in. per 6 feet; 96 in. per e feet

14. 60 miles per hour; a mi. in 5 hours; 900 mi. in e hours

Data: The **average**, **median**, **mode**, and **range** are numbers used to describe data.

15. Name the average, median, mode and range for:
 $\{58, 42, 35, 59, 47, 42, 35, 45, 42\}$



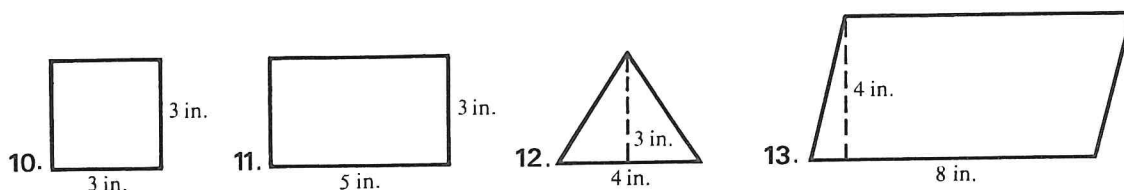
LOOKING WAY BACK

For $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and
 $B = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ name

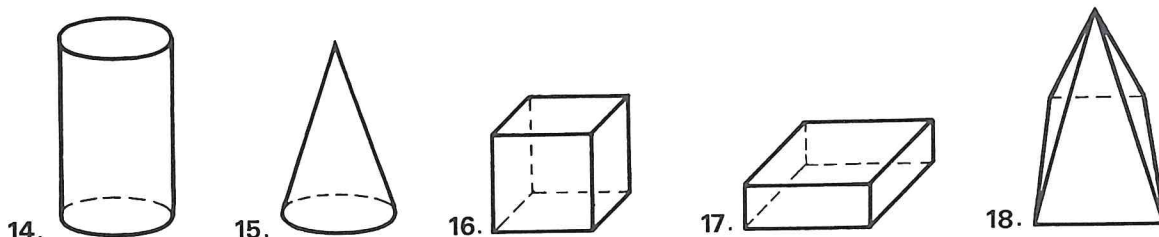
1. a universal set for A
2. a universal set for B
3. $A \cup B$
4. $A \cap B$
5. $n(A)$
6. $n(B)$
7. $n(A \cup B)$
8. $n(A \cap B)$

****9.** Name a rule to match the members of A with the members of B , write the rule as an equation, and write the set of ordered pairs.

Name the area of each polygonal region.



Name these closed surfaces.



Complete the exercises.

$$\begin{array}{r} 19. \quad 5784 \\ + 6928 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 9056 \\ - 5429 \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 6020 \\ - 4928 \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad 8002 \\ - 3508 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 6784 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 583 \\ \times 89 \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 4096 \\ \times 76 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 1216 \\ - 849 \\ \hline \end{array}$$

Complete the sentences. Replace the $\bigcirc$ with $<$, $>$ or $=$.

$$27. \quad 58 \times 39 \bigcirc 43^2$$

$$30. \quad 42^2 \bigcirc 14 \times 126$$

$$28. \quad 8^2 + 12^2 \bigcirc (8 + 12)^2$$

$$31. \quad 41^2 - 39^2 \bigcirc 2 \times 80$$

$$29. \quad 10^2 + 24^2 \bigcirc 26^2$$

$$32. \quad 71^2 - 69^2 \bigcirc 2 \times 140$$



1. From this pattern we see that we can name the square of a number if we know the square of the number one greater or one less. Can you name the rule? Use it to make a list of the squares of the numbers from 1 to 30.

$$\begin{aligned} 12^2 &= 144 \\ 13^2 &= 169 = 144 + 25 \\ 14^2 &= 196 = 169 + 27 \\ 15^2 &= 225 = 196 + 29 \\ 16^2 &= 256 = 225 + 31 \end{aligned}$$

To square a number between 10 and 100 whose ones' digit is 5, we may use a short method. For example, to square 65, we notice that:

$$\begin{aligned} 5 \times 65 &= (5 \times 60) + (5 \times 5) \\ 60 \times 65 &= (60 \times 5) + (60 \times 60) \\ \hline 65 \times 65 &= (5 \times 60) + (60 \times 5) + (60 \times 60) + (5 \times 5) \\ &= (10 \times 60) + (60 \times 60) + (5 \times 5) \\ &= (70 \times 60) + (5 \times 5) \\ &= 4200 + 25 \\ &= 4225 \end{aligned}$$

- The rule is:
1. Multiply the tens' digit by a number one greater, $7 \times 6 = 42$;
 2. Square the ones' digit, $5 \times 5 = 25$.
 3. Write the numeral from step 1, followed by the numeral from step 2, that is, 4225.

Use this idea to name the squares of

2. 75 3. 35 4. 85 5. 95

Name the products.

6. 473×72865 7. 5725×104728 8. $(987642)^2$

Check each of the first three equations to see if they are true. Then use these equations to help you name the products.

$$19 \times 21 = 20^2 - 1$$

$$18 \times 22 = 20^2 - 2^2$$

$$17 \times 23 = 20^2 - 3^2$$

9. $16 \times 24 = a$
10. $29 \times 31 = b$

11. $42 \times 38 = c$
12. $24 \times 26 = d$

13. $33 \times 37 = e$
14. $14 \times 16 = f$

Solutions and replacements

$$\begin{aligned} 3 \times n &= 15 \\ n &< 6 \\ n &> 4 \end{aligned}$$

These number sentences are **open**. We cannot tell whether they are true or false until n has been **replaced**. The set from which we choose replacements for n is called a **replacement set**.

$$\begin{aligned} 3 \times 5 &= 15 \\ 5 &< 6 \\ 5 &> 4 \end{aligned}$$

These number sentences are **closed**. The sentences are now **true**. Five is a **solution** of the open number sentences $3 \times n = 15$, $n < 6$ and $n > 4$.

An equation like $3 \times n = 15$ has only one solution. An inequality like $x < 6$ may have more than one solution. Solutions for this inequality include 0, 1, 2, 3, 4, 5. We call the set of all solutions the **solution set**.

If our replacement set is {whole numbers}, the solution set for $x < 6$ is {0, 1, 2, 3, 4, 5}. If our replacement set is {even numbers}, the solution set for $x < 6$ is {0, 2, 4}. A member of the solution set must be a member of the replacement set.

EXERCISES

Name the solution set for each of the open number sentences. In each case the replacement set is {whole numbers}.

1. $7 + 14 = m$

4. $7 + a = 14$

*7. $3 \times s < 28$

2. $3 \times 19 = n$

5. $v \div 3 = 18$

*8. $5 + u < 6$

3. $m + 1 = 8$

6. $k - 7 = 14$

*9. $5 \times v < 28$

If the replacement set is {whole numbers}, which of the open sentences are true for every replacement?

10. $a + 5 = 5 + a$

13. $1 \times n = n$

16. $4 \times a = a + 4$

11. $r + 0 = r$

14. $n \div 2 = 2 \div n$

17. $t + 5 = 7$

12. $9 + n = 9$

15. $6 \times c = 2 \times c$

18. $6 \times n = n \times 6$



Complete the sentence.

19. The solution set is a subset of the ____?____ set.

Name the solution set for each of the open sentences if the replacement set is {whole numbers}.

20. $m < 6$

24. $n + 2 = 7$

28. $5 \times a = 5$

21. $n \div 6 = 7$

25. $5 \times m < 21$

29. $t \times 0 = 0$

22. $7 \times n = 28$

26. $45 \div n = 5$

*30. $6 + n > 10$

23. $1 \times x = 52$

27. $13 \div x = 1$

**31. $c \times 0 > 0$

Name the solution set for each of the open sentences if the replacement set is $A = \{1, 3, 5, 7, 9, 11\}$.

32. $2 \times a = 14$

36. $n < 4$

40. $3 \times n < 6$

33. $2 \times n = 102$

37. $a + 10 < 14$

41. $4 \times n < 13$

34. $3 \times c = 9$

38. $e + 4 < 6$

42. $2 \times x < 5$

*35. $16 - n = 8$

*39. $16 - n < 8$

*43. $16 - n > 8$

Name the solution sets for the boxed sentences using the replacement sets below.

$n < 8$

$5 \times n < 17$

44. {whole numbers}

47. {whole numbers}

45. {odd numbers}

48. {even numbers}

46. {multiples of 3}

49. {multiples of 6}

Name the

50. odd numbers less than 8.

54. odd numbers between 10 and 20.

51. multiples of 3, less than 8.

55. even numbers between 20 and 30.

52. even numbers less than 4.

56. multiples of 10, less than 40.

53. multiples of 3, less than 4.

57. multiples of 10, less than 10.

● Equivalent equations

The solution for each of the equations in A is 7. They have the same solution set. They are **equivalent**. The equations in B are also **equivalent**: they have the same addends and sum.

A	B
$4 + n = 11$	$4 + 7 = 11$
$n + 4 = 11$	$7 + 4 = 11$
$11 - 4 = n$	$11 - 4 = 7$
$11 - n = 4$	$11 - 7 = 4$

EXERCISES

Complete the sentence.

1. The equations $3 \times 9 = 27$, $9 \times 3 = 27$, $27 \div 3 = 9$ and $27 \div 9 = 3$ are equivalent because they have the same _____ and product.

The basic addition and multiplication facts are learned in families of equivalent equations. Write the other equations which belong in the same family as

2. $3 + 9 = 12$ 4. $8 + 4 = 12$ 6. $9 + 8 = 17$ 8. $6 + 6 = 12$
3. $8 \times 7 = 56$ 5. $36 \div 2 = 18$ 7. $8 \times 9 = 72$ 9. $4 \times 4 = 16$

Use the equation on the left to help you complete the equivalent equations on the right.

10. $n = 4$: $n + 2 = a$, $2 \times n = c$, $n \div 2 = e$
11. $n = 3$: $n + 1 = a$, $n + 0 = c$, $n - 1 = e$
12. $n = 2$: $n + 4 = a$, $n - 2 = c$, $n - 1 = e$
13. $n = 4$: $3 \times n = a$, $5 \times n = c$, $7 \times n = e$
14. $n = 48$: $n \div 3 = a$, $n \div 8 = c$, $n \div 24 = e$

Solve the equations. Write **a**, **s**, **f**, or **p** to show whether your solution is an addend, sum, factor, or product.

15. $n + 3 = 16$ 17. $7 \times n = 63$ 19. $57 + 38 = n$ 21. $53 - n = 28$
16. $n - 7 = 17$ 18. $n \div 4 = 5$ 20. $7 \times 14 = n$ 22. $45 \div n = 5$

Study these number patterns.

A	B
$n = 3$	$n + 4 = 7$
$n + 1 = 4$	$n + 3 = 6$
$n + 2 = 5$	$n + 2 = 5$
$n + 3 = 6$	$n + 1 = 4$
$n + 4 = 7$	$n = 3$

C	D
$n = 5$	$16 \times n = 80$
$2 \times n = 10$	$8 \times n = 40$
$3 \times n = 15$	$4 \times n = 20$
$4 \times n = 20$	$2 \times n = 10$
$5 \times n = 25$	$n = 5$

The equations in each pattern are **equivalent**. The solution for the equations in **A** and **B** is 3; in **C** and **D** the solution is 5.

EXERCISES

Complete the equations in each of the sets of equivalent equations.

- $\{n = 7, n + 3 = 10, n + 4 = a, n + 6 = r\}$
- $\{n = 9, n - 4 = i, n - 6 = c, n - 8 = a\}$
- $\{n = 8, 8 \times n = c, 4 \times n = v, n + n = w\}$
- $\{n = 24, n \div 2 = a, n \div 3 = c, n \div 6 = e\}$

Complete the sentences.

- $n = 5$, so $3 \times n = a$
- $n = 11$, so $n + 11 = c$
- $n + 7 = 16$, so $n = i$
- $4 \times n = 36$, so $n = m$
- $n \div 8 = 5$, so $n = r$
- $15 \times n = 180$, so $n = u$

Use the rule to write a second equation equivalent to the first equation. The first is done for you.

	first equation	rule	second equation
11.	$n = 4$	$+2$	$n + 2 = 6$
12.	$n = 7$	-3	a
13.	$n = 6$	$\times 4$	c
14.	$n = 21$	$+5$	i

	first equation	rule	second equation
15.	$2 \times n = 14$	$\div 2$	$n = 7$
16.	$n \div 3 = 18$	$\times 3$	r
17.	$6 + n = 12$	-6	s
18.	$(2 \times n) + 3 = 17$	-3	v

Equivalent inequalities

Study these number patterns.

A	B
$n < 5$	$n + 6 < 11$
$n + 2 < 7$	$n + 4 < 9$
$n + 4 < 9$	$n + 2 < 7$
$n + 6 < 11$	$n < 5$

C	D
$n < 3$	$8 \times n < 24$
$2 \times n < 6$	$4 \times n < 12$
$3 \times n < 9$	$2 \times n < 6$
$4 \times n < 12$	$n < 3$

The inequalities in each pair of patterns have the same solution set. If the replacement set is {whole numbers}, the solution set for the inequalities in **A** and **B** is {0, 1, 2, 3, 4}; in **C** and **D** the solution set is {0, 1, 2}. The inequalities in each pattern are **equivalent**.

EXERCISES

Use the inequality on the left to complete the equivalent inequalities on the right.

- $n < 3$: $n + 2 < a$, $n + 4 < c$, $n + 9 < e$
- $n < 7$: $n - 3 < a$, $n - 1 < c$, $n - 5 < e$
- $n < 8$: $2 \times n < a$, $3 \times n < c$, $6 \times n < e$
- $n > 10$: $n + 2 > a$, $n - 4 > c$, $3 \times n > e$
- $n > 30$: $n \div 2 > a$, $n \div 3 > c$, $n \div 10 > e$

Complete the sentences.

- $n < 4$, so $2 \times n < a$
- $n - 3 < 7$, so $n < c$
- $n + 4 < 10$, so $n < e$
- $3 \times n > 15$, so $n > i$
- $8 \times n > 16$, so $n > m$
- $n \div 4 < 4$, so $n < r$

Use the rule to write a second inequality equivalent to the first one. The first exercise in each chart is done for you.

	first inequality	rule	second inequality
12.	$n < 4$	$+2$	$n + 2 < 6$
13.	$n < 7$	$+1$	a
14.	$n < 6$	-2	b
15.	$n < 8$	$\times 5$	c

	first inequality	rule	second inequality
16.	$n + 2 < 6$	-2	$n < 4$
17.	$n + 1 < 8$	-1	f
18.	$n - 2 < 4$	$+2$	g
19.	$2 \times n < 10$	$\div 2$	h

Study this method for solving the inequality $n \times 3 < 23$ if the replacement set is {whole numbers}.

Step 1: Name the greatest multiple of 3, less than 23. Use division to help you.

$$\begin{array}{r} 7 \\ 3 \overline{)23} \\ \underline{21} \\ 2 \end{array}$$

Step 2: Since $7 \times 3 = 21$ and $21 < 23$, 7 and all whole numbers less than 7 form the solution set for $n \times 3 < 23$.

Step 3: The solution set is $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

EXERCISES

Name these quotients.

1. $4 \overline{)13}$

3. $6 \overline{)34}$

5. $3 \overline{)55}$

7. $5 \overline{)64}$

2. $7 \overline{)53}$

4. $12 \overline{)100}$

6. $15 \overline{)500}$

8. $23 \overline{)197}$

Use the quotients in exercises 1–8 to help name the solution set of the inequalities. The replacement set is {whole numbers}.

9. $4 \times n < 13$

12. $12 \times n < 100$

15. $5 \times n < 64$

10. $7 \times n < 53$

13. $3 \times n < 55$

16. $23 \times n < 197$

11. $6 \times n < 34$

14. $15 \times n < 500$

17. $23 \times n > 197$

To solve the inequality $n \times 3 > 23$ we name the least multiple of 3 that is greater than 23. Since $8 \times 3 = 24$ and $23 < 24$, 8 and all whole numbers greater than 8 form the solution set for $n \times 3 > 23$. The solution set is $\{8, 9, 10, \dots\}$.

Name the least multiple of

18. 4, that > 25

20. 6, that > 32

19. 7, that > 17

21. 5, that > 24

Use exercises 18–21 to help name the solution set of each of the inequalities. The replacement set is {whole numbers}.

22. $4 \times n > 25$

23. $6 \times n > 32$

24. $7 \times n > 17$

25. $5 \times n > 24$



Using equations

If a 12 foot board is cut into 3 pieces of the same length, how long is each piece? We solve this problem by using any of these equivalent equations:

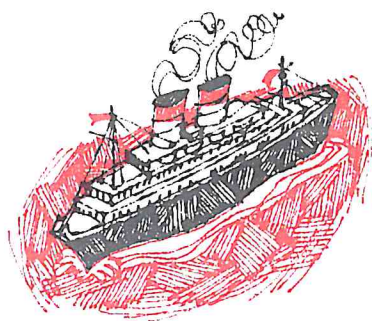
$$\begin{array}{ll} 3 \times n = 12, & 12 \div n = 3, \\ n \times 3 = 12, & 12 \div 3 = n. \end{array}$$

The solution of each of these equations is 4. The solution of the problem is: The length of each piece is 4 feet.

EXERCISES

Write and solve equations to answer the problems.

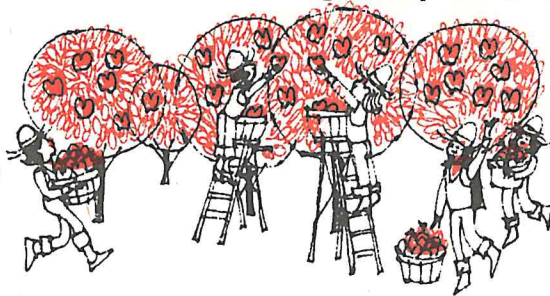
1. In 3 days the liner on which Carol was traveling steamed 1536 miles. If it steamed the same distance each day, how far did it go each day?
2. If it costs \$24 to clothe and feed one boy for a week, how much does it cost to clothe and feed 8 boys at the same rate?
3. Tim and Terry charge 25¢ each to shine shoes. If Tim shined 8 pairs of shoes and Terry shined 7 pairs, how much money did the boys earn?
4. In a dart game Sam scored 25, 10, 25 and 35. Penny scored 25, 25, 15 and 20. Who won the game and by how much?



Write and solve equations to name the numbers.

5. Three times a number plus 8 is equal to 14.
6. Five times a number plus 4 is equal to 34.
7. Two times a number plus 7 is equal to 19.
- *8. A friend told Jim to choose a number, multiply it by 4, add 8, divide the result by 4, and then subtract the original number from the quotient. Jim's result was 2. What number did he start with?

If you want to share 23 apples evenly among 5 girls, what is the greatest number of apples each girl can get? (You do not cut up any of the apples.)



We solve this problem by using the inequality $n \times 5 < 23$ for the replacement set {whole numbers}. The solution set is {0, 1, 2, 3, 4} and the greatest member of this set is 4. The solution of the problem is: The greatest number of apples each girl can get is 4.

EXERCISES

Write inequalities to solve the problems. In each case the replacement set is {whole numbers}.

1. About what time is it now if 3 hours from now it must be just before 9 A.M.?
2. To make a profit on the class play the Sixth Grade at Elmwood School must collect more than \$60. If tickets cost \$2 each, at least how many people must buy tickets if there is to be a profit?
3. If you bought 25 hot dogs for a cook-out and each person at the cook-out has to have 3 hot dogs, what is the greatest number of people you could have at the cook-out?



Write and solve the sentences using inequalities.

4. Four more than the number is less than 7.
5. Three less than the number is greater than 10.
6. Six more than the number is greater than 7.
7. Six less than the number is less than 1.
8. Three times the number is less than 24.
9. Six times the number is greater than 20.



Graphing solutions



If the replacement set is {whole numbers}, the solution set of the equation $n + 2 = 8$ may be **graphed** on the number line as the point 6.



If the replacement set is {whole numbers}, the solution set of the inequality $n < 9$ may be **graphed** as the points 0, 1, 2, 3, 4, 5, 6, 7, 8.

EXERCISES

Name the solution set of each of the following sentences. The replacement set is {whole numbers}. Graph each solution set on a number line.

1. $a < 5$

6. $a + 4 = 10$

11. $2 \times b < 14$

2. $c < 6$

7. $a + 2 < 9$

12. $b + 6 = 8$

3. $a < \frac{7}{2}$

8. $c - 4 = 8$

13. $n \div 2 < 9$

4. $b < \frac{13}{4}$

9. $3 \times m < 18$

14. $b - 5 = 16$

5. $a + 4 < 10$

10. $4 \times k = 12$

15. $k - 4 > 20$



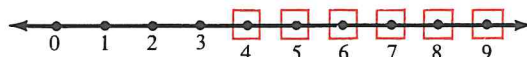
This number line may be used to graph the problem: Jane made three stops on her trip, one at A, 25 miles from home, one at B, 45 miles from home, and one at C, 50 miles from home. What was the distance between stops?

$$45 - 25 = 20, \quad 50 - 45 = 5$$

Solve the problems.

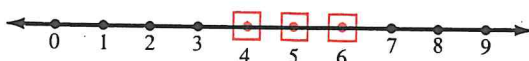
16. How many miles would there be between Jane's first and third stops?
17. Name the miles that would be showing on the mile post halfway between the first and second stops.

Study this example for naming the solution set of the sentence, $a < 7$ and $a > 3$. The replacement set is {whole numbers}.



$$a < 7$$

$$a > 3$$

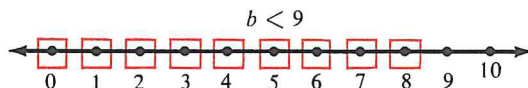
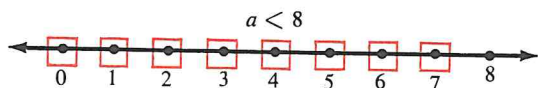
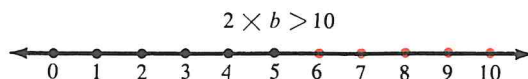
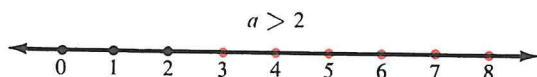


$$a < 7 \text{ and } a > 3$$

We may write the sentence $a < 7$ and $a > 3$ more briefly as $3 < a < 7$. The solution set of $3 < a < 7$ is the *intersection* of the solution sets of $a > 3$ and $a < 7$.

EXERCISES

Use the number lines to help you name the solution sets. The replacement set is {whole numbers}.



1. $a > 2$ and $a < 8$

2. $2 \times b > 10$ and $b < 9$

Name the solution set for each of the following number sentences. In each case the replacement set is {whole numbers}.

3. $b > 4$ and $b < 10$

8. $13 < c < 15$

4. $b > 2$ and $b < 4$

9. $6 < 3 \times a < 12$

5. $a > 0$ and $a + 5 < 10$

10. $12 < 4 \times y < 24$

6. $a < 4$ and $a + 1 > 3$

*11. $a > 4$ and $a < 4$

7. $5 < a < 10$

*12. $a > 7$ and $a < 5$



Ordered pairs of numbers

Ordered pairs of numbers such as $\{(10, 25), (20, 45), (30, 90), (40, 150), (50, 225)\}$, can be used to show the data in this table.

automobile speeds and stopping distances

speed in miles per hour	10	20	30	40	50
stopping distance in feet	25	45	90	150	225

EXERCISES

Complete the sentence

1. Bob said, "I think it would be better to name the distances first." Bill disagreed. He said, "I think it is better to name the speeds before the distances, because ____?"

Write the data from each of the tables as a set of ordered pairs.

2.

time in seconds	distance traveled (in feet) by a falling body
1	16
2	64
3	144
4	256
5	400

3.

time in seconds	distance traveled (in yards) by a boy running
2	14
4	28
6	42
10	70
30	210

Answer the questions.

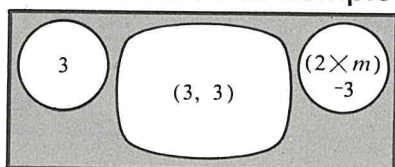
4. In the data shown at the top of the page, does the stopping distance depend on the speed?
5. In exercise 2, does the distance traveled by a falling body depend on the time?
6. In exercise 3, does the distance traveled by a boy running depend on the time?
7. In the ordered number pairs in exercises 2 and 3, does the second number depend on the first number?

Functions — Equations with 2 placeholders

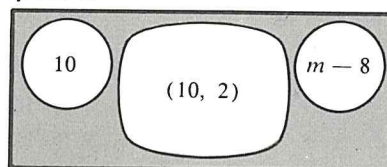
As you know, we may use an equation to state a rule for writing a set of ordered pairs of numbers. For example, we may give the rule $(3 \times m) + 2 = f(m)$. If m is replaced by 1, $(3 \times 1) + 2 = 5$, $f(m) = 5$. If m is replaced by 2, $(3 \times 2) + 2 = 8$, $f(m) = 8$. This equation gives the set of ordered pairs: $\{(1, 5), (2, 8), (3, 11), (4, 14), \dots\}$.

EXERCISES

Use the function TV to complete the ordered pairs.



1. $\{(3, 3), (4, a), (6, b), (9, c), (10, d), (20, e), (33, f)\}$



2. $\{(10, 2), (8, a), (12, c), (15, e), (21, i), (32, r), (40, s)\}$

Use the equation to help you complete the tables.

3. $(4 \times m) + 2 = f(m)$

m	0	1	2	4	7	8
$f(m)$	a	c	e	r	s	u

5. $(m \div 3) + 3 = f(m)$

m	0	3	6	9	12	21
$f(m)$	a	c	e	r	s	u

4. $20 - (2 \times m) = f(m)$

m	0	1	3	4	6	10
$f(m)$	a	c	e	r	s	u

*6. $2 \times m^2 = f(m)$

m	0	1	2	3	4	5
$f(m)$	a	c	e	r	s	u

Use the rule to complete the ordered pairs.

7. $(5 \times m) - 7 = f(m)$: $\{(2, a), (3, b), (4, c), (7, d), (10, e), (21, f), (30, g)\}$
8. $(10 \times m) - 8 = f(m)$: $\{(1, a), (2, c), (3, e), (5, r), (6, s), (9, w), (10, u)\}$
9. $(m \div 3) + 4 = f(m)$: $\{(6, a), (9, b), (12, c), (15, d), (18, e), (21, f), (48, g)\}$
10. $(4 \times p) - 1 = f(p)$: $\{(1, a), (2, b), (3, c), (4, d), (5, e), (7, f), (9, g)\}$
11. $16 \times t^2 = f(t)$: $\{(0, a), (1, b), (2, c), (3, d), (4, e), (5, f), (6, g)\}$

Formulas

We have used equations with 2 or more placeholders in writing **formulas** such as the formulas for area and volume. Among the formulas we have worked with are:

$$A = l \times w \quad \text{and} \quad V = l \times w \times h$$

EXERCISES

Use the formula $A = l \times w$ to name the missing measures for each of the rectangles.

1.

area	a sq. in.	42 sq. in.	108 sq. in.	d sq. in.	140 sq. ft.	198 sq. ft.
length	7 in.	14 in.	c in.	30 ft.	20 ft.	f ft.
width	4 in.	b in.	12 in.	17 ft.	e ft.	18 ft.

Use the formula P (*Perimeter*) $= 2 \times (l + w)$ to name the missing measures for each of the rectangles.

2.

perimeter	a in.	b in.	c ft.	d in.	28 ft.	42 ft.
length	8 in.	23 in.	15 ft.	7 in.	e ft.	f ft.
width	4 in.	18 in.	5 ft.	21 in.	8 ft.	12 ft.

The formula to name a *Profit* (P) is $P = S - C$ where S stands for *Selling Price* and C for *Cost Price*. Complete the table.

3.

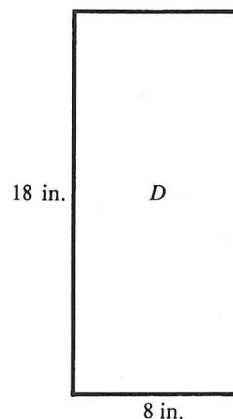
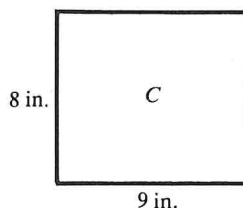
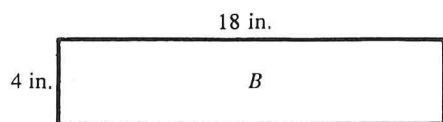
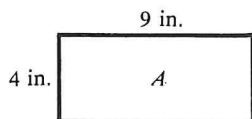
selling price	\$8	\$19.50	\$25.74	d	e	\$28	\$37.50	\$42.50
cost	\$7	\$16.00	\$19.98	\$15	\$18.00	f	g	\$17.00
profit	a	b	c	\$ 5	\$ 3.50	\$ 4	\$ 5.75	h

The formula to name a *Distance* (D) traveled is $D = R \times T$ where R stands for the *Rate* and T for the *Time*. In this table the rates are given in miles per hour, the time in hours and the distance in miles.

4.

rate (mph)	12	16	35	25	40	6	45	36
time (hr)	5	7	4	d	e	f	g	h
distance (m)	a	b	c	150	160	162	180	144

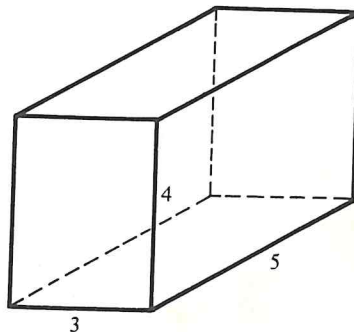
Answer the questions about the rectangular regions.



1. Name the area of A .
2. Name the area of B .
3. The length of B is twice the length of A . Is the area of B twice the area of A ?
4. Name the area of C .
5. The width of C is twice the width of A . Is the area of C twice the area of A ?
6. Name the area of D .
7. The length and width of D are twice that of A . Is the area of D twice the area of A ?
8. Name the perimeter of A .
9. Name the perimeter of B .
10. Name the perimeter of C .
11. Name the perimeter of D .

The area in the base of the prism on the right is 15 square units.

12. What is the volume in the prism on the right?
13. What is the volume in a prism with the same base and twice the height of the prism on the right?
14. Write a formula for the volume in a prism using A for the area of the base and h for the height.

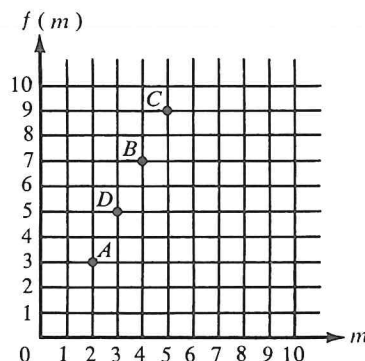


Ordered pairs on the number plane

We may show sets of ordered pairs on the number plane. Some of the ordered pairs for the function with the rule $(2 \times m) - 1 = f(m)$ are:

$$\{(1, 1), (2, 3), (3, 5), (4, 7), \dots\}.$$

Some of these ordered pairs are shown on the number plane on the right.



EXERCISES

Name as an ordered pair the point on the number plane above which is labeled

1. *A*
2. *B*
3. *C*
4. *D*

For the function $(2 \times m) - 1 = f(m)$ name the value of $f(m)$ when m is replaced by

5. 5
6. 7
7. 9
8. 10
9. 11
10. 12

Use the function equation to help you complete the ordered pairs.

11. $m + 3 = f(m)$: $A = \{(0, 3), (1, 4), (2, a), (3, c), (5, e), (7, i)\}$
12. $m - 5 = f(m)$: $B = \{(5, 0), (7, a), (10, c), (12, u), (17, v), (20, x)\}$
13. $4 \times m = f(m)$: $C = \{(1, 4), (3, a), (4, c), (0, e), (9, r), (15, s)\}$
14. $m = f(m)$: $D = \{(1, 1), (3, a), (7, c), (13, e), (0, r), (25, s)\}$
15. $(3 \times m) - 4 = f(m)$: $E = \{(2, 2), (4, a), (6, s), (7, r), (8, u), (13, v)\}$

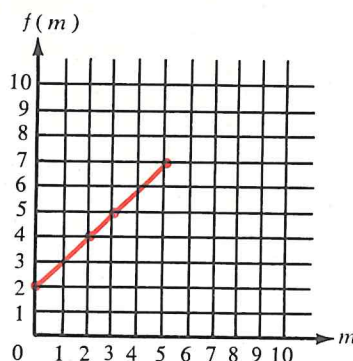
Answer the questions.

16. Draw a number plane with 10 units on each number line. Locate and join each ordered pair in the set A in exercise 11 on your number plane. Does your picture show a line segment?
17. Do the points $(4, 7)$, $(6, 9)$, and $(7, 10)$ lie on your line segment?
18. Do the ordered pairs $(4, 7)$, $(6, 9)$ and $(7, 10)$ make the equation $m + 3 = f(m)$ a true statement?

Graphs of function equations

The number pairs in the set $\{(0, 2), (2, 4), (3, 5), (5, 7), \dots\}$ are solutions of the function equation $m + 2 = f(m)$.

If we join the points on a number plane labeled by these number pairs, we draw a line segment. We call the line segment the **graph** of the equation $m + 2 = f(m)$.



EXERCISES

Use the function equations to complete the tables.

1. $m - 2 = f(m)$

m	3	4	5	6	7	8
$f(m)$	1	a	b	c	d	e

3. $10 - m = f(m)$

m	5	3	4	6	0	8
$f(m)$	5	l	m	n	w	p

2. $m + 3 = f(m)$

m	0	1	2	3	4	5
$f(m)$	3	f	g	h	i	j

4. $2 \times m = f(m)$

m	0	1	2	3	4	5
$f(m)$	0	q	r	s	t	u

5–8. Draw a number plane for each equation above and graph the equation.

Use the function equations to complete the tables.

9. $(2 \times m) + 1 = f(m)$

m	1	2	3	4	5	0
$f(m)$	3	5	a	b	c	d

11. $(3 \times a) + 2 = f(a)$

a	1	2	3	4	0	5
$f(a)$	5	8	i	j	k	l

10. $(2 \times m) - 2 = f(m)$

m	3	5	1	2	4	6
$f(m)$	4	8	e	f	g	b

12. $(4 \times a) - 1 = f(a)$

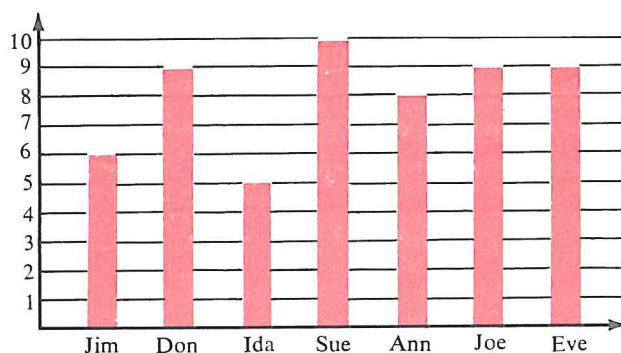
a	1	2	3	4	5	6
$f(a)$	3	m	n	p	q	r

13–16. Draw a number plane for each equation above and graph the equation.

Data and bar graphs

Seven students used a **bar graph** to show the number of books they read in a week.

The total number of books read was 56, so the **average** number of books read was $56 \div 7$, or 8.



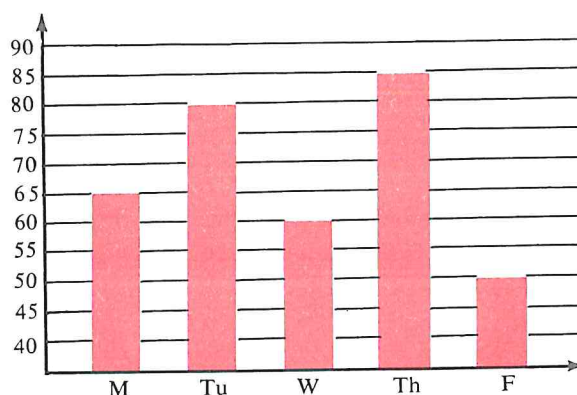
EXERCISES

Study the bar graph above.

- Write the data on the bar graph as ordered pairs like (Jim, 6).
- Which student read the same number of books as the average?

This bar graph shows the number of students who visited the school library after school in one week.

- Name the ordered pairs for the data shown on the graph.
- Name the average number of students who visited the library after school during the week.



Draw a bar graph for each set of data.

5.	student	Tom	Pat	Sam	Ida	Sue	Joe	Ira	Eve
	height (in inches)	64	58	56	57	61	65	67	60

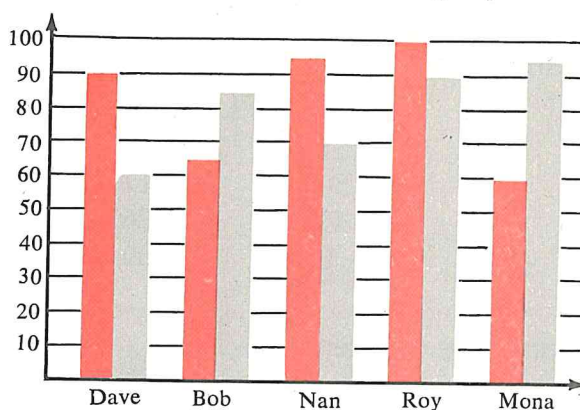
6.	date (January)	18	19	20	21	22	23	24	25
	high temperature	32	34	39	30	22	24	29	36

7.	date (August)	20	21	22	23	24	25	26	27
	high temperature	83	88	94	97	92	90	85	81

8-10. Name the averages for the data in exercises 5-7.

Double bar graphs

The data shown on this **double bar graph** are the scores of 5 students on a math test (in red) and a spelling test (in black).



EXERCISES

Use the bar graph above to

1. name the average score on the math test.
2. name the average score on the spelling test.

From the data on the bar graph above, name

3. how many students did better on the math test than the spelling test.
4. the average score of Bob.
5. the range of scores on the math test.
6. the range of scores on the spelling test.

Draw a double bar graph showing the number of runs in 8 games that the Bluejays played in the Girls' Softball League.

7.

game	1	2	3	4	5	6	7	8
runs for Bluejays	6	4	8	13	10	3	7	13
runs for other team	3	9	7	5	8	4	9	3

Use the data in exercise 7 to

8. name the number of games won and lost by the Bluejays.
9. name the average number of runs scored by the Bluejays.
10. name the average number of runs scored by the other team.
11. name the game won by the greatest margin.

Practice page

Name the sums.

- | | | | | | |
|---|---|---|--|---|--|
| 1. $\begin{array}{r} 6 \\ 7 \\ 9 \\ 8 \\ + 5 \\ \hline \end{array}$ | 2. $\begin{array}{r} 26 \\ 37 \\ 50 \\ 42 \\ + 8 \\ \hline \end{array}$ | 3. $\begin{array}{r} 597 \\ 486 \\ 317 \\ 426 \\ + 709 \\ \hline \end{array}$ | 4. $\begin{array}{r} 6107 \\ 5324 \\ 8075 \\ 4311 \\ + 9206 \\ \hline \end{array}$ | 5. $\begin{array}{r} 37825 \\ 40961 \\ 29383 \\ 65418 \\ + 79249 \\ \hline \end{array}$ | 6. $\begin{array}{r} 576392 \\ 485176 \\ 620013 \\ 509281 \\ + 435018 \\ \hline \end{array}$ |
|---|---|---|--|---|--|

Name the differences.

- | | | | | |
|--|--|--|---|---|
| 7. $\begin{array}{r} 34 \\ - 18 \\ \hline \end{array}$ | 8. $\begin{array}{r} 927 \\ - 305 \\ \hline \end{array}$ | 9. $\begin{array}{r} 7906 \\ - 5498 \\ \hline \end{array}$ | 10. $\begin{array}{r} 859213 \\ - 476549 \\ \hline \end{array}$ | 11. $\begin{array}{r} 5378026 \\ - 2697394 \\ \hline \end{array}$ |
|--|--|--|---|---|

Name the sums.

- | | | |
|------------------------|------------------------|------------------------|
| 12. $(3 \times 7) + 4$ | 16. $(6 \times 5) + 4$ | 20. $(8 \times 6) + 3$ |
| 13. $(9 \times 5) + 2$ | 17. $(7 \times 8) + 3$ | 21. $(4 \times 6) + 3$ |
| 14. $(6 \times 7) + 3$ | 18. $(8 \times 8) + 5$ | 22. $(6 \times 6) + 7$ |
| 15. $(5 \times 5) + 4$ | 19. $(7 \times 5) + 4$ | 23. $(7 \times 7) + 5$ |

Name the products.

- | | | | | | |
|---|--|---|---|--|--|
| 24. $\begin{array}{r} 56 \\ \times 9 \\ \hline \end{array}$ | 25. $\begin{array}{r} 871 \\ \times 6 \\ \hline \end{array}$ | 26. $\begin{array}{r} 926 \\ \times 13 \\ \hline \end{array}$ | 27. $\begin{array}{r} 3716 \\ \times 5 \\ \hline \end{array}$ | 28. $\begin{array}{r} 2905 \\ \times 23 \\ \hline \end{array}$ | 29. $\begin{array}{r} 546 \\ \times 342 \\ \hline \end{array}$ |
|---|--|---|---|--|--|

Name the quotients and remainders.

- | | | | | |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 30. $9 \overline{)396}$ | 31. $8 \overline{)4208}$ | 32. $13 \overline{)299}$ | 33. $27 \overline{)598}$ | 34. $56 \overline{)8765}$ |
| 35. $23 \overline{)1968}$ | 36. $42 \overline{)2753}$ | 37. $56 \overline{)1834}$ | 38. $48 \overline{)9657}$ | |

- 39–50.** Name the sum of each of the numbers on the right and the number on the left.

23: 46, 28, 39, 47, 63, 44

87: 36, 58, 74, 68, 57, 61

- 51–62.** Name the product of the number on the left and each number on the right for exercises 39–50.

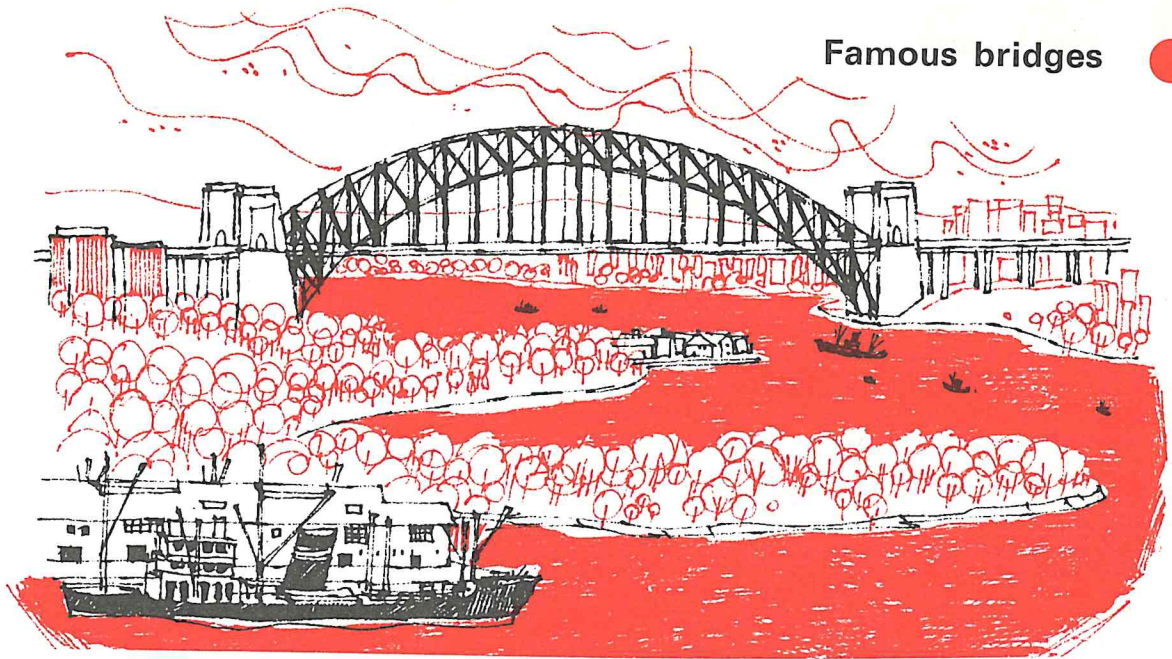
Name the quotient and remainder with divisor 38 and dividend

63. 564

64. 7234

65. 84,691

66. 21,384

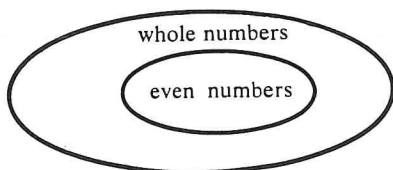


The Sydney Harbour Bridge in Australia is the largest steel arch bridge in the world. It has a span of about 1700 feet and is about 175 feet above the water. The bridge has 4 railroad tracks, 6 lanes of highway, and 2 sidewalks.

1. The Golden Gate Bridge in California is one of the longest suspension bridges in the world. It has a span of 4200 feet, is 90 feet wide and 240 feet above the water. It has 6 lanes of traffic and 2 sidewalks.
2. One of the largest floating bridges in the world is the Lake Washington Bridge in Seattle, Washington. It has a span of about 6566 ft.
3. The world's longest railroad bridge is the Lower Zambezi Bridge in Mozambique. It has a total length of about 12,000 feet.
4. The Royal Gorge Bridge in Colorado is the highest suspension bridge in the world. It is about 1050 feet above the Arkansas River and has a span of 880 feet.
5. The world's widest bridge is the Crawford Street Bridge in Providence, Rhode Island with a width of 1147 feet.
6. The world's longest single span bridge is the Verrazano-Narrows Bridge stretching across the entrance to New York Harbor. It measures 6690 feet between supports and carries two decks each of six lanes of traffic.

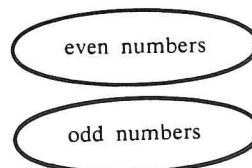


All, some, no



This diagram shows:

All even numbers are whole numbers.
Some whole numbers are even numbers.

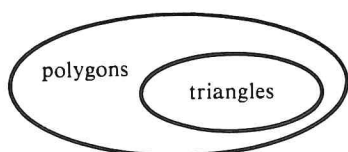


This diagram shows:

No even numbers are odd numbers.

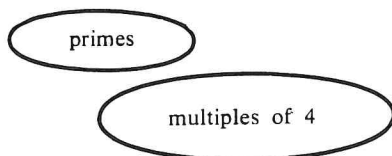
EXERCISES

Study the diagram, then write the word *all*, *some*, or *no* to complete the sentence.

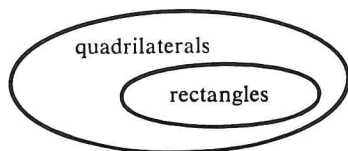


1. ___?___ triangles are polygons.

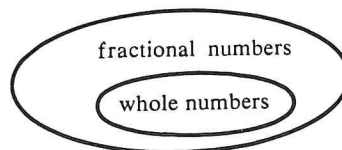
2. ___?___ polygons are triangles.



5. ___?___ prime numbers are multiples of 4.

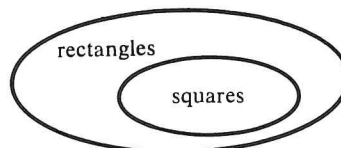


7. ___?___ quadrilaterals are rectangles.

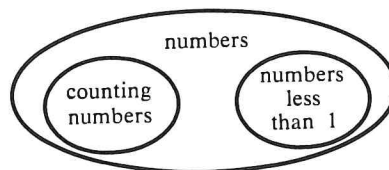


3. ___?___ fractional numbers are whole numbers.

4. ___?___ whole numbers are fractional numbers.



6. ___?___ squares are rectangles.

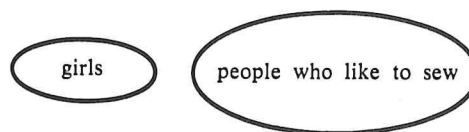
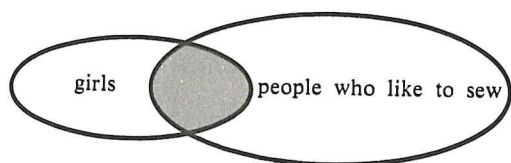


8. ___?___ counting numbers are less than 1.

Draw diagrams to show each of the statements.

9. No polygons are circles.
10. Some counting numbers are greater than 9.

11. All straight lines are curves.
12. No triangles are quadrilaterals.



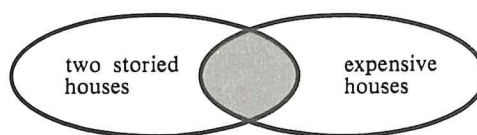
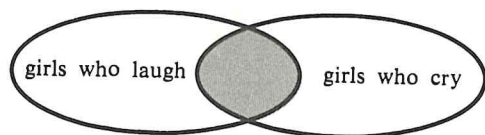
The first diagram shows:

A. **Some** girls like to sew. B. **Some** girls do **not** like to sew.

Statement B does not disagree with A. To disagree with A we must write “No girls like to sew.” We show this in the second diagram.

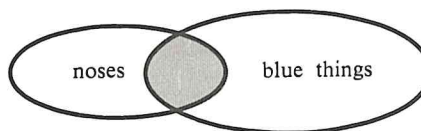
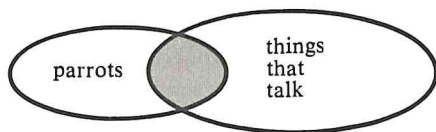
EXERCISES

Use the diagrams to help you. Write *some* or *no* to complete each statement. Then write a statement to disagree and draw a diagram to show this statement.



1. ? girls laugh and also cry.

3. ? two storied houses are expensive.



2. ? parrots talk.

4. ? noses are blue.

Write a statement to disagree with each of these

5. Some counting numbers are even.
6. Some counting numbers are prime.
7. Some whole numbers are less than 8.
8. Some triangles are right triangles.
9. Some quadrilaterals are squares.
10. Some boys ride bicycles.

● To disagree with all or no

A. All birds fly.	A. No birds swim.
B. Some birds do not fly. There is at least 1 bird that does not fly.	B. Some birds swim. There is at least 1 bird that swims.

The statements in B disagree with the statements in A.

EXERCISES

Pair each statement on the left with the statement on the right with which it disagrees. For example, write (1, c).

- | | |
|--|---|
| 1. All girls like to play basketball. | a. No dogs chase cats. |
| 2. Some dogs chase cats. | b. No boys in sixth grade like to swim. |
| 3. No boys in sixth grade like to swim. | c. Some girls do not like to play basketball. |
| 4. Some apples are ripe. | d. No apples are ripe. |
| 5. There is at least 1 boy in sixth grade who likes to swim. | e. Some boys in sixth grade like to swim. |
| 6. All apples are ripe. | f. There is at least 1 dog that chases cats. |
| 7. No girls like to play basketball. | g. Some apples are not ripe. |
| 8. No dogs chase cats. | h. Some girls like to play basketball. |
| 9. All triangles are equilateral. | i. There is at least one triangle which is not equilateral. |

Write an *all*, *some*, *some-not*, or *no* statement to disagree with each of these statements.

10. All even numbers have 4 as a factor.
11. Some quadrilaterals are rectangles.
12. All horses love to run.
13. No prime numbers greater than 2 are even.
14. Some schools are made of brick.



Pair each statement on the left with the statement on the right with which it disagrees.

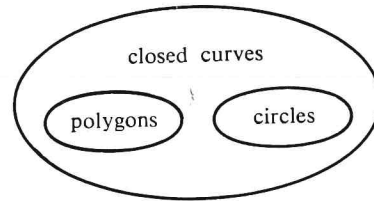
- | | |
|---|--|
| 1. All rectangles are quadrilaterals. | a. There is at least 1 quadrilateral that does not have 4 sides. |
| 2. Some odd numbers are counting numbers. | b. There is at least 1 quadrilateral that has 4 sides. |
| 3. No odd numbers are counting numbers. | c. No odd numbers are counting numbers. |
| 4. All quadrilaterals have 4 sides. | d. There is at least 1 rectangle that is not a quadrilateral. |
| 5. No quadrilaterals have 4 sides. | e. There is at least 1 counting number that is an odd number. |
| 6. All equations are number sentences. | f. Some equations are number sentences. |
| 7. No equations are number sentences. | g. Some equations are not number sentences. |

Write a *some-not* statement to disagree with each of these statements.

8. All rectangles are squares.
9. All triangles are quadrilaterals.
10. All odd numbers are less than 13.
11. All numbers are addends.
12. All numerals are fractions.
13. All odd numbers are multiples of 3.
14. All numbers with 5 as a factor are odd.
15. All grebs are kane.
16. All whys are zee.
- 17–25. Write a *there is at least 1* statement to disagree with each of the statements in exercises 8–16.
- 26–39. Write the numerals 26–39. Next to each numeral write *T* or *F* to show whether the statements in exercises 1–14 are *True* or *False*.

● All-some, if-then

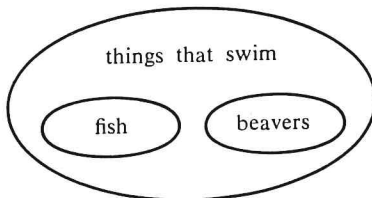
The diagram shows that **all** polygons are closed curves and **all** circles are closed curves. **If** a figure is a polygon, **then** it is a closed curve; **if** a figure is a circle, **then** it is a closed curve.



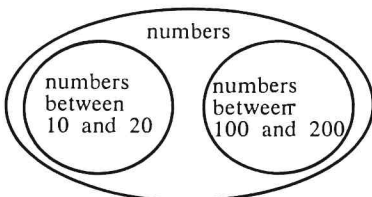
The diagram shows that *some* but *not all* closed curves are polygons or circles. We cannot be sure that if a figure is a closed curve, then it is a polygon or a circle. It may be some other kind of closed curve.

EXERCISES

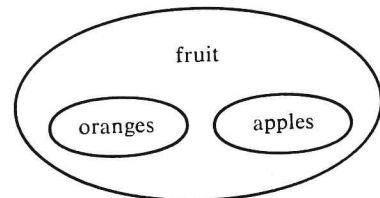
Use the diagrams to help you. Write *T* or *F* to show whether each statement is *True* or *False*.



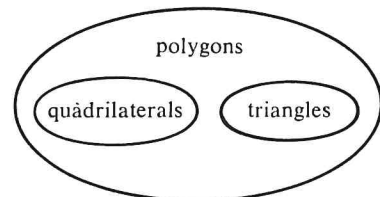
1. All fish swim.
2. If it is a fish, then it swims.
3. Some things that swim are beavers.
4. If it swims, then it is a beaver.



9. All numbers between 10 and 20 are numbers.
10. If it is a number, then it is a number between 10 and 20.
11. Some numbers are numbers between 100 and 200.
12. If it is a number between 100 and 200, then it is a number.

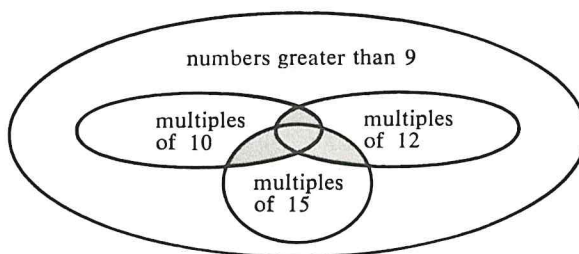


5. All oranges are fruit.
6. All fruit are apples.
7. If it is a fruit, then it is an orange.
8. If it is fruit, then it is an apple.



13. All triangles are polygons.
14. All polygons are quadrilaterals or triangles.
15. If it is a polygon, then it is a quadrilateral or a triangle.
16. If it is a quadrilateral, then it is a polygon.

The diagram shows that *if* a number is a multiple of 10, *then* it is greater than 9.



EXERCISES

Use the diagram above to help you name the correct statement in each of these pairs.

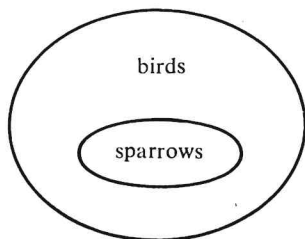
1. a. If a number is greater than 9, then it is a multiple of 12.
b. If a number is a multiple of 12, then it is greater than 9.
2. a. If a number is a multiple of 15, then it is greater than 9.
b. If a number is greater than 9, then it is a multiple of 15.

Write A or B to show which sentence is correct.

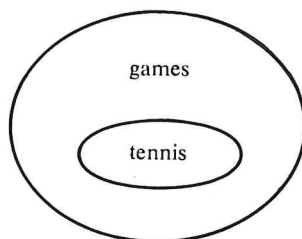
3. All boys are human, so
A. if Pat is a boy, then Pat is human.
B. if Pat is human, then Pat is a boy.
4. All apples grow on trees, so
A. if a Howzat grows on a tree, then it is an apple.
B. if a Howzat is an apple, then it grows on a tree.
5. All people who laugh are happy, so
A. if Brenda laughs, then she is happy.
B. if Brenda is happy, then she laughs.
6. All addends are numbers, so
A. if n is a number, then it is an addend.
B. if n is an addend, then it is a number.

Use each of the diagrams to write a correct *if-then* statement.

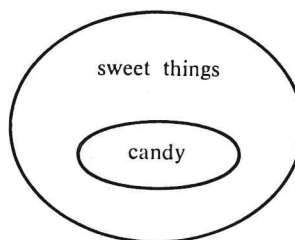
7.



8.



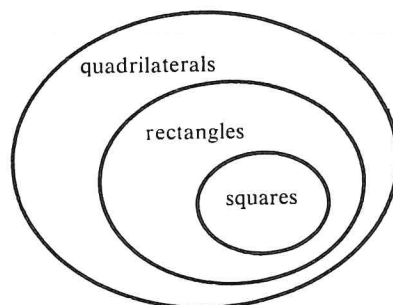
9.



● If-then statements

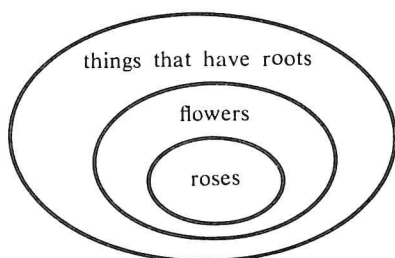
The diagram shows:

- A. *If it is a square,
then it is a rectangle.*
- B. *If it is a rectangle,
then it is a quadrilateral.*
- C. *If it is a square,
then it is a quadrilateral.*

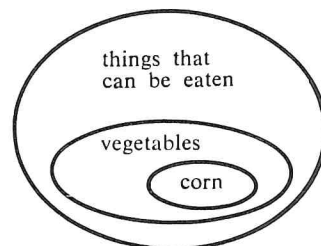


EXERCISES

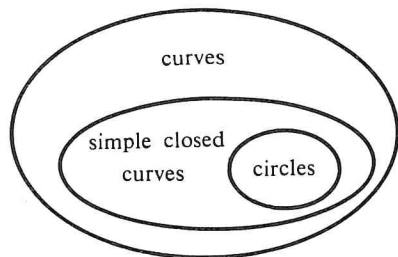
Study the diagrams, then complete the *if-then* statements.



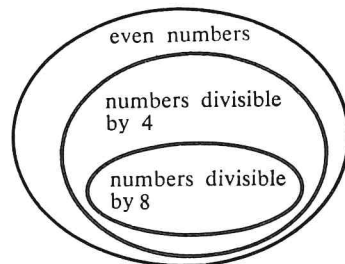
1. If it is a rose, then it has _____?_____ or is _____?_____.
2. If it is a _____?_____, then it is a flower.
3. If it is a flower, then it _____?_____.



4. If it is corn, then it can be _____?_____ or _____?_____.
5. If it is _____?_____, then it is a vegetable.
6. If it is a vegetable, then it can be _____?_____.



7. If it is a circle, then it is a _____?_____ or a _____?_____.
8. If it is a _____?_____, then it is a simple closed curve.
9. If it is a simple closed curve, then it is a _____?_____.

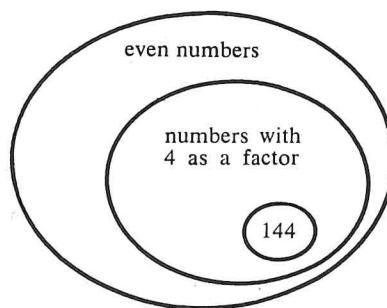


10. If it is divisible by 4 then it is an _____?_____.
11. If it is divisible by 8, then it is divisible by _____?_____.
12. If it is divisible by 8, then it is an _____?_____.



The diagram shows:

If all numbers with 4 as a factor are even *and* 144 is a number with 4 as a factor, *then* 144 is an even number.



EXERCISES

Study the first 2 statements in each exercise, then complete the *then* statement.

If all boys like sports, *and* Bob is a boy,

1. then ____?

If all numbers that have 9 as a factor are divisible by 3, *and* 117 has 9 as a factor,

4. then ____?

If all squares are rectangles, *and* $ABCD$ is a square,

2. then ____?

If all quadrilaterals are polygons, *and* $MNOP$ is a quadrilateral,

5. then ____?

If all parallel lines in a plane do not intersect, *and* $\overline{AB}$ is parallel to $\overline{CD}$,

3. then ____?

If all equilateral triangles are isosceles *and* triangle ABC is equilateral,

6. then ____?

Complete these sentences.

7. If all numerals are names for numbers and $7 + 5$ is a numeral, then $7 + 5$ is a ____?

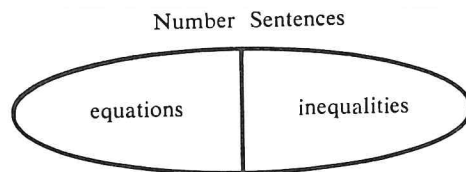
8. If all addends are numbers and 26 is an addend, then ____? is a number.

9. If all numbers with 8 as a factor are even and 184 is a number with 8 as a factor, then 184 is ____?

10. If all lines are curves and CD is a line, then ____? is a curve.

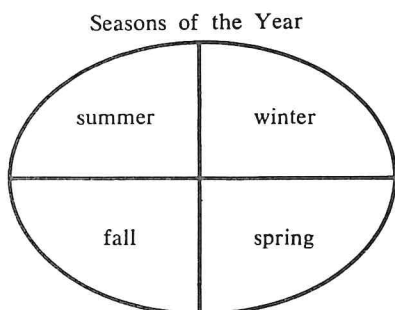
If not-then

The diagram shows that all number sentences are either equations or inequalities. **If** a number sentence is **not** an equation, **then** it is an inequality.

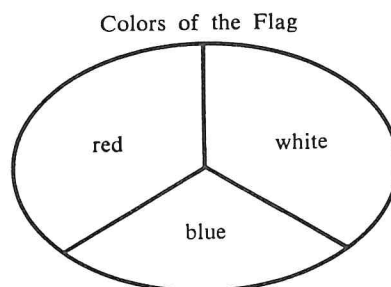


EXERCISES

Use the diagrams to help you complete the sentences.



1. If a season of the year is not summer, winter, or fall, then it is ____?
2. If a season of the year is not spring, winter, or fall, then it is ____?



3. If a color on the flag is not red or white, then it is ____?
4. If a color on the flag is not blue or red, then it is ____?

Complete the sentences.

5. If you must travel by land, sea, or air, and you are not traveling by land or sea, then you are traveling by ____?
6. If you spend your time eating, sleeping, playing, or working, and if you are not working, sleeping, or eating, then you are ____?
7. There are 4 different coins less than a half-dollar in value. If you have one of these coins and it is not a penny, a nickel, or a quarter, then it must be a ____?
8. If you study mathematics, spelling, science, or music in the morning and are not studying mathematics, music, or science in the morning, then you must be studying ____?

If someone said to you, "I am thinking of a whole number between 1 and 10. It is not an even number and it is not prime," you could name the number by listing the whole numbers between 1 and 10, then checking off the even numbers and the primes. The list will look like this: {~~2~~, ~~3~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~8~~, 9}. The number is 9.

EXERCISES

Solve the problems.

1. Name a whole number between 10 and 20 that is not an even number and not a prime.
2. Name a whole number between 40 and 50 that is not an odd number or a multiple of 3 or 4.
3. Name a whole number between 70 and 80 that is not a prime, not an even number, and not a multiple of 7.
4. If a number in a division is either the dividend, the divisor, the quotient, or the remainder, and if a number in a particular division is not the dividend, the divisor, or the remainder, then it must be the ____? ____.
5. Name a factor of 42, other than 1, that is not an even number or a multiple of 7.
6. Name a factor of 54 that is not a multiple of 2 or 3.
7. A triangle may have 2 sides congruent, 3 sides congruent or no sides congruent. If the triangle ABC does not have 2 or 3 sides congruent, then triangle ABC has ____? ____ sides congruent.
8. In the mystery story, the villain can be Tom, Dick or Harry. If you learn that the villain is not Dick or Harry, it must be ____? ____.
9. If you choose a number from $A = \{53, 24, 71, 48, 29, 36\}$, and the number you choose is less than 50 and is not a multiple of 4, it must be ____? ____.
10. Ellen brought a box to school. She said, "In this box I have a fish, a bird, or a hamster. All fish have scales and all birds have feathers. The thing I have in this box does not have scales or feathers." What does Ellen have in the box?
How can you tell?

CHECKING UP

Name the solution set and draw a graph for each of the number sentences. The replacement set is {whole numbers}.

1. $m < 7$

2. $n + 3 < 11$

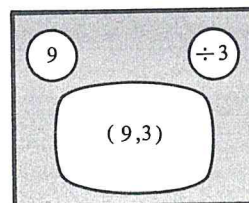
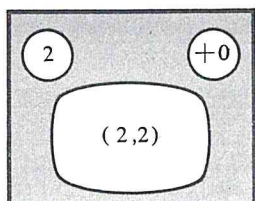
3. $(5 \times r) + 2 > 17$

4. $s < 9$ and $s > 2$

5. $x > 0$ and $x + 3 < 8$

6. $6 < z < 9$

Use the function TV to complete the ordered pairs.



7. $\{(2, 2), (3, a), (4, b), (7, c), (10, d), (12, e)\}$

8. $\{(9, 3), (15, a), (27, b), (45, c), (87, d), (147, e)\}$

Solve these problems using the formulas, $P = (2 \times l) + (2 \times w)$, $A = l \times w$ or $V = l \times w \times h$.

9. Name the area in a rectangle with length 7 inches and width 5 inches.
10. Name the area in a square with the length of a side 8 inches.
11. Name the perimeter of the rectangle in exercise 9.
12. Name the perimeter of the square in exercise 10.
13. Name the volume in a rectangular prism with length 6 inches, width 3 inches, and height 2 inches.

Pair the statements on the right that disagree with the statements on the left.

- | | |
|--|---|
| 14. All odd numbers are counting numbers. | a. There is at least 1 quadrilateral that is not a polygon. |
| 15. Some of the odd numbers are greater than 10. | b. None of the quadrilaterals are rectangles. |
| 16. All quadrilaterals are polygons. | c. Some of the odd numbers are not counting numbers. |
| 17. Some quadrilaterals are rectangles. | d. None of the odd numbers are greater than 10. |



Number sentences that have the same solution set are **equivalent**.

Complete the sets of equivalent equations.

1. $\{(2 \times n) + 3 = 11, (4 \times n) + 6 = a, n = b, (5 \times n) + 1 = c\}$
2. $\{(4 \times n) - 1 = 7, (2 \times n) = d, (8 \times n) - 2 = e, (4 \times n) = f\}$

A set from which a replacement for n can be chosen is called the **replacement set**.

If $A = \{2, 4, 6, 8, 10, 12\}$ is the replacement set, name the solution set for each sentence.

3. $2 \times n = 14$
- *5. $n + 2 < 12$
7. $(3 \times n) + 2 = 20$
4. $n < 10$
6. $12 \div 3 = n$
- *8. $n + 1 > 4$

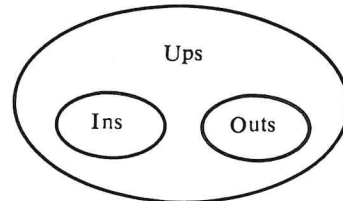
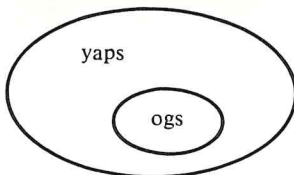
The solution set of an equation or inequality may be graphed on a number line.

Draw a graph for each solution set if the replacement set is {whole numbers}.

9. $a < 6$
11. $2 \times a = 14$
13. $2 \times n < 14$
10. $a + 3 < 6$
12. $15 < 5 \times a < 30$
14. $9 \div n = 1$

Set diagrams may be used to show the meaning of *some-all-no* and *if-then* statements.

Use the diagrams to help you write *T* or *F* to show whether each statement is *True* or *False*.



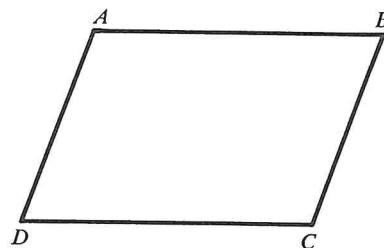
15. All ogs are yaps.
16. If it is a yap, then it is an og.
17. If it is an og, then it is a yap.
18. Some Ins are Outs.
19. If it is an In, then it is an Up.
20. If it is an Up, then it is an Out.



LOOKING WAY BACK

Complete these questions for parallelogram $ABCD$.
Name, in inches or degrees,

1. $m(\overline{AB})$
2. $m(\overline{BC})$
3. $m(\overline{DC})$
4. $m(\overline{AD})$
5. $m(\angle DAB)$
6. $m(\angle ABC)$
7. $m(\angle BCD)$
8. $m(\angle CDA)$
9. Name the congruent sides and angles.



Write a compact numeral for each of the following.

10. $(6 \times 10^3) + (5 \times 10^2) + (1 \times 10) + 7$
11. 382 million, 467 thousand, 95
12. $7 + (3 \times 10^6) + (4 \times 10^3)$
13. $9,000,000 + 300,000 + 70,000 + 8000 + 32$

Solve the equations.

14. $3872 + 5968 = a$
15. $3 \times 5798 = b$
16. $549 \times 6795 = c$
17. $6975 \div 5 = d$
18. $9876 - 5993 = e$
19. $64 \times 4376 = f$
20. $347,689 + 58,341 = g$
21. $9558 \div 27 = h$

Complete the questions for the sets.

$$A = \{a, b, c, d, e, f, \}$$

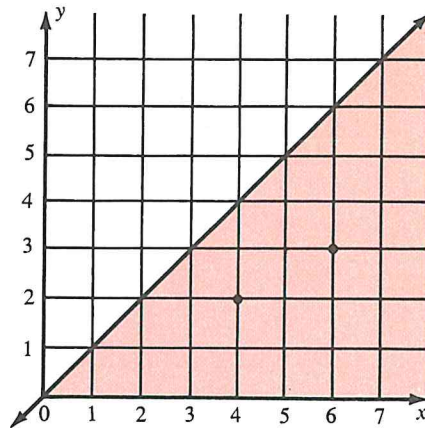
$$B = \{c, a, r, s\}$$

22. $n(A) = \underline{\quad ? \quad}$
23. $n(B) = \underline{\quad ? \quad}$
24. $A \cup B = \underline{\quad ? \quad}$
25. $A \cap B = \underline{\quad ? \quad}$
26. $n(A \cup B) = \underline{\quad ? \quad}$
27. $n(A \cap B) = \underline{\quad ? \quad}$
28. $n(A \times B) = \underline{\quad ? \quad}$
29. $n(A) \times n(B) = \underline{\quad ? \quad}$

Name the quotients and remainders.

30. $23 \overline{)486}$
31. $49 \overline{)598}$
32. $51 \overline{)706}$
33. $37 \overline{)729}$
34. $47 \overline{)3478}$
35. $36 \overline{)1265}$
36. $84 \overline{)2335}$
37. $69 \overline{)3339}$
38. $26 \overline{)4644}$
39. $19 \overline{)2528}$
40. $83 \overline{)9670}$
41. $87 \overline{)9999}$
42. $26 \overline{)37465}$
43. $45 \overline{)80191}$
44. $79 \overline{)49964}$
45. $97 \overline{)42517}$

The graph of the equation $x = y$ is the line shown on the right. What would be the graph of $x > y$? Consider the pairs $(4, 2)$ and $(6, 3)$. They do not satisfy the equation $x = y$ but they do satisfy the inequality $x > y$ since $4 > 2$ and $6 > 3$ are true statements. In fact, any point in the shaded region of our number plane will correspond to an ordered pair which will satisfy $x > y$. So the graph of the inequality $x > y$ is all of the points in the shaded region of the number plane.



EXERCISES

For exercises 1 and 2 draw 10 by 10 number planes.

1. a. Graph $x + 3 = y$.
b. Shade red the region for the graph of $x + 3 > y$.
c. Shade blue the region for the graph of $x + 3 < y$.
2. a. Graph $2 \times x = y$.
b. Shade red the region for the graph of $2 \times x > y$.
c. Shade blue the region for the graph of $2 \times x < y$.

Complete the sentences.

3. The graphs of $x + 3 = y$, $x + 3 > y$, and _____ cover the number plane.
4. The graphs of _____, $2 \times x > y$ and $2 \times x < y$ cover the number plane.

For each sentence draw a 10 by 10 number plane and graph the sentence.

5. $x - 2 < y$
6. $x + 1 > y$
7. $x < y + 2$
8. $x > y - 3$

Draw a 10 by 10 number plane and graph these sentences.

9. $x = 4$
10. $x = 7$
11. $y = 1$
12. $y = 8$

Draw a 10 by 10 number plane and graph these sentences.

13. $x = 3$
14. $x < 3$
15. $x > 3$



Factors

$$1 \times 24 = 24 \quad 2 \times 12 = 24 \quad 3 \times 8 = 24 \quad 4 \times 6 = 24$$

The whole number factors of 24 are $\{1, 2, 3, 4, 6, 8, 12, 24\}$.

$$24 \div 2 = 12 \quad 24 \div 3 = 8 \quad 24 \div 4 = 6$$

To name the whole number factors of 24 we divide by 2, 3, 4, and so on until we come to a number whose square is greater than 24. $5 \times 5 = 5^2 = 25$, $25 > 24$. We do not have to divide by a number greater than 5.

EXERCISES

Use the equations to name each set of factors.

$2 \times 9 = 18$

$2 \times 6 = 12$

$2 \times 14 = 28$

$4 \times 8 = 32$

$3 \times 6 = 18$

$3 \times 4 = 12$

$4 \times 7 = 28$

$2 \times 16 = 32$

$1 \times 18 = 18$

$1 \times 12 = 12$

$1 \times 28 = 28$

$1 \times 32 = 32$

1. $A = \{\text{factors of } 18\}$

3. $C = \{\text{factors of } 28\}$

2. $B = \{\text{factors of } 12\}$

4. $D = \{\text{factors of } 32\}$

Complete the set of factors for the number on the left.

5. 42: $\{1, 2, a, c, e, n, r, s\}$

6. 36: $\{1, 2, 3, a, c, 9, n, r, 36\}$

7. 54: $\{1, 2, a, 6, c, e, n, 54\}$

8. 60: $\{1, 2, 3, 4, a, c, e, n, r, 20, s, 60\}$

9. 75: $\{1, 3, a, c, e, n\}$

10. 48: $\{1, 2, 3, 4, a, 8, c, e, n, 48\}$

Name the set of factors for each number.

11. 58

13. 81

15. 56

17. 50

19. 17

12. 64

14. 144

16. 63

18. 27

20. 256

Name the whole number which is

21. a factor of every whole number. 22. never a factor of a counting number.



$$A = \{\text{factors of } 24\} = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

$$B = \{\text{factors of } 30\} = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

$$A \cap B = \{1, 2, 3, 6\}$$

The **common factors**
of 24 and 30 are
1, 2, 3, and 6.

EXERCISES

Study the sets and name the intersections.

$$A = \{1, 2, 4, 8, 16\}$$

$$B = \{1, 2, 3, 4, 6, 12\}$$

$$C = \{1, 2, 4, 8\}$$

1. $A \cap B$

3. $B \cap A$

5. $C \cap B$

2. $A \cap C$

4. $B \cap C$

6. $C \cap A$

Use the intersections in exercises 1–6 to name the common factors of

7. 16 and 12

8. 16 and 8

9. 12 and 8

Use the sets of factors on the left to name the common factors on the right.

factors of

common factors of

18: $\{1, 2, 3, 6, 9, 18\}$

10. 18 and 20

15. 20 and 30

20: $\{1, 2, 4, 5, 10, 20\}$

11. 18 and 24

16. 20 and 36

24: $\{1, 2, 3, 4, 6, 8, 12, 24\}$

12. 18 and 30

17. 24 and 30

30: $\{1, 2, 3, 5, 6, 10, 15, 30\}$

13. 18 and 36

18. 24 and 36

36: $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

14. 20 and 24

19. 30 and 36

Study the number lines which show the factors of 12, 18, and 24; then name the common factors of

20. 12 and 18

23. 6 and 24

26. 6 and 18

29. 24 and 9

21. 12 and 24

24. 8 and 24

27. 9 and 18

30. 8 and 18

22. 4 and 12

25. 18 and 24

28. 3 and 18

*31. 9 and 4





Multiples

To name the **multiples** of 3, we multiply 3 by each member of the set of whole numbers, $W = \{0, 1, 2, 3, 4, 5, 6, \dots\}$.
 $3 \times 0 = 0$, $3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$, and so on.
 The set of multiples of 3 = $\{0, 3, 6, 9, 12, 15, \dots\}$.

EXERCISES

Complete the sets of multiples.

1. {multiples of 2 which are less than 20} = $\{0, 2, a, 6, b, c, 12, d, e, 18\}$
2. {multiples of 4 which are less than 34} = $\{0, 4, a, b, 16, 20, c, d, e\}$
3. {multiples of 9 which are less than 79} = $\{a, 9, 18, b, c, 45, d, 63, e\}$
4. {multiples of 7 which are less than 63} = $\{a, 7, 14, b, 28, c, d, e, 56\}$
5. {multiples of 5 which are less than 45} = $\{0, a, b, 15, c, 25, d, e, f\}$

List the set of multiples of

6. 6, less than 52

7. 3, less than 27

8. 8, less than 72

Complete the sentences.

9. ____? is a multiple of every number.
10. Every number is a multiple of ____?

A number may be a multiple of many numbers. For example, 24 is a multiple of 1, 2, 3, 4, 6, 8, 12, and 24, because $8 \times 3 = 24$, $4 \times 6 = 24$, $2 \times 12 = 24$, and $1 \times 24 = 24$.

Use the equations to complete the sets.

$9 \times 4 = 36$, $3 \times 12 = 36$, $2 \times 18 = 36$, $6 \times 6 = 36$, $1 \times 36 = 36$
$9 \times 6 = 54$, $27 \times 2 = 54$, $18 \times 3 = 54$, $54 \times 1 = 54$,
$6 \times 8 = 48$, $4 \times 12 = 48$, $3 \times 16 = 48$, $2 \times 24 = 48$, $1 \times 48 = 48$

11. 36 is a multiple of $\{1, 2, a, b, c, d, e, f, g\}$.
12. 54 is a multiple of $\{1, 2, a, b, c, d, e, 54\}$.
13. 48 is a multiple of $\{1, 2, a, b, c, d, e, f, g, h\}$.

$$A = \{\text{multiples of 2 which are less than 20}\} = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

$$B = \{\text{multiples of 3 which are less than 20}\} = \{0, 3, 6, 9, 12, 15, 18\}$$

$$A \cap B = \{0, 6, 12, 18\}$$

The **common multiples** of 2 and 3 which are less than 20 are 0, 6, 12, and 18.

EXERCISES

Use the sets on the left to complete the description on the right.

1. $M = \{0, 4, 8, 12, 16, 20, 24\} = \{\text{multiples of } a \text{ which are less than 25}\}$
2. $N = \{0, 6, 12, 18, 24\} = \{\text{multiples of } b \text{ which are less than 25}\}$
3. $M \cap N = \{0, 12, 24\} = \{\text{common multiples of } c \text{ and } d \text{ which are less than 25}\}$

Name the members of the sets.

4. $A = \{\text{multiples of 3 up to 42}\}$
5. $B = \{\text{multiples of 6 up to 42}\}$
6. $C = \{\text{common multiples of 3 and 6 up to 42}\}$
7. $D = \{\text{multiples of 2 up to 24}\}$
8. $E = \{\text{multiples of 4 up to 24}\}$
9. $F = \{\text{common multiples of 2 and 4 up to 24}\}$
10. $G = \{\text{multiples of 10 up to 60}\}$
11. $H = \{\text{multiples of 4 up to 60}\}$
12. $I = \{\text{common multiples of 10 and 4 up to 60}\}$
13. $J = \{\text{multiples of 3 between 30 and 50}\}$
14. $K = \{\text{multiples of 5 between 30 and 50}\}$
15. $L = \{\text{common multiples of 3 and 5 between 30 and 50}\}$
16. $M = \{\text{multiples of 4 between 40 and 70}\}$
17. $N = \{\text{multiples of 6 between 40 and 70}\}$
18. $P = \{\text{common multiples of 4 and 6 between 40 and 70}\}$

Complete the sentence.

19. ____ is a common multiple of every pair of numbers.

● Numbers with only 2 factors

$1 \times 2 = 2$	{factors of 2} = {1, 2}
$1 \times 3 = 3$	{factors of 3} = {1, 3}
$1 \times 5 = 5$	{factors of 5} = {1, 5}
$1 \times 7 = 7$	{factors of 7} = {1, 7}
$1 \times 11 = 11$	{factors of 11} = {1, 11}
$1 \times 13 = 13$	{factors of 13} = {1, 13}

A number with only 2 factors, itself and 1,
is called a **prime number**.

EXERCISES

List the set of factors for the numbers. Then write
P or *NP* to tell whether they are *Primes* or *Not Primes*.

- | | | | | |
|---------------------------|--------------------------------|-------|-------|--------|
| 1. 6 <i>1, 2, 3, 6 NP</i> | 3. 16 <i>1, 2, 4, 8, 16 NP</i> | 5. 24 | 7. 36 | 9. 37 |
| 2. 17 <i>1, 17 P</i> | 4. 23 <i>1, 23 P</i> | 6. 31 | 8. 29 | 10. 42 |

Name all the prime numbers

11. less than 50. 12. between 60 and 80. 13. between 80 and 100.

Use exercises 11–13 to help you.

14. Name the 25 prime numbers less than 100.

Name 2 prime numbers with sum

- | | | | | | |
|--------|--------|--------|--------|--------|--------|
| 15. 40 | 17. 16 | 19. 42 | 21. 56 | 23. 36 | 25. 20 |
| 16. 24 | 18. 18 | 20. 12 | 22. 28 | 24. 48 | 26. 80 |

(There may be more than 1 pair of primes for some of these.)

Name 2 prime numbers with product

- | | | | | | |
|--------|--------|--------|--------|--------|--------|
| 27. 10 | 29. 21 | 31. 22 | 33. 35 | 35. 46 | 37. 86 |
| 28. 15 | 30. 77 | 32. 38 | 34. 57 | 36. 91 | 38. 65 |

(There is only 1 pair of primes for each of these products.)

Complete the sentence.

39. ____? is the only prime number that is an even number.

$$\begin{array}{c} 20 \\ | \\ \text{even} \end{array} = \begin{array}{c} 17 \\ \diagdown \quad \diagup \\ \text{prime} \end{array} + 3 \quad (\text{or } \begin{array}{c} 13 \\ \diagdown \quad \diagup \\ \text{prime} \end{array} + 7)$$

$$\begin{array}{c} 44 \\ | \\ \text{even} \end{array} = \begin{array}{c} 41 \\ \diagdown \quad \diagup \\ \text{prime} \end{array} + 3 \quad (\text{or } \begin{array}{c} 37 \\ \diagdown \quad \diagup \\ \text{prime} \end{array} + 7)$$

A famous guess, not yet proven, states that any even number greater than 2 may be expressed as the sum of 2 primes.

EXERCISES

Express the numbers as the sum of 2 primes.

- | | | | | | |
|-------|-------|-------|-------|--------|---------|
| 1. 18 | 3. 32 | 5. 70 | 7. 58 | 9. 62 | 11. 100 |
| 2. 24 | 4. 68 | 6. 54 | 8. 50 | 10. 96 | 12. 84 |

It can be proven that any prime greater than 3 is one more than, or one less than, a multiple of 6.

$$\begin{aligned} 31 &= (5 \times 6) + 1 \\ 41 &= (7 \times 6) - 1 \\ 17 &= (3 \times 6) - 1 \end{aligned}$$

Show that the above statement is true for each of these prime numbers.

- | | | | | |
|--------|--------|--------|--------|--------|
| 13. 37 | 15. 19 | 17. 53 | 19. 73 | 21. 67 |
| 14. 23 | 16. 47 | 18. 77 | 20. 83 | 22. 91 |

Name the answers.

23. 11, 19, 43, and 67 are prime numbers that are greater than 10 and less than 100. The digit in the ones' place is 1, 3, 7, or 9. Check the other primes greater than 10 and less than 100 to see whether they also have this property.
24. (3, 5) and (5, 7) are pairs of prime numbers whose difference is 2. Name 5 other pairs of prime numbers less than 100 whose difference is 2.
25. (19, 29) and (61, 71) are pairs of prime numbers whose difference is 10. Name 5 other pairs of prime numbers less than 100 whose difference is 10.
- **26. 41 and 29 are primes; $41 = 5^2 + 4^2$; $29 = 5^2 + 2^2 = 2^2 + 3^2 + 4^2$. Name 6 other primes less than 100 that can be expressed as the sum of 2 or 3 squares.

Prime factors

The prime factors of 24 are 2 and 3. A numeral which shows a number as a product of its prime factors is a **prime factorization**.

$$\begin{aligned}24 &= 8 \times 3 \\&= 2 \times 4 \times 3 \\&= 2 \times 2 \times 2 \times 3\end{aligned}$$

$2 \times 2 \times 2 \times 3$ is the *prime factorization* of 24

EXERCISES

Solve the equations.

$$\begin{aligned}1. \quad 36 &= 3 \times 12 \\&= 3 \times 3 \times 4 \\&= 3 \times 3 \times 2 \times a\end{aligned}$$

$$\begin{aligned}2. \quad 54 &= 2 \times 27 \\&= 2 \times 3 \times 9 \\&= 2 \times 3 \times b \times c\end{aligned}$$

$$\begin{aligned}3. \quad 63 &= 3 \times 21 \\&= 3 \times n \times u\end{aligned}$$

$$\begin{aligned}4. \quad 36 &= 2 \times 18 \\&= 2 \times 2 \times 9 \\&= 2 \times 2 \times f \times g\end{aligned}$$

$$\begin{aligned}5. \quad 54 &= 9 \times 6 \\&= 3 \times 3 \times 6 \\&= 3 \times 3 \times h \times i\end{aligned}$$

$$\begin{aligned}6. \quad 48 &= 3 \times 16 \\&= 3 \times 2 \times 8 \\&= 3 \times 2 \times 2 \times i \times k\end{aligned}$$

$$\begin{aligned}7. \quad 42 &= 7 \times 6 \\&= 7 \times l \times m\end{aligned}$$

$$\begin{aligned}8. \quad 60 &= 2 \times 30 \\&= 2 \times 3 \times 10 \\&= 2 \times 3 \times d \times e\end{aligned}$$

Name the factorizations that are prime factorizations.

Rewrite the other factorizations as prime factorizations.

$$9. \quad 28 = 2 \times 2 \times 7$$

$$10. \quad 45 = 5 \times 3 \times 3$$

$$11. \quad 24 = 2 \times 2 \times 6$$

$$12. \quad 72 = 2 \times 3 \times 3 \times 4$$

$$13. \quad 66 = 2 \times 11 \times 3$$

$$14. \quad 96 = 3 \times 8 \times 2 \times 2$$

$$15. \quad 102 = 3 \times 17 \times 2$$

$$16. \quad 144 = 2 \times 2 \times 3 \times 12$$

Name the prime factorization of

$$17. \quad 32$$

$$19. \quad 20$$

$$21. \quad 64$$

$$23. \quad 56$$

$$25. \quad 90$$

$$27. \quad 72$$

$$18. \quad 28$$

$$20. \quad 16$$

$$22. \quad 76$$

$$24. \quad 100$$

$$26. \quad 84$$

$$28. \quad 121$$

From the commutative property we know that the order of the factors is not important. For example,
 $2 \times 3 \times 5 = 3 \times 2 \times 5 = 30$.

Complete the sentence.

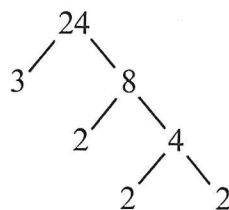
29. There is only _____ prime factorization of any number.

We may show the prime factorization of 24 by a **factor tree**.

Step 1: Rename 24 as 3×8 .

Step 2: Rename 8 as 2×4 .

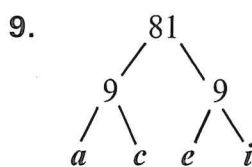
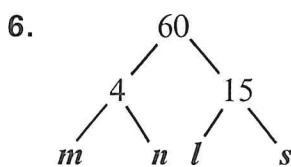
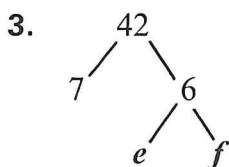
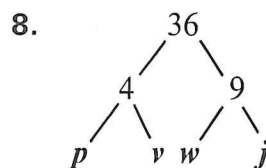
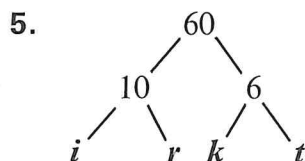
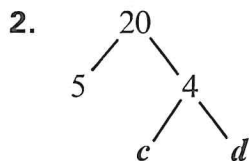
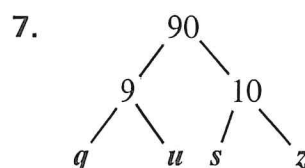
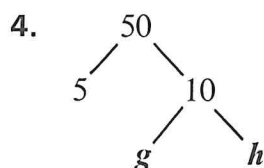
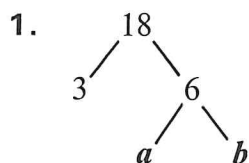
Step 3: Rename 4 as 2×2 .



The numbers at the end of the branches are primes. The *prime factorization* of 24 is $3 \times 2 \times 2 \times 2$. We may use exponents and write this as 3×2^3 .

EXERCISES

Complete the factor trees and name the prime factorizations.



Draw a factor tree for each number, then name the prime factorization using exponents.

10. 28 11. 16 12. 51 13. 45 14. 44 15. 105

Answer the questions.

16. Does a prime number have a factor tree?
 17. Does every whole number, greater than 1, which is not a prime number have a factor tree?

Prime factorizations

We may name the prime factorization of 24 by repeated division, as here.

step 1

$$\begin{array}{r} 2 \overline{)24} \\ 12 \end{array}$$

step 2

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 6 \end{array}$$

step 3

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

The prime factorization is $2 \times 2 \times 2 \times 3 = 2^3 \times 3$

EXERCISES

Complete the examples and name the factorizations.

$$\begin{array}{r} 1. \quad 3 \overline{)36} \\ 3 \overline{)a} \\ 2 \overline{)c} \\ e \end{array}$$

$$\begin{array}{r} 3. \quad 2 \overline{)42} \\ 3 \overline{)n} \\ r \end{array}$$

$$\begin{array}{r} 5. \quad 2 \overline{)50} \\ 5 \overline{)u} \\ v \end{array}$$

$$\begin{array}{r} 7. \quad 2 \overline{)100} \\ 2 \overline{)z} \\ 5 \overline{)a} \\ c \end{array}$$

$$\begin{array}{r} 9. \quad 2 \overline{)90} \\ 3 \overline{)n} \\ 3 \overline{)r} \\ s \end{array}$$

$$\begin{array}{r} 11. \quad 3 \overline{)162} \\ 3 \overline{)z} \\ 3 \overline{)a} \\ 3 \overline{)c} \\ e \end{array}$$

$$\begin{array}{r} 2. \quad 3 \overline{)45} \\ 3 \overline{)e} \\ i \end{array}$$

$$\begin{array}{r} 4. \quad 7 \overline{)70} \\ 5 \overline{)r} \\ s \end{array}$$

$$\begin{array}{r} 6. \quad 3 \overline{)75} \\ 5 \overline{)w} \\ x \end{array}$$

$$\begin{array}{r} 8. \quad 2 \overline{)84} \\ 2 \overline{)e} \\ 3 \overline{)i} \\ m \end{array}$$

$$\begin{array}{r} 10. \quad 2 \overline{)80} \\ 2 \overline{)u} \\ 2 \overline{)v} \\ 2 \overline{)w} \\ x \end{array}$$

$$\begin{array}{r} 12. \quad 2 \overline{)120} \\ 2 \overline{)i} \\ 2 \overline{)m} \\ 3 \overline{)n} \\ r \end{array}$$

Use repeated division to name the prime factorization of

13. 18

14. 20

15. 60

16. 30

17. 48

18. 54

On page 123 you learned that naming the square root is the opposite of squaring. A square is the product of 2 equal factors: $5^2 = 5 \times 5 = 25$, so a square root is 1 of 2 equal factors. $\sqrt{25} = \sqrt{5 \times 5} = 5$; $\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3} = 2 \times 3 = 6$

Use prime factorizations to name the square roots.

19. $\sqrt{9}$

22. $\sqrt{100}$

25. $\sqrt{64}$

28. $\sqrt{256}$

20. $\sqrt{16}$

23. $\sqrt{81}$

26. $\sqrt{144}$

29. $\sqrt{169}$

21. $\sqrt{36}$

24. $\sqrt{49}$

27. $\sqrt{324}$

30. $\sqrt{225}$



$$A = \{\text{factors of } 24\} = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

$$B = \{\text{factors of } 30\} = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

$$A \cap B = \{\text{common factors of } 24 \text{ and } 30\} = \{1, 2, 3, 6\}$$

The **Greatest Common Factor (GCF)** of 24 and 30 is **6**.

EXERCISES

Name the common factors and the greatest common factor for each pair of numbers.

1. 18 and 24

3. 40 and 60

5. 12 and 60

2. 30 and 48

4. 36 and 27

6. 14 and 42

Study this method for using prime factorizations to name the greatest common factor of 24 and 30.

The prime factorization of 24 is

$$2 \times 2 \times 2 \times 3$$

The prime factorization of 30 is

$$2 \times 3 \times 5$$

The Greatest Common Factor of 24 and 30 is

$$2 \times 3 = 6$$

Use prime factorizations to name the greatest common factor. The first one is done for you.

7. $30 = 2 \times 3 \times 5$

$$18 = 2 \times 3 \times 3$$

$$\text{GCF} = 2 \times 3 = 6$$

11. $60 = 2 \times 2 \times 3 \times 5$

$$20 = 2 \times 2 \times 5$$

$$\text{GCF} = d$$

8. $27 = 3 \times 3 \times 3$

$$30 = 2 \times 3 \times 5$$

$$\text{GCF} = a$$

12. $66 = 2 \times 3 \times 11$

$$88 = 2 \times 2 \times 2 \times 11$$

$$\text{GCF} = e$$

9. $24 = 2 \times 2 \times 2 \times 3$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{GCF} = b$$

13. $70 = 2 \times 5 \times 7$

$$42 = 2 \times 3 \times 7$$

$$\text{GCF} = f$$

10. $27 = 3 \times 3 \times 3$

$$54 = 2 \times 3 \times 3 \times 3$$

$$\text{GCF} = c$$

14. $125 = 5 \times 5 \times 5$

$$75 = 3 \times 5 \times 5$$

$$\text{GCF} = g$$



Least common multiple

$$A = \{\text{multiples of 2 which are less than 20}\} = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$$

$$B = \{\text{multiples of 3 which are less than 20}\} = \{0, 3, 6, 9, 12, 15, 18\}$$

$$A \cap B = \{\text{common multiples of 2 and 3 which are less than 20}\} = \{0, 6, 12, 18\}$$

The **Least Common Multiple (LCM)** of 2 and 3 is 6. Notice that although 0 is the least member of $A \cap B$, it is never used as the LCM.

EXERCISES

Name the common multiples, less than 50, and the least common multiples (LCM) of the pairs of numbers.

1. 3 and 6

3. 4 and 10

5. 3 and 9

2. 2 and 4

4. 5 and 15

6. 3 and 5

Study this method for using prime factorizations to name the least common multiple (LCM) of 24 and 30.

The prime factorization of 24 is

$$2 \times 2 \times 2 \times 3$$

The prime factorization of 30 is

$$2 \times 3 \times 5$$

The least common multiple of 24 and 30 is

$$2 \times 2 \times 2 \times 3 \times 5 = 120.$$

Use the prime factorizations to name the least common multiples. The first one is done for you.

7. $20 = 2 \times 2 \times 5$

$18 = 2 \times 3 \times 3$

$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 5 = 180$

9. $27 = 3 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

$\text{LCM} = c$

8. $18 = 2 \times 3 \times 3$

$30 = 2 \times 3 \times 5$

$\text{LCM} = a$

10. $60 = 2 \times 2 \times 3 \times 5$

$20 = 2 \times 2 \times 5$

$\text{LCM} = d$

Use the prime factorization method to name the least common multiples of each pair of numbers.

11. 6 and 5

13. 20 and 24

15. 9 and 15

12. 20 and 10

14. 36 and 18

16. 8 and 24

$A = \{\text{factors of } 30\}$
 $= \{1, 2, 3, 5, 6, 10, 15, 30\}$

$B = \{\text{factors of } 45\}$
 $= \{1, 3, 5, 9, 15, 45\}$

$A \cap B = \{1, 3, 5, 15\}$

The GCF of 30 and 45 is 15.

$C = \{\text{multiples of } 6 \text{ which are less than } 50\}$
 $= \{0, 6, 12, 18, 24, 30, 36, 42, 48\}$

$D = \{\text{multiples of } 8 \text{ which are less than } 50\}$
 $= \{0, 8, 16, 24, 32, 40, 48\}$

$C \cap D = \{0, 24, 48\}$

The LCM of 6 and 8 is 24.

EXERCISES

Use the set method to name the GCF of

- | | | | |
|--------------|--------------|--------------|--------------|
| 1. 21 and 28 | 3. 16 and 28 | 5. 24 and 56 | 7. 27 and 36 |
| 2. 30 and 42 | 4. 15 and 21 | 6. 36 and 54 | 8. 40 and 24 |

Use the set method to name the LCM of

- | | | | |
|--------------|---------------|---------------|---------------|
| 9. 6 and 10 | 11. 8 and 12 | 13. 16 and 40 | 15. 30 and 20 |
| 10. 4 and 10 | 12. 14 and 21 | 14. 9 and 12 | 16. 12 and 15 |

Use the prime factorization method to name the GCF of

- | | | | |
|---------------|---------------|---------------|----------------|
| 17. 25 and 30 | 19. 48 and 40 | 21. 24 and 32 | 23. 42 and 48 |
| 18. 60 and 36 | 20. 72 and 96 | 22. 12 and 30 | 24. 60 and 105 |

Use the prime factorization method to name the LCM of

- | | | | |
|---------------|---------------|---------------|---------------|
| 25. 15 and 20 | 27. 20 and 16 | 29. 12 and 18 | 31. 9 and 15 |
| 26. 12 and 30 | 28. 20 and 25 | 30. 8 and 28 | 32. 35 and 21 |

If we wish to arrange a set of 6 dots and a set of 8 dots in rows of the same size, the greatest number of dots in each row is the GCF of 6 and 8, that is, 2.

Use the GCF idea to name the greatest number of dots in each row. Arrange the dots in rows of the same size for sets of

- | | | |
|--------------|---------------|---------------|
| 33. 8 and 12 | 34. 10 and 15 | 35. 12 and 16 |
|--------------|---------------|---------------|

*Use the LCM idea to name the smallest number of dots that can be arranged exactly into rows of either

- | | | | | |
|------------|------------|------------|--------------|--------------|
| 36. 4 or 5 | 37. 4 or 6 | 38. 6 or 8 | 39. 12 or 18 | 40. 12 or 54 |
|------------|------------|------------|--------------|--------------|



Practice page

Name the answers.

1. Divide 10,920 by a. 2 b. 5 c. 3 d. 8.
2. From 7896 subtract a. 1869 b. 7468 c. 2009 d. 4789.
3. To 14,679 add a. 2309 b. 1964 c. 3089 d. 3546.
4. Multiply 4769 by a. 7 b. 9 c. 38 d. 56.

Use the function rule on the left to complete the ordered pairs.

5. $(3 \times m) + 2 = f(m)$: $\{(2, 8), (7, 23), (8, a), (9, e), (11, c), (14, r)\}$
6. $(5 \times m) + 3 = f(m)$: $\{(3, 18), (4, 23), (7, a), (8, e), (12, c), (21, r)\}$
7. $(7 \times m) + 4 = f(m)$: $\{(2, 18), (4, 32), (6, a), (7, e), (12, c), (15, r)\}$
8. $(8 \times m) + 6 = f(m)$: $\{(2, 22), (3, 30), (5, a), (9, e), (15, c), (21, r)\}$
9. $(9 \times m) + 5 = f(m)$: $\{(3, 32), (4, 41), (5, a), (7, e), (16, c), (28, r)\}$
10. $(8 \times m) + 7 = f(m)$: $\{(2, 23), (5, 47), (9, a), (12, e), (15, c), (24, r)\}$

Complete and check this number pattern.

$$\begin{array}{rcl}
 11. & (1 \times 9) + 2 & = 11 \\
 & (12 \times 9) + 3 & = 111 \\
 & (123 \times 9) + 4 & = 1111 \\
 & \vdots & \vdots \\
 & (123,456,789 \times 9) + 10 & = 1,111,111,111
 \end{array}$$

*Name the numerals to replace the letters in the examples.

$$\begin{array}{r}
 12. \quad ae9c \\
 - 7c2e \\
 \hline
 ce62
 \end{array}$$

$$\begin{array}{r}
 13. \quad 90c8 \\
 - a8ec \\
 \hline
 acae
 \end{array}$$

$$\begin{array}{r}
 14. \quad aaba \\
 + 2ba2 \\
 \hline
 b88b
 \end{array}$$

$$\begin{array}{r}
 15. \quad 7aaa \\
 - aabb \\
 \hline
 a9b7
 \end{array}$$

$$\begin{array}{r}
 16. \quad a23 \\
 \times 4a \\
 \hline
 3c3e \\
 n4rn0 \\
 \hline
 nea5e
 \end{array}$$

$$\begin{array}{r}
 17. \quad 2n \\
 a7 \overline{)3ea} \\
 \hline
 nr \\
 51 \\
 \hline
 51
 \end{array}$$

$$\begin{array}{r}
 18. \quad bb \\
 3a \overline{)79b} \\
 \hline
 7b \\
 7b \\
 \hline
 7b
 \end{array}$$

$$\begin{array}{r}
 19. \quad ba \\
 ab \overline{)57a} \\
 \hline
 ab \\
 b6a \\
 \hline
 b6a
 \end{array}$$

More about factors

$$A = \{\text{factors of } 10\} = \{1, 2, 5, 10\}$$

The sum of the factors of 10, other than 10 itself, is

$$1 + 2 + 5 = 8 \\ 8 < 10$$

$$B = \{\text{factors of } 24\} = \{1, 2, 3, 4, \\ 6, 8, 12, 24\}$$

The sum of the factors of 24, other than 24 itself, is

$$1 + 2 + 3 + 4 + 6 + 8 + 12 = 36 \\ 36 > 24$$

The sum of the factors of a number, other than the number itself, may be less than or greater than the number itself.

EXERCISES

Name the answers.

1. Name the numbers in the set $\{14, 18, 27, 30, 32, 42, 62, 64, 65\}$ which have the sum of their factors, other than the number itself, greater than the number.

A number is said to be “**perfect**” if the sum of its factors, not counting the number, is equal to the number itself. For instance, the factors of 6 are 1, 2, 3, 6, and $1 + 2 + 3 = 6$.

2. The only other perfect number less than 100 is 28. Check that 28 is a perfect number.

Name the factors of the numbers.

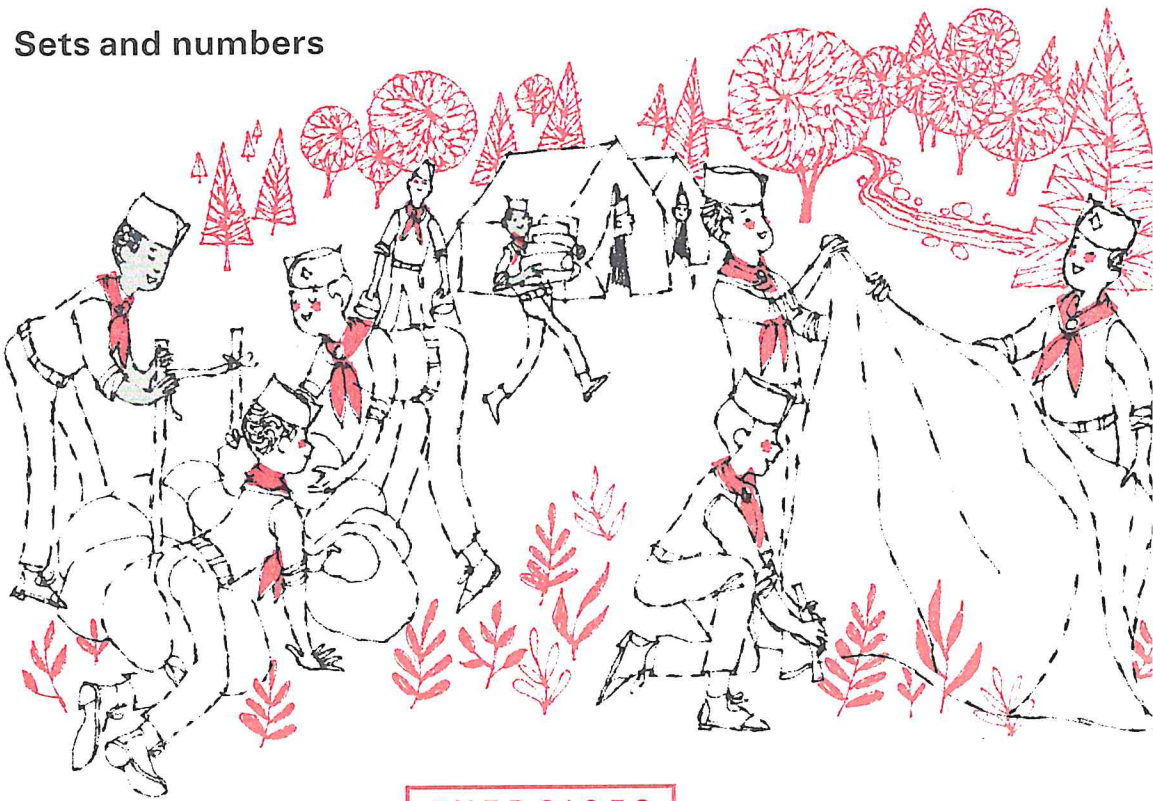
- | | | | | | |
|-------|-------|--------|---------|---------|---------|
| 3. 5 | 5. 7 | 7. 11 | 9. 13 | 11. 17 | 13. 19 |
| 4. 25 | 6. 49 | 8. 121 | 10. 169 | 12. 289 | 14. 361 |

Write the numerals to complete the sentences.

15. A prime has ? factors.
16. The square of a prime has ? factors.

Write *odd* or *even* to complete the sentences.

17. A number that is a square has an ? number of factors.
18. A number that is not a square has an ? number of factors.



EXERCISES

1. Twenty-seven Boy Scouts are going on a camping trip over the weekend. If 4 boys can sleep in each tent, how many tents will be needed?
2. Tom and Jim are in charge of preparing the first breakfast. If they have 5 dozen eggs to share among the boys, what is the greatest number of eggs each boy can have? How many will be left over?
3. The boys brought 8 cases of soda drink. Each case contains 24 bottles. If each cooler bag can hold 9 bottles, how many cooler bags will be needed to hold the soda drink?
4. If there are 80 apples, are there enough to give each boy 3 apples?
5. Clean up duty at the end of each meal requires 5 boys. If 6 meals are served during the weekend, will any of the boys be required to serve on clean up duty more than once?
6. Tony is in charge of recreation. If one of the games requires 6 boys on a team, what is the greatest number of teams that can be formed?



Answer the questions in A as quickly as you can. Then see if you can answer the questions in B even more quickly.

A

1. $84 + 6 + 9 + 7 + 4 = n$
2. Name the product of 673 and 9.
3. Which is greater:
 13×7 or 12×8 ?
4. Name the remainder when 704 is divided by 6.
5. What is the cost of
 - a. 7 items at 53¢ each?
 - b. 9 items at 59¢ each?
6. What is the cost of 1 item if 7 of these items cost \$4.69?
7. How many inches are there in 13 feet 4 inches?
8. Name the whole number which is greater than 23,483 and less than 23,485.
9. Name the least whole number you can with 4 digits.
10. What is $A \cup B$ if
 $A = \{1, 2, 3, 4, 5, 6\}$
 $B = \{3, 4, 5, 6, 7, 8\}$?

B

1. $3 \times 487 = n$
2. $703 - 687 = n$
3. Which is less:
 5×24 or 7×17 ?
4. Name the quotient when 903 is divided by 7.
5. If travel by bus costs 4¢ a mile, what is the cost of traveling 529 miles by bus?
6. Name the difference between
 - a. 713 and 568.
 - b. (7×12) and (5×16) .
7. How many pints are there in 12 gallons 2 quarts?
8. What is the total value of the 6 in 56,838,297?
9. Name the greatest whole number you can with 5 digits.
10. What is $C \cap D$ if
 $C = \{\text{odd numbers less than } 20\}$
 $D = \{\text{multiples of } 3\}$?

● **Multiples of 2, 3, 5 and 10.**

$$P = \{\text{multiples of 2}\} = \{0, 2, 4, 6, 8, \dots\}$$

$$Q = \{\text{multiples of 5}\} = \{0, 5, 10, 15, 20, \dots\}$$

$$R = \{\text{multiples of 10}\} = \{0, 10, 20, 30, 40, \dots\}$$

Name the next 10 members of

1. P

2. Q

3. R

Name the digits in the ones' place of the first 15 members of

4. P

5. Q

6. R

Complete the sentences.

7. A number is a multiple of 2 if the digit in the ones' place is a, b, c, d , or e .
8. A number is a multiple of 5 if the digit in the ones' place is f or g .
9. A number is a multiple of 10 if the digit in the ones' place is h .

$$S = \{\text{multiples of 3}\} = \{0, 3, 6, 9, 12, \dots\}.$$

10. Name the next 10 members of S .

11. Name the sum of the digits of each of the first 15 members of S . If any sum is greater than 9, find the sum of its digits.

12. Divide each sum in exercise 11 by 3.

Complete the sentence.

13. A number is a multiple of 3 if the sum of its digits is a ____ of 3.

Name the remainder when the divisor is 3 and the dividend is

14. 73

15. 65

16. 157

17. 834

18. 729

Use the rule for recognizing multiples to find which of these numbers are multiples of 2, 3, 5, or 10.

(Some are multiples of more than one.)

19. 93,423

21. 729

23. 315

25. 726,846

27. 730

29. 2226

20. 602

22. 792

24. 215

26. 112

28. 4836

30. 9183

The multiples of 4 (less than 100) may be arranged as follows:

0	4	8	12	16
20	24	28	32	36
40	44	48	52	56
60	64	68	72	76
80	84	88	92	96

or

0	4	8	12	16
$20 + 0$	$20 + 4$	$20 + 8$	$20 + 12$	$20 + 16$
$40 + 0$	$40 + 4$	$40 + 8$	$40 + 12$	$40 + 16$
$60 + 0$	$60 + 4$	$60 + 8$	$60 + 12$	$60 + 16$
$80 + 0$	$80 + 4$	$80 + 8$	$80 + 12$	$80 + 16$

1. Name the digits in the ones' place in these multiples of 4.
2. If the digit in the ones' place is 0, 4 or 8, is the digit in the tens' place even or odd?
3. If the digit in the ones' place is 2 or 6, is the digit in the tens' place even or odd?
4. Write a rule for recognizing a multiple of 4, less than 100.

Study the equations.

$$132 = 100 + 32$$

$$= (4 \times 25) + 32$$

$$332 = 300 + 32$$

$$= (3 \times 4 \times 25) + 32$$

5. Is 100 a multiple of 4?
6. Is any multiple of 100 a multiple of 4?
7. Are 132 and 332 multiples of 4?
8. Write a rule for recognizing multiples of 4, greater than 100.

Study the equations.

$$42 = 2 \times 21$$

$$= 2 \times 3 \times 7$$

$$= 6 \times 7$$

$$42 = 3 \times 14$$

$$= 3 \times 2 \times 7$$

$$= 6 \times 7$$

42 is a multiple of 2 and of 3.

42 is a multiple of 6.

9. Write a rule for recognizing a multiple of 6.
10. Use the rules from exercises 4, 8 and 9 to name the multiples of 4 or 6 in this set.
{52, 78, 102, 126, 128, 144, 155, 324, 768}
11. Name the multiples of 2, 3, 4, 5, 6, or 10 in this set.
{75, 275, 258, 456, 760, 583, 278, 924, 650}

● Rules for divisibility

$V = \{\text{multiples of } 9\} = \{0, 9, 18, 27, 36, \dots\}$

1. Name the next 6 multiples of 9.
2. Name the sum of the digits of the multiples of 9.
3. A number is a multiple of 9 if the sum of its digits is ____?

If the sum of the digits of a number is a multiple of 9, then the number, itself, is a multiple of 9. For example, 6282: $6 + 2 + 8 + 2 = 18$ or 2×9 , thus 6282 is a multiple of 9.

Name the numbers in $\{24, 83, 189, 243, 562, 558, 63, 254, 82, 215\}$ that are divisible by (are multiples of)

4. 9 5. 6 6. 3

Name the numbers in $\{672, 345, 822, 346, 782, 593, 277, 91, 325, 474\}$ that are divisible by

7. 2 9. 3 11. 5
8. 4 10. 6 12. 9

Study this set of multiples of 8: $\{144, 5144, 6144, 8144, 14,144\}$.

13. Use this set to name a multiple of 8 between 5000 and 6000; between 6000 and 7000; between 8000 and 9000.
14. Name these quotients: $1000 \div 8$; $2000 \div 8$; $3000 \div 8$.
15. Use the ideas in 13 and 14 to write a rule for deciding whether a number is divisible by 8.

If a number is not divisible by 2, 3, 4, 5, 6, 8, or 9, we can name the remainder from the rules for divisibility on these pages. For example, if the sum of the digits is 2 more than a multiple of 3, the number is 2 more than a multiple of 3. If the tens and ones in any number are 3 more than a multiple of 4, the number is 3 more than a multiple of 4.

Use the rules for divisibility on these pages to name the remainders when the numbers are divided by

- a. 3 b. 9 c. 4 d. 5

16. 73 17. 107 18. 143 19. 867 20. 962 21. 1639 22. 5727

Checking by "casting out nines"

876 leaves a remainder of 3 when divided by 9; the sum of the digits of 876 is 3 more than a multiple of 9, that is, $3 = 21 - (2 \times 9)$. We will call the 3 the **9-sum** of 876. It names the remainder when we **cast out** the multiples of 9.

$\begin{array}{r} 347 \\ + 236 \\ \hline \end{array}$	$3 + 4 + 7 = 14,$ $2 + 3 + 6 = 11,$	$14 - (1 \times 9) = 5$ $11 - (1 \times 9) = +2$ $\hline 7$	The <i>9-sum</i> of the sum of the <i>9-sums</i> of 347 and 236 equals the <i>9-sum</i> of 583.
$\begin{array}{r} 583 \\ \hline \end{array}$	$5 + 8 + 3 = 16,$	$16 - (1 \times 9) = 7$	
$\begin{array}{r} 723 \\ - 236 \\ \hline \end{array}$	$7 + 2 + 3 = 12,$ $2 + 3 + 6 = 11,$	$12 - (1 \times 9) = 3$ $11 - (1 \times 9) = -2$ $\hline 1$	The <i>9-sum</i> of the difference of the <i>9-sums</i> of 723 and 236 equals the <i>9-sum</i> of 487.
$\begin{array}{r} 487 \\ \hline \end{array}$	$4 + 8 + 7 = 19,$	$19 - (2 \times 9) = 1$	
$\begin{array}{r} 296 \\ \times 21 \\ \hline 296 \\ 592 \\ \hline 6216 \end{array}$	$2 + 9 + 6 = 17,$ $2 + 1 = 3,$	$17 - (1 \times 9) = 8$ $3 = \times 3$ $\hline 24$	The <i>9-sum</i> of the product of the <i>9-sums</i> of 296 and 21 equals the <i>9-sum</i> of 6216.
$\begin{array}{r} 6216 \\ \hline \end{array}$	$6 + 2 + 1 + 6 = 15,$	$24 - (2 \times 9) = 6$ $15 - (1 \times 9) = 6$	

EXERCISES

Name the sums and differences. Check your work by "casting out nines."

1. $\begin{array}{r} 344 \\ + 698 \\ \hline \end{array}$	3. $\begin{array}{r} 514 \\ - 392 \\ \hline \end{array}$	5. $\begin{array}{r} 293 \\ + 786 \\ \hline \end{array}$	7. $\begin{array}{r} 5043 \\ - 2981 \\ \hline \end{array}$	9. $\begin{array}{r} 5146 \\ + 2178 \\ \hline \end{array}$
2. $\begin{array}{r} 617 \\ + 309 \\ \hline \end{array}$	4. $\begin{array}{r} 870 \\ - 516 \\ \hline \end{array}$	6. $\begin{array}{r} 3542 \\ + 6718 \\ \hline \end{array}$	8. $\begin{array}{r} 6273 \\ + 4119 \\ \hline \end{array}$	10. $\begin{array}{r} 3941 \\ - 2176 \\ \hline \end{array}$

Name the products. Check your work by "casting out nines."

11. $\begin{array}{r} 23 \\ \times 7 \\ \hline \end{array}$	12. $\begin{array}{r} 54 \\ \times 9 \\ \hline \end{array}$	13. $\begin{array}{r} 687 \\ \times 2 \\ \hline \end{array}$	14. $\begin{array}{r} 954 \\ \times 6 \\ \hline \end{array}$	15. $\begin{array}{r} 318 \\ \times 73 \\ \hline \end{array}$	*16. $\begin{array}{r} 48269 \\ \times 47 \\ \hline \end{array}$
---	---	--	--	---	--



Base two numerals

If we counted objects in sets of 2, 2×2 , $2 \times 2 \times 2$, and so on, we would have a base two system of numeration. This is called the **binary** system.

EXERCISES

Copy and complete the last column of this chart.

1.

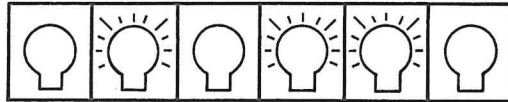
base ten grouping	base ten numeral	base two grouping	base two numeral
x	1	x	1
xx	2	x x	10_{two}
xxx	3	x x x	11_{two}
xxxx	4	xx xx	100_{two}
xxxxx	5	xx x xx	<i>a</i>
xxxxx x	6	xx x xx x	<i>e</i>
xxxxx xx	7	xx x xx x x	<i>c</i>
xxxxx xxx	8	xxxx xxxx	1000_{two}
xxxxx xxxx	9	xxxx xxxx x	<i>n</i>
xxxxx xxxx	10	xxxx x xxxx x	<i>r</i>

Complete this chart which shows place values in the binary system (named in the decimal system).

2.	2^5	2^4	2^3	2^2	2	1
	32	<i>a</i>	<i>e</i>	4	<i>c</i>	1
	thirty-twos'	sixteens'	eights'	fours'	twos'	ones'

More on binary numerals

There are only 2 digits in the binary system — 0 and 1. Because an electric current has 2 possibilities — either it flows or it doesn't — the binary system is suitable for electric computers. We may show the digits in each place by a light. If the light is on for any place, there is 1 in the place; if the light is not on, there is 0.



The diagram shows 10110_{two} or

$$(1 \times 2 \times 2 \times 2 \times 2) + (0 \times 2 \times 2 \times 2) + (1 \times 2 \times 2) + (1 \times 2) + 0.$$

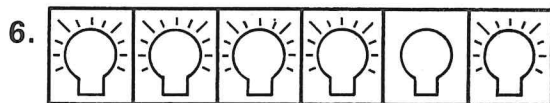
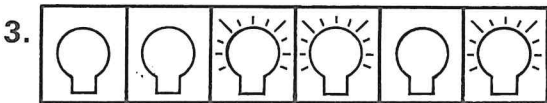
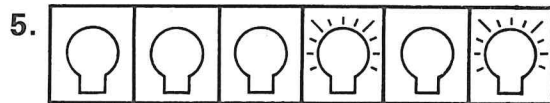
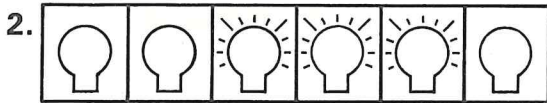
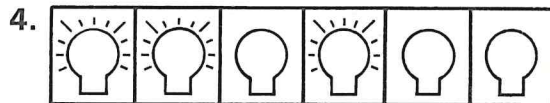
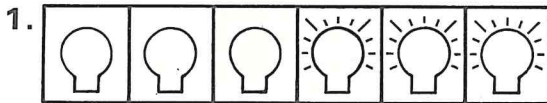
In the decimal system this is:

$$(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + 0$$

or $(1 \times 16) + (1 \times 4) + 2 = 22_{\text{ten}}.$

EXERCISES

Name each number shown in the diagram as a binary numeral.



Name the binary numerals with decimal numerals.

7. 10_{two}

9. 111_{two}

11. 1100_{two}

13. 10000_{two}

15. 10111_{two}

8. 101_{two}

10. 1001_{two}

12. 1111_{two}

14. 10011_{two}

16. 100011_{two}

* Use binary numerals to name the decimal numerals.

17. 4_{ten}

19. 10_{ten}

21. 18_{ten}

23. 24_{ten}

25. 41_{ten}

18. 7_{ten}

20. 11_{ten}

22. 21_{ten}

24. 32_{ten}

26. 50_{ten}

Binary numerals

Here are all the basic addition and multiplication facts for binary numerals.

Addition: $0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$, $1 + 1 = 10_{\text{two}}$

Multiplication: $0 \times 0 = 0$, $0 \times 1 = 0$, $1 \times 0 = 0$, $1 \times 1 = 1$

We can use the properties of addition and multiplication to complete any of the addition or multiplication, as here:

$$\begin{aligned} 101_{\text{two}} + 1 &= (100_{\text{two}} + 1) + 1 \\ &= 100_{\text{two}} + (1 + 1) \\ &= 100_{\text{two}} + 10_{\text{two}} \\ &= 110_{\text{two}} \end{aligned}$$

Associative property

Renaming $1 + 1$ as 10_{two}

Renaming $100_{\text{two}} + 10_{\text{two}}$ as 110_{two}

EXERCISES

Copy and complete the binary system addition-multiplication charts.

1.

+	0	1
0	0	<i>a</i>
1	<i>e</i>	<i>c</i>

2.

$\times$	0	1
0	0	<i>r</i>
1	<i>n</i>	<i>s</i>

Solve each equation below, then name the property (or properties) of addition or multiplication shown in the equation.

a. The commutative property of addition.

b. The commutative property of multiplication.

c. The associative property of addition.

d. The associative property of multiplication.

e. The distributive property of multiplication with respect to addition.

f. Zero as an addend.

g. One as a factor.

3. $11_{\text{two}} + 10_{\text{two}} = 10_{\text{two}} + a_{\text{two}}$

4. $(101_{\text{two}} \times 11_{\text{two}}) \times 10_{\text{two}} = 101_{\text{two}} \times (11_{\text{two}} \times b_{\text{two}})$

5. $110_{\text{two}} + (11_{\text{two}} + 101_{\text{two}}) = (110_{\text{two}} + c_{\text{two}}) + 101_{\text{two}}$

6. $1_{\text{two}} \times 1001_{\text{two}} = d_{\text{two}}$

7. $111_{\text{two}} + 0 = e_{\text{two}}$

8. $10_{\text{two}} \times 11_{\text{two}} = f_{\text{two}} \times 10_{\text{two}}$

9. $11_{\text{two}} \times (10_{\text{two}} + 110_{\text{two}}) = (11_{\text{two}} \times 10_{\text{two}}) + (g_{\text{two}} \times h_{\text{two}})$

Study these examples of addition, multiplication, and subtraction using binary numerals.

$$\begin{array}{r} 111_{\text{two}} \\ + 10_{\text{two}} \\ \hline 1 \\ 100 \\ 100 \\ \hline 1001_{\text{two}} \end{array}$$

$$\begin{array}{r} 101_{\text{two}} \\ \times 11_{\text{two}} \\ \hline 101 \\ 1010 \\ \hline 1111_{\text{two}} \end{array}$$

$$\begin{array}{r} 1011_{\text{two}} \\ - 101_{\text{two}} \\ \hline 110_{\text{two}} \end{array} \quad (\text{remember } 1 + 1 = 10_{\text{two}})$$

EXERCISES

Name the sums, products or differences.

1. $\begin{array}{r} 10_{\text{two}} \\ + 10_{\text{two}} \\ \hline \end{array}$

4. $\begin{array}{r} 1101_{\text{two}} \\ + 1001_{\text{two}} \\ \hline \end{array}$

7. $\begin{array}{r} 100_{\text{two}} \\ \times 11_{\text{two}} \\ \hline \end{array}$

10. $\begin{array}{r} 1101_{\text{two}} \\ \times 101_{\text{two}} \\ \hline \end{array}$

13. $\begin{array}{r} 1001_{\text{two}} \\ - 100_{\text{two}} \\ \hline \end{array}$

2. $\begin{array}{r} 101_{\text{two}} \\ + 11_{\text{two}} \\ \hline \end{array}$

5. $\begin{array}{r} 11011_{\text{two}} \\ + 1101_{\text{two}} \\ \hline \end{array}$

8. $\begin{array}{r} 111_{\text{two}} \\ \times 10_{\text{two}} \\ \hline \end{array}$

11. $\begin{array}{r} 10_{\text{two}} \\ - 1_{\text{two}} \\ \hline \end{array}$

14. $\begin{array}{r} 1101_{\text{two}} \\ - 111_{\text{two}} \\ \hline \end{array}$

3. $\begin{array}{r} 100_{\text{two}} \\ + 111_{\text{two}} \\ \hline \end{array}$

6. $\begin{array}{r} 10_{\text{two}} \\ \times 10_{\text{two}} \\ \hline \end{array}$

9. $\begin{array}{r} 1001_{\text{two}} \\ \times 100_{\text{two}} \\ \hline \end{array}$

12. $\begin{array}{r} 101_{\text{two}} \\ - 11_{\text{two}} \\ \hline \end{array}$

15. $\begin{array}{r} 10100_{\text{two}} \\ - 1011_{\text{two}} \\ \hline \end{array}$

Study these short methods for naming sums and products (remember $10_{\text{two}} = 2_{\text{ten}}$).

$$\begin{array}{r} 111_{\text{two}} \\ + 10_{\text{two}} \\ \hline 1001_{\text{two}} \end{array}$$

$$\begin{array}{l} 101_{\text{two}} \times 10_{\text{two}} = 1010_{\text{two}} \\ 101_{\text{two}} \times 100_{\text{two}} = 10100_{\text{two}} \end{array}$$

$$\begin{array}{l} 101_{\text{two}} \times 1000_{\text{two}} = 101000_{\text{two}} \\ 101_{\text{two}} \times 10000_{\text{two}} = 1010000_{\text{two}} \end{array}$$

Use the short methods above to name the sums or products in the binary system.

16. $\begin{array}{r} 1011_{\text{two}} \\ + 1011_{\text{two}} \\ \hline \end{array}$

18. $\begin{array}{r} 10101_{\text{two}} \\ + 11011_{\text{two}} \\ \hline \end{array}$

20. $\begin{array}{r} 11_{\text{two}} \\ \times 10_{\text{two}} \\ \hline \end{array}$

22. $\begin{array}{r} 1000_{\text{two}} \\ \times 111_{\text{two}} \\ \hline \end{array}$

17. $\begin{array}{r} 10011_{\text{two}} \\ + 1011_{\text{two}} \\ \hline \end{array}$

19. $\begin{array}{r} 1001011_{\text{two}} \\ + 1011101_{\text{two}} \\ \hline \end{array}$

21. $\begin{array}{r} 110_{\text{two}} \\ \times 100_{\text{two}} \\ \hline \end{array}$

23. $\begin{array}{r} 1011_{\text{two}} \\ \times 1000_{\text{two}} \\ \hline \end{array}$



Clock arithmetic

Three hours after 8 o'clock is 11 o'clock.

$$3 + 8 \stackrel{12}{=} 11$$

Nine hours after 8 o'clock is 5 o'clock.

$$9 + 8 \stackrel{12}{=} 5$$

Three hours before 11 o'clock is 8 o'clock.

$$11 - 3 \stackrel{12}{=} 8$$

Nine hours before 5 o'clock is 8 o'clock.

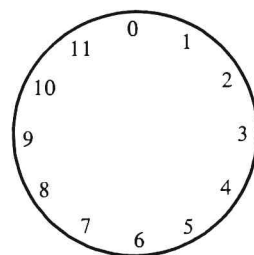
$$5 - 9 \stackrel{12}{=} 8$$

The number system here is not the same as the number system we have been using. In this system there are only 12 numbers — 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. Notice that we call 12 o'clock zero. The number sentences are not the same as the equations we have been using. We will call them **clock equations**. We read the second one for example, as “Nine plus eight equals five on the twelve clock.”

EXERCISES

Complete the sentences and then write them as clock equations.

1. Ten hours after 6 o'clock is ____?____ o'clock.
2. Six hours after 9 o'clock is ____?____ o'clock.
3. Six hours after 4 o'clock is ____?____ o'clock.
4. Three hours after 11 o'clock is ____?____ o'clock.
5. Eight hours before 5 o'clock is ____?____ o'clock.
6. Nine hours before 6 o'clock is ____?____ o'clock.



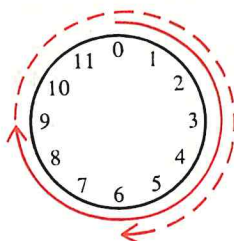
Complete the chart.

7.	time at first	3	8	4	7	9	2	8	6	3
	hours later	6	6	9	7	8	11	5	2	10
	time later	9	2	<i>a</i>	<i>e</i>	<i>c</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>

Solve the clock equations.

- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| 8. $6 + 8 \stackrel{12}{=} a$ | 14. $4 - 5 \stackrel{12}{=} g$ | 20. $3 - m \stackrel{12}{=} 8$ |
| 9. $4 + 9 \stackrel{12}{=} b$ | 15. $6 - 10 \stackrel{12}{=} h$ | 21. $n - 6 \stackrel{12}{=} 8$ |
| 10. $9 + 9 \stackrel{12}{=} c$ | 16. $4 + i \stackrel{12}{=} 2$ | 22. $2 - 4 \stackrel{12}{=} p$ |
| 11. $5 - 9 \stackrel{12}{=} d$ | 17. $j + 8 \stackrel{12}{=} 2$ | 23. $3 + r \stackrel{12}{=} 1$ |
| 12. $1 - 10 \stackrel{12}{=} e$ | 18. $6 + 9 \stackrel{12}{=} k$ | 24. $8 - 10 \stackrel{12}{=} s$ |
| 13. $3 + 11 \stackrel{12}{=} f$ | 19. $2 - l \stackrel{12}{=} 5$ | 25. $t + 6 \stackrel{12}{=} 6$ |

If we call 12 o'clock zero,
2 sets of 9 hours after zero
is 6 o'clock.



$$2 \times 9 \stackrel{12}{=} 6$$

EXERCISES

Complete the sentences, then write them as clock equations.

- Three sets of 5 hours after zero hour is ____?____ o'clock.
- Four sets of 4 hours after zero hour is ____?____ o'clock.
- Five sets of 2 hours after zero hour is ____?____ o'clock.

Solve the clock equations.

- $2 \times 8 \stackrel{12}{=} a$
- $4 \times 5 \stackrel{12}{=} e$
- $3 \times 9 \stackrel{12}{=} r$
- $2 \times 10 \stackrel{12}{=} u$
- $3 \times 5 \stackrel{12}{=} c$
- $2 \times 7 \stackrel{12}{=} n$
- $5 \times 7 \stackrel{12}{=} s$
- $5 \times 6 \stackrel{12}{=} v$

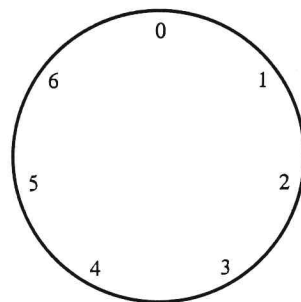
Copy and complete the addition and multiplication tables for 12-clock arithmetic.

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10				
4	4	5	6	7	8	9						
5	5	6	7	8	9							
6	6	7	8	9								
7	7	8	9	10								
8	8	9	10	11	0							
9	9	10	11	0	1							
10	10	11	0	1	2							
11	11	0	1	2	3	4						

×	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	0	2	4	6	8	10
3	0	3	6	9	0	3	6					
4	0	4	8	0	4	8						
5	0	5	10	3	8	1						
6	0	6	0	6	0	6						
7	0	7	2	9	4							
8	0	8	4	0	8							
9	0	9	6	3	0							
10	0	10	8	6	4							
11	0	11	10	9	8							

The seven-clock

This clock face shows the days of the week. Sunday is day zero and we write 1–6 for the other days of the week. We may think of this as a clock system using only the numbers in $\{0, 1, 2, 3, 4, 5, 6\}$. Since there are 7 numbers in the system, we will call this a **7-clock system**.



EXERCISES

Complete the sentences, then solve the 7-clock equations. The first two are done for you.

1. Four days after Friday is *Tuesday*. $5 + 4 \stackrel{7}{=} 2$
2. Three days before Monday is *Friday*. $1 - 3 \stackrel{7}{=} 5$
3. Two days after Wednesday is _____. $2 + 3 \stackrel{7}{=} a$
4. Four days after Thursday is _____. $4 + 4 \stackrel{7}{=} c$
5. Six days after Saturday is _____. $6 + 6 \stackrel{7}{=} e$
6. Three days before Tuesday is _____. $2 - 3 \stackrel{7}{=} n$
7. Four days before Wednesday is _____. $3 - 4 \stackrel{7}{=} r$
8. Five days before Sunday is _____. $0 - 5 \stackrel{7}{=} s$

Solve the 7-clock equations and then write “days of the week” sentences for each one.

- | | | |
|-------------------------------|-------------------------------|-------------------------------|
| 9. $3 + 2 \stackrel{7}{=} a$ | 14. $4 - 5 \stackrel{7}{=} f$ | 19. $6 - k \stackrel{7}{=} 3$ |
| 10. $4 + 3 \stackrel{7}{=} b$ | 15. $2 - 5 \stackrel{7}{=} g$ | 20. $4 - l \stackrel{7}{=} 5$ |
| 11. $6 + 1 \stackrel{7}{=} c$ | 16. $4 - 4 \stackrel{7}{=} h$ | 21. $4 + m \stackrel{7}{=} 1$ |
| 12. $6 + 4 \stackrel{7}{=} d$ | 17. $5 + i \stackrel{7}{=} 2$ | 22. $1 - n \stackrel{7}{=} 2$ |
| 13. $5 - 3 \stackrel{7}{=} e$ | 18. $3 + j \stackrel{7}{=} 1$ | 23. $2 - p \stackrel{7}{=} 6$ |

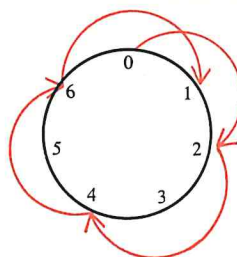
Solve the 7-clock equations.

- | | |
|-------------------------------------|---|
| 24. $(3 + 2) + 4 \stackrel{7}{=} a$ | 29. $5 + (6 + 3) \stackrel{7}{=} f$ |
| 25. $(5 + 6) + 3 \stackrel{7}{=} b$ | 30. $(6 + 2) + 5 \stackrel{7}{=} g$ |
| 26. $6 + (2 + 5) \stackrel{7}{=} c$ | 31. $(6 + 6) + (6 + 6) \stackrel{7}{=} h$ |
| 27. $5 + (5 + 5) \stackrel{7}{=} d$ | 32. $4 \times 6 \stackrel{7}{=} i$ |
| 28. $(4 + 4) + 4 \stackrel{7}{=} e$ | 33. $3 \times 4 \stackrel{7}{=} j$ |

Multiplication on the 7-clock

We may define multiplication on the 7-clock by repeated addition of the same addend.

$$2 + 2 + 2 + 2 \stackrel{7}{=} 1, \text{ so } 4 \times 2 \stackrel{7}{=} 1$$



EXERCISES

Complete the 7-clock sentences.

1. $5 + 5 \stackrel{7}{=} 3$, so $2 \times 5 \stackrel{7}{=} a$
2. $3 + 3 + 3 + 3 \stackrel{7}{=} 5$, so $4 \times 3 \stackrel{7}{=} b$
3. $4 + 4 \stackrel{7}{=} c$, so $2 \times 4 \stackrel{7}{=} d$
4. $6 + 6 + 6 \stackrel{7}{=} e$, so $3 \times 6 \stackrel{7}{=} f$

Copy and complete the addition and multiplication tables for the 7-clock numbers.

addition

+	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

multiplication

$\times$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

Solve the 7-clock equations.

5. $3 \times 4 \stackrel{7}{=} a$
6. $6 \times 5 \stackrel{7}{=} b$
7. $5 \times 4 \stackrel{7}{=} c$
8. $(2 \times 3) \times 4 \stackrel{7}{=} d$
9. $5 \times 3 \times 2 \stackrel{7}{=} e$
10. $6 \times (6 \times 4) \stackrel{7}{=} f$
11. $(6 \times 6) \times 4 \stackrel{7}{=} g$
12. $2 \times (3 \times 4) \stackrel{7}{=} h$
13. $4 \times i \stackrel{7}{=} 5$
14. $3 \times j \stackrel{7}{=} 2$
15. $2 \times k \stackrel{7}{=} 1$
16. $5 \times l \stackrel{7}{=} 4$
17. $2 \times 6 \stackrel{7}{=} m$
18. $6 \times 2 \stackrel{7}{=} n$
19. $6 \times 6 \stackrel{7}{=} p$
20. $5 \times 5 \stackrel{7}{=} q$
21. $4 \times 4 \stackrel{7}{=} r$
22. $3 \times 3 \stackrel{7}{=} s$



Properties

Now that we have arithmetic systems for the 7-clock and the 12-clock we may check the addition and multiplication properties. For example, if we add or multiply 2 numbers on the 12-clock we will always get a number on the 12-clock. We say that the numbers on the 12-clock (or 7-clock) are **closed** with respect to addition and multiplication. This is also a property of the whole numbers.

EXERCISES

Solve the 7-clock and 12-clock equations.

- | | | |
|---|---|--|
| 1. $6 + 4 \stackrel{7}{=} a$ | 11. $(6 \times 4) \times 3 \stackrel{7}{=} p$ | 21. $5 \times 6 \stackrel{12}{=} j$ |
| 2. $4 + 6 \stackrel{7}{=} b$ | 12. $6 \times (4 \times 3) \stackrel{7}{=} q$ | 22. $9 \times 6 \stackrel{12}{=} k$ |
| 3. $3 + (4 + 5) \stackrel{7}{=} e$ | 13. $5 + 0 \stackrel{7}{=} r$ | 23. $6 \times 9 \stackrel{12}{=} l$ |
| 4. $(3 + 4) + 5 \stackrel{7}{=} f$ | 14. $0 + 3 \stackrel{7}{=} s$ | 24. $4 \times (3 \times 6) \stackrel{12}{=} m$ |
| 5. $4 \times 3 \stackrel{7}{=} i$ | 15. $1 \times 6 \stackrel{7}{=} t$ | 25. $(4 \times 3) \times 6 \stackrel{12}{=} n$ |
| 6. $3 \times 4 \stackrel{7}{=} j$ | 16. $5 + 9 \stackrel{12}{=} a$ | 26. $(7 \times 8) \times 4 \stackrel{12}{=} p$ |
| 7. $6 \times 2 \stackrel{7}{=} k$ | 17. $9 + 5 \stackrel{12}{=} b$ | 27. $7 \times (8 \times 4) \stackrel{12}{=} q$ |
| 8. $2 \times 6 \stackrel{7}{=} l$ | 18. $5 + (10 + 6) \stackrel{12}{=} e$ | 28. $4 + 0 \stackrel{12}{=} r$ |
| 9. $4 \times (3 \times 2) \stackrel{7}{=} m$ | 19. $(5 + 10) + 6 \stackrel{12}{=} f$ | 29. $1 \times 8 \stackrel{12}{=} t$ |
| 10. $(4 \times 3) \times 2 \stackrel{7}{=} n$ | 20. $6 \times 5 \stackrel{12}{=} i$ | 30. $5 \times 1 \stackrel{12}{=} u$ |

Using the answers to exercises 1–30, write *yes* or *no* to indicate whether these properties hold for arithmetic on a. the 7-clock, and b. the 12-clock.

- | | |
|----------------------------------|----------------------------------|
| 31. commutative (addition) | 34. associative (multiplication) |
| 32. commutative (multiplication) | 35. addition of zero |
| 33. associative (addition) | 36. multiplication by one |

Solve the clock equations.

- | | |
|--|--|
| 37. $3 \times (4 + 3) \stackrel{7}{=} a$ | 39. $6 \times (8 + 5) \stackrel{12}{=} e$ |
| 38. $(3 \times 4) + (3 \times 3) \stackrel{7}{=} b$ | 40. $(6 \times 8) + (6 \times 5) \stackrel{12}{=} f$ |
| 41. From your results in exercises 37–40 do you think the distributive property works on the 7-clock and on the 12-clock? Try some other examples. | |



We have discovered that some of the properties which work for the addition and multiplication of whole numbers also work for the numbers on the 7-clock and the 12-clock. But there are differences between the arithmetic of whole numbers and arithmetic on a clock. Study these examples.

For whole numbers,
 $4 \times a = 0$ means that a
 must be 0.

For numbers on the 12-clock,
 $2 \times 6 \stackrel{12}{=} 0$, but neither of the
 factors is 0.

If the product of 2 whole numbers is 0, then one of the numbers *must* be 0.

If the product of two 12-clock numbers is 0 it may be that *neither* of the numbers is 0.

EXERCISES

1. Study the multiplication table for the 12-clock on page 189, and list all the pairs of factors whose product is 0, with neither factor being 0. For example, $6 \times 10 \stackrel{12}{=} 0$.

For the equation $3 \times a = 21$, there is only one whole number, 7, which will make the sentence true. For the equation $4 \times a \stackrel{12}{=} 8$ there are 4 solutions, 2, 5, 8, and 11.

List *all* of the solutions for each of the following equations. The table on page 189 will help you.

- | | | | |
|------------------------------------|------------------------------------|------------------------------------|-------------------------------------|
| 2. $9 \times a \stackrel{12}{=} 6$ | 4. $3 \times c \stackrel{12}{=} 9$ | 6. $7 \times e \stackrel{12}{=} 6$ | 8. $6 \times g \stackrel{12}{=} 6$ |
| 3. $2 \times b \stackrel{12}{=} 6$ | 5. $5 \times d \stackrel{12}{=} 2$ | 7. $4 \times f \stackrel{12}{=} 4$ | 9. $10 \times h \stackrel{12}{=} 8$ |

Answer the questions.

10. If a and b are clock numbers and are shown on a circle, is it possible to go from a to b and also from b to a in the same direction on the clock?
11. If a and b are clock numbers, is it always possible to add to a to get b and also to add to b to get a ?



CHECKING UP

Name the set of factors for each number.

1. 12 2. 18 3. 24 4. 36 5. 56 6. 96 7. 144 8. 400

Use the sets in exercises 1–8 to name the common factors of

9. 12 and 18 11. 24 and 36 13. 36 and 96 15. 24 and 144
10. 12 and 24 12. 18 and 56 14. 96 and 144 16. 144 and 400

List the members of

17. the set of multiples of 2 which are less than 36.
18. the set of multiples of 3 which are less than 36.
19. the set of common multiples of 2 and 3 which are less than 36.

Name the prime numbers.

20. All the prime numbers less than 60.
21. 2 prime numbers with sum: a. 42 b. 28 c. 80 d. 36
22. 2 prime numbers with product: a. 21 b. 86 c. 57 d. 77

Use the prime factorizations to name the greatest common factors and least common multiples of the pairs of numbers.

23. $12 = 2 \times 2 \times 3$
 $24 = 2 \times 2 \times 2 \times 3$
GCF = *a* LCM = *b*
24. $18 = 2 \times 3 \times 3$
 $54 = 2 \times 3 \times 3 \times 3$
GCF = *c* LCM = *d*
25. $20 = 2 \times 2 \times 5$
 $50 = 2 \times 5 \times 5$
GCF = *e* LCM = *f*
26. $30 = 2 \times 3 \times 5$
 $70 = 2 \times 5 \times 7$
GCF = *g* LCM = *h*

Name the numbers in $S = \{822, 24, 91, 144, 296, 60, 96\}$ which are divisible by

27. 2 28. 3 29. 4 30. 5 31. 6 32. 8

Name the sums, differences, or products, then use “casting out nines” to check your work.

33. $\begin{array}{r} 3278 \\ - 2586 \\ \hline \end{array}$ 34. $\begin{array}{r} 3415 \\ + 6278 \\ \hline \end{array}$ 35. $\begin{array}{r} 582763 \\ + 405167 \\ \hline \end{array}$ 36. $\begin{array}{r} 54 \\ \times 27 \\ \hline \end{array}$ 37. $\begin{array}{r} 6137 \\ \times 84 \\ \hline \end{array}$



To name the **Greatest Common Factor** of 2 numbers, name the greatest factor in the intersection of their sets of factors.

To name the **Least Common Multiple** of 2 numbers, name the least multiple greater than zero in the intersection of their sets of multiples.

Name the common factors and greatest common factor of each pair of numbers.

1. 18 and 30

2. 24 and 60

3. 144 and 256

Name the common multiples, less than 30, and the least common multiple of each pair of numbers.

4. 2 and 3

5. 4 and 5

6. 6 and 9

Prime factorizations may be used to name the GCF and the LCM. Study these examples.

$$60 = 5 \times 2 \times 2 \times 3$$

$$\text{GCF} = 2 \times 2 \times 3 = 12$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{LCM} = 5 \times 2 \times 2 \times 3 \times 3 = 180$$

Use prime factorizations to name the LCM and GCF of each pair of numbers.

7. 24 and 36

8. 44 and 18

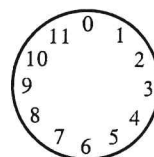
9. 15 and 75

A clock may be used to show a special kind of arithmetic. Study these 12-clock equations.

$$7 + 6 \stackrel{12}{=} 1$$

$$4 - 9 \stackrel{12}{=} 7$$

$$3 \times 6 \stackrel{12}{=} 6$$



Solve the 12-clock equations.

10. $3 + 7 \stackrel{12}{=} a$

12. $5 - 11 \stackrel{12}{=} c$

14. $2 \times 3 \stackrel{12}{=} e$

16. $4 \times 7 \stackrel{12}{=} g$

11. $8 + 5 \stackrel{12}{=} b$

13. $9 - 3 \stackrel{12}{=} d$

15. $5 \times 6 \stackrel{12}{=} f$

17. $10 \times 6 \stackrel{12}{=} h$



LOOKING WAY BACK

Name the sums.

$$\begin{array}{r} 1. \quad 52 \\ 67 \\ 93 \\ 74 \\ + 26 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 394 \\ 865 \\ 322 \\ 509 \\ + 600 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 6171 \\ 8242 \\ 3154 \\ 6895 \\ + 9999 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 48263 \\ 64002 \\ 70138 \\ 55615 \\ + 23984 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 32754609 \\ + 20350085 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 198768975 \\ + 432411982 \\ \hline \end{array}$$

Name the differences.

$$\begin{array}{r} 7. \quad 54 \\ - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 623 \\ - 109 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 5015 \\ - 2677 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 32986 \\ - 19993 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 839627 \\ - 541549 \\ \hline \end{array}$$

Name the products.

$$\begin{array}{r} 12. \quad 5476 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 3107 \\ \times 25 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 92 \\ \times 54 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 618 \\ \times 576 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 3095 \\ \times 176 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 5472 \\ \times 2000 \\ \hline \end{array}$$

Name the quotients and remainders.

$$18. \quad 81 \overline{)7234}$$

$$20. \quad 546 \overline{)28421}$$

$$22. \quad 9 \overline{)756}$$

$$24. \quad 4 \overline{)13892}$$

$$19. \quad 23 \overline{)10976}$$

$$21. \quad 109 \overline{)54321}$$

$$23. \quad 5 \overline{)1260}$$

$$25. \quad 20 \overline{)5430}$$

Use the function rule on the left to complete the ordered pairs.

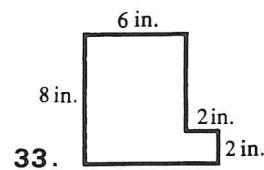
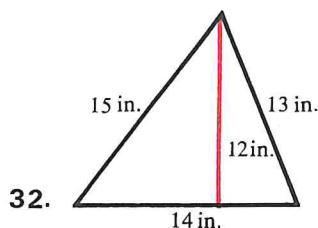
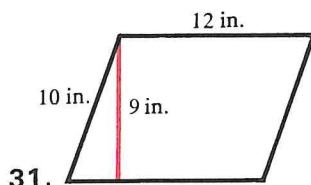
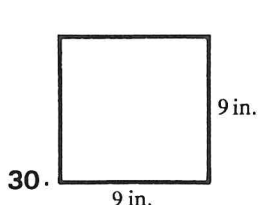
$$26. \quad (3 \times m) + 2 = f(m): \{(0, 2), (1, 5), (2, a), (4, c), (7, e), (9, n), (11, r)\}$$

$$27. \quad (4 \times m) - 1 = f(m): \{(1, 3), (2, 7), (3, a), (5, c), (6, e), (8, n), (10, r)\}$$

$$28. \quad (9 \times m) + 4 = f(m): \{(0, 4), (2, 22), (3, a), (4, c), (6, e), (9, n), (10, r)\}$$

$$29. \quad (2 \times m) - 2 = f(m): \{(1, 0), (3, 4), (4, a), (7, c), (8, e), (11, n), (13, r)\}$$

Name the perimeter and area of each plane figure.



Copy and complete the magic squares.

The *sum* of any 4 numbers in a line is the same.

1.

1	17	16	6
<i>a</i>	8	7	<i>e</i>
<i>c</i>	<i>n</i>	13	5
15	<i>r</i>	<i>s</i>	<i>u</i>

2.

8	13	17	4
6	15	11	<i>a</i>
<i>c</i>	<i>e</i>	<i>n</i>	<i>r</i>
21	<i>s</i>	<i>u</i>	9

3.

2	18	17	7
<i>a</i>	9	8	<i>c</i>
11	13	14	<i>s</i>
<i>e</i>	<i>n</i>	<i>r</i>	<i>u</i>

The difference between the middle number and the sum of opposite outside numbers is the same.

4.

12	<i>a</i>	14
<i>e</i>	11	13
8	7	<i>c</i>

5.

12	9	10
15	13	11
<i>a</i>	<i>c</i>	<i>e</i>

6.

87	84	85
<i>a</i>	88	86
<i>c</i>	<i>e</i>	89

The product of any 3 numbers in a line is the same.

7.

12	1	18
<i>a</i>	6	4
2	<i>e</i>	<i>c</i>

8.

20	<i>a</i>	50
25	10	<i>e</i>
<i>c</i>	100	5

*9.

45	1	75
25	15	<i>n</i>
<i>a</i>	<i>c</i>	<i>e</i>

Some measuring problems.

10. How would you measure 6 pints of water, using 2 cans, if one can holds 4 pints and the other holds 7 pints?
11. How would you find the thickness of a piece of string?
12. How could you measure up to 15 lb. using 4 weights: 1 lb., 2 lb., 4 lb., and 8 lb.?

Just For Fun

You are used to thinking of arithmetic with an infinite set — the whole numbers. But in this chapter you have worked arithmetic problems with a finite set, $\{0, 1, 2, 3, 4, 5, 6\}$. What differences do you see with this “new” arithmetic? Try forming an arithmetic using a 5-clock or an 8-clock or any other finite set of numbers.

CHAPTER 7 • Fractional Numbers

Fractional numbers

$\frac{1}{4}$ of the objects in A are red.

$$A = \{\bullet \bullet \bullet \bullet\}$$

$2 \times \frac{1}{4} = \frac{2}{4}$ of the objects in B are red.

$$B = \{\bullet \bullet \bullet \bullet\}$$

$3 \times \frac{1}{4} = \frac{3}{4}$ of the objects in C are red.

$$C = \{\bullet \bullet \bullet \bullet\}$$

$4 \times \frac{1}{4} = \frac{4}{4}$ of the objects in D are red.

$$D = \{\bullet \bullet \bullet \bullet\}$$

A **fraction** is a numeral for a fractional number: $\frac{1}{4}$, $2 \times \frac{1}{4}$,

$\frac{2}{4}$, $3 \times \frac{1}{4}$, $\frac{3}{4}$, $4 \times \frac{1}{4}$, $\frac{4}{4}$ are fractions.

EXERCISES

Name the fractional numbers.

1. $\{\blacklozenge \blacklozenge \blacklozenge\}$

$$a \times \frac{1}{3} = b$$

2. $\{\bullet \bullet \bullet \bullet \bullet\}$

$$3 \times \frac{1}{c} = d$$

3. $\{\bullet \bullet \bullet \bullet \bullet\}$

$$4 \times \frac{1}{e} = f$$

4. $\{\bullet \bullet \bullet \bullet \bullet\}$

$$g \times \frac{1}{5} = h$$

5. $\{\blacklozenge \blacklozenge \blacklozenge \blacklozenge \blacklozenge\}$

$$i \times \frac{1}{6} = j$$

6. $\{\bullet \bullet \bullet \bullet \bullet \bullet\}$

$$k \times \frac{1}{7} = l$$

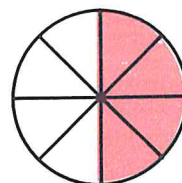
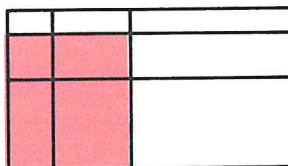
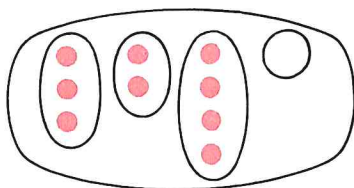


The rod is separated into 4 congruent parts. $\frac{3}{4}$ of the rod is red.



The parts of the rod are not congruent. We cannot say that $\frac{3}{4}$ of the rod is red.

Name the fractional numbers. Write *No* for those sets or regions which cannot be used to show fractional numbers.



Numerator and denominator



$$A = \{\triangle \triangle \triangle \blacktriangle\}$$

$\frac{3}{4}$ of the triangles are red.

The 4 is the **denominator**.

The 3 is the **numerator**.

The 4 names the number of equivalent subsets into which A is separated. The 3 names the number of subsets being compared with the total set.

EXERCISES

Complete this chart.

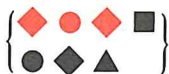
1.	numerator	3	6	3	2	d	e	11	7	i	k	4
	denominator	4	7	5	c	6	f	13	h	j	10	l
	fractional number	$\frac{3}{4}$	a	b	$\frac{2}{9}$	$\frac{5}{6}$	$\frac{7}{8}$	g	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{8}{10}$	$\frac{4}{9}$

Write fractions using the numerators and denominators suggested by the red objects in these sets.

2.



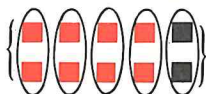
3.



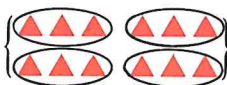
4.



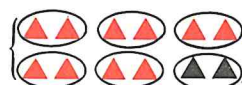
5.



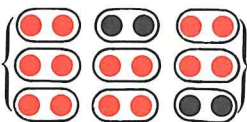
6.



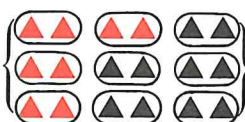
7.



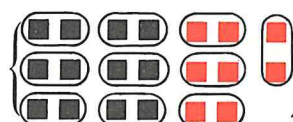
8.



9.



10.



Name the numerator and denominator suggested by the sentences, then write the fractions.

11. Five out of every 9 children were boys.

12. Three out of every 8 dogs were collies.

13. Seven out of every 13 apples were ripe.

14. Four out of every 15 apples were green.



Fractional numbers and division

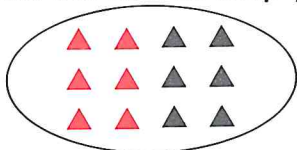
These 4 sentences all state that 12 is divided by 4.

$$\frac{1}{4} \text{ of } 12 \text{ is } 3 \quad \frac{1}{4} \times 12 = 3 \quad 12 \div 4 = 3 \quad \frac{12}{4} = 3$$

$$\frac{\text{dividend}}{\text{divisor}} \rightarrow \frac{12}{4} = 12 \div 4$$

EXERCISES

Use the sets to help you solve the equations.



1. $\frac{1}{2} \times 12 = a$

2. $12 \div b = 6$

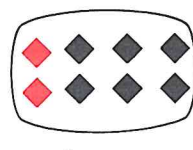
3. $\frac{12}{2} = c$



4. $\frac{1}{3} \times 9 = d$

5. $9 \div e = 3$

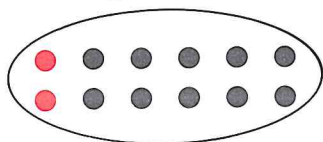
6. $\frac{9}{3} = f$



7. $\frac{1}{4} \times 8 = g$

8. $8 \div h = 2$

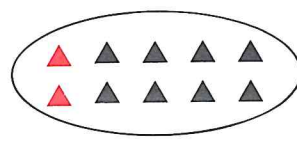
9. $\frac{8}{4} = i$



10. $\frac{1}{6} \times 12 = j$

11. $k \div 6 = 2$

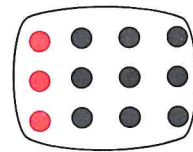
12. $\frac{12}{6} = l$



13. $\frac{1}{5} \times 10 = m$

14. $n \div 5 = 2$

15. $\frac{r}{5} = 2$



16. $\frac{1}{3} \times 12 = s$

17. $t \div 3 = 4$

18. $\frac{12}{u} = 4$

Draw sets, if necessary, to help you solve the equations.

19. $\frac{1}{4} \times 12 = a$

20. $12 \div b = 3$

21. $\frac{12}{4} = c$

22. $\frac{1}{2} \times 10 = d$

23. $10 \div e = 5$

24. $\frac{10}{2} = f$

25. $\frac{1}{4} \times 16 = g$

26. $16 \div h = 4$

27. $\frac{16}{4} = i$

Name the dividend and divisor for each of the fractions.

28. $\frac{20}{5}$

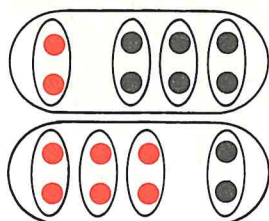
29. $\frac{6}{13}$

30. $\frac{2}{3}$

31. $\frac{18}{3}$

32. $\frac{4}{5}$

33. $\frac{7}{10}$

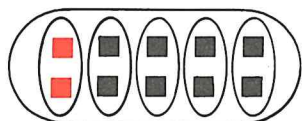


$$\frac{1}{4} \text{ of } 8 = 2$$

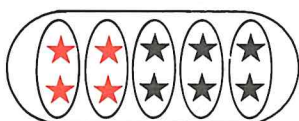
$$\begin{aligned} \frac{3}{4} \text{ of } 8 &= \left(3 \times \frac{1}{4}\right) \times 8 \\ &= 3 \times \left(\frac{1}{4} \times 8\right) \\ &= 3 \times 2 = 6 \end{aligned}$$

EXERCISES

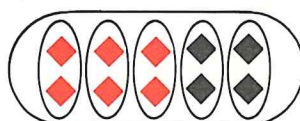
Use the sets to solve the equations.



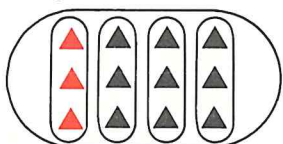
1. $\frac{1}{5} \text{ of } 10 = a$



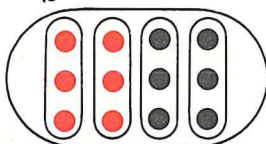
2. $\frac{2}{5} \text{ of } 10 = b$



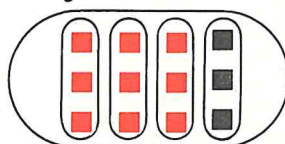
3. $\frac{3}{5} \text{ of } 10 = c$



4. $\frac{1}{4} \text{ of } 12 = d$



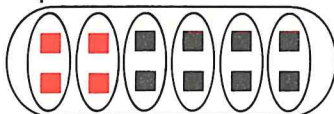
5. $\frac{2}{4} \text{ of } 12 = e$



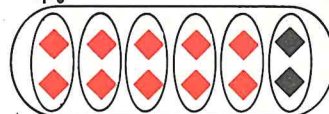
6. $\frac{3}{4} \text{ of } 12 = f$



7. $\frac{1}{6} \text{ of } 12 = g$



8. $\frac{2}{6} \text{ of } 12 = h$



9. $\frac{5}{6} \text{ of } 12 = i$

Solve the equations.

10. $\frac{1}{4} \text{ of } 16 \text{ is } 4, \text{ so } \frac{3}{4} \text{ of } 16 = a$

14. $\frac{1}{4} \text{ of } 20 \text{ is } 5, \text{ so } \frac{3}{4} \text{ of } 20 = b$

11. $\frac{1}{3} \text{ of } 12 \text{ is } 4, \text{ so } \frac{2}{3} \text{ of } 12 = c$

15. $\frac{1}{3} \text{ of } 18 \text{ is } 6, \text{ so } \frac{2}{3} \text{ of } 18 = d$

12. $\frac{1}{5} \text{ of } 15 \text{ is } 3, \text{ so } \frac{3}{5} \text{ of } 15 = e$

16. $\frac{1}{5} \text{ of } 20 \text{ is } 4, \text{ so } \frac{4}{5} \text{ of } 20 = f$

13. $\frac{1}{6} \text{ of } 24 \text{ is } 4, \text{ so } \frac{2}{6} \text{ of } 24 = g$

17. $\frac{1}{6} \text{ of } 24 \text{ is } 4, \text{ so } \frac{5}{6} \text{ of } 24 = h$



Fractional numbers and division

dividend	divisor	quotient
192	4	$\frac{192}{4} = 48$
48	4	$\frac{48}{4} = 12$
12	4	$\frac{12}{4} = 3$
3	4	$\frac{3}{4}$



The number pattern shows that $\frac{3}{4}$ is a quotient.

The arrows show:

$$3 \div 4 = \frac{3}{4}$$

In the fractional number $\frac{3}{4}$, the 3 is the dividend (or numerator), the 4 is the divisor (or denominator).

EXERCISES

Complete the number patterns.

1.

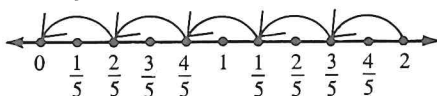
dividend	54	18	6	2	375	75	15	3
divisor	3	3	3	3	5	5	5	5
quotient	18	<i>a</i>	<i>b</i>	<i>c</i>	75	<i>d</i>	<i>e</i>	<i>f</i>

Complete this chart.

2.

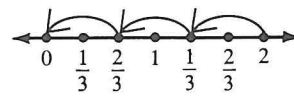
dividend	24	5	16	2	14	3	30	7	9	8	24	169
divisor	8	6	2	3	7	7	5	8	11	13	6	13
quotient	3	$\frac{5}{6}$	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>h</i>	<i>i</i>	<i>k</i>	<i>m</i>

Study the number lines and solve the equations.



3. $2 \times \frac{1}{5} = a$

4. $2 \div 5 = b$



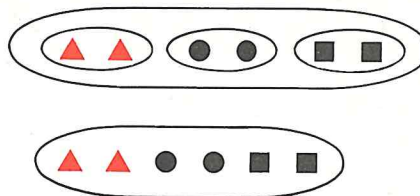
5. $2 \times \frac{1}{3} = c$

6. $2 \div 3 = d$

Two names for the same fractional number

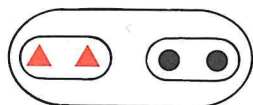
The subset of red objects is $\frac{1}{3}$ of the subsets.

The red objects are $\frac{2}{6}$ of the objects. $\frac{1}{3}$ and $\frac{2}{6}$ name the same fractional number.

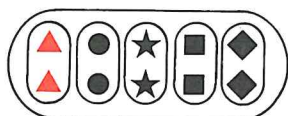


EXERCISES

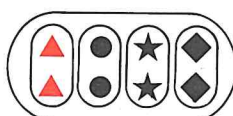
Study the sets and solve the equations.



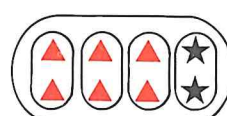
1. $\frac{1}{2} = \frac{a}{4}$



2. $\frac{1}{5} = \frac{c}{10}$

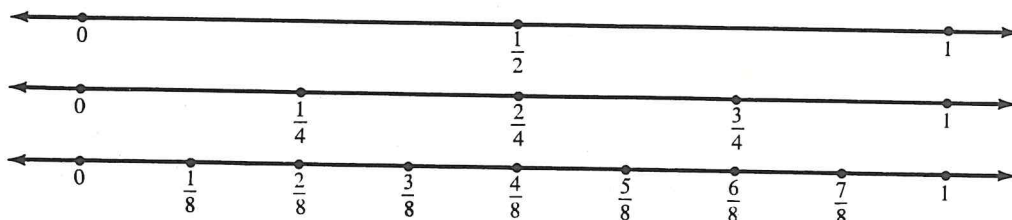


3. $\frac{1}{4} = \frac{e}{8}$



4. $\frac{3}{4} = \frac{i}{8}$

Study the number lines and solve the equations.



5. $\frac{1}{2} = \frac{a}{8}$

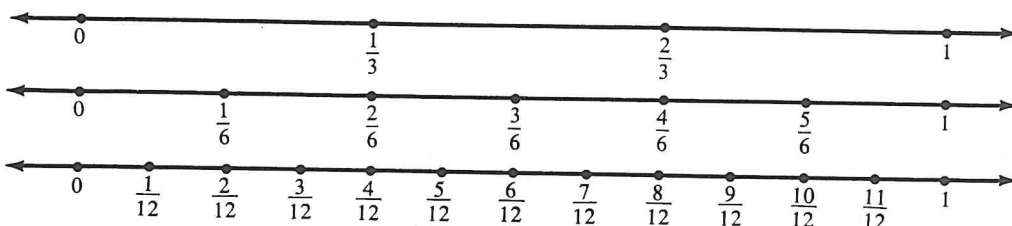
7. $\frac{1}{4} = \frac{c}{8}$

9. $\frac{2}{2} = \frac{e}{4}$

6. $\frac{2}{2} = \frac{b}{8}$

8. $\frac{3}{4} = \frac{d}{8}$

10. $\frac{4}{4} = \frac{f}{8} = \frac{h}{2}$



11. $\frac{2}{3} = \frac{a}{6}$

13. $\frac{2}{3} = \frac{c}{12}$

15. $\frac{6}{6} = \frac{e}{12}$

12. $\frac{1}{3} = \frac{b}{12}$

14. $\frac{1}{6} = \frac{d}{12}$

16. $\frac{3}{6} = \frac{f}{12} = \frac{h}{2}$

Equivalent fractions

$$\frac{1}{3} \times 12 = 4$$

$$\begin{aligned}\frac{2}{6} \times 12 &= \left(2 \times \frac{1}{6}\right) \times 12 \\ &= 2 \times \left(\frac{1}{6} \times 12\right) \\ &= 2 \times 2 = 4\end{aligned}$$

$$\frac{1}{3} \times 12 = \frac{2}{6} \times 12, \text{ so } \frac{1}{3} = \frac{2}{6}.$$

The equations show that $\frac{1}{3}$ and $\frac{2}{6}$ are two names for the same fractional number.

EXERCISES

Solve the equations.

1. $\frac{1}{2} \times 8 = a$

2.
$$\begin{aligned}\frac{2}{4} \times 8 &= \left(2 \times \frac{1}{4}\right) \times 8 \\ &= 2 \times \left(\frac{1}{4} \times 8\right) \\ &= 2 \times b \\ &= c\end{aligned}$$

so $\frac{1}{2} = \frac{d}{4}$

3. $\frac{1}{3} \times 18 = m$

4.
$$\begin{aligned}\frac{3}{9} \times 18 &= \left(3 \times \frac{1}{9}\right) \times 18 \\ &= 3 \times \left(\frac{1}{9} \times 18\right) \\ &= 3 \times n \\ &= p\end{aligned}$$

so $\frac{1}{3} = \frac{r}{9}$

5. $\frac{1}{2} \times 16 = s$

6.
$$\begin{aligned}\frac{4}{8} \times 16 &= \left(4 \times \frac{1}{8}\right) \times 16 \\ &= 4 \times \left(\frac{1}{8} \times 16\right) \\ &= 4 \times t \\ &= u\end{aligned}$$

so $\frac{1}{2} = \frac{v}{8}$

7. $\frac{1}{5} \times 20 = i$

8.
$$\begin{aligned}\frac{2}{10} \times 20 &= \left(2 \times \frac{1}{10}\right) \times 20 \\ &= 2 \times \left(\frac{1}{10} \times 20\right) \\ &= 2 \times j \\ &= k\end{aligned}$$

so $\frac{1}{5} = \frac{l}{10}$

Answer this question.

9. Rita said, " $\frac{1}{4}$ of the puppies are black." Sally said, " $\frac{2}{8}$ of the puppies are black." Could they both be right? Write an equation to explain this.



B is the set of **equivalent fractions** for the fractional number whose simplest name is $\frac{1}{3}$. Any member of B may be

named by multiplying $\frac{1}{3}$ by 1 in one of the forms from A .

$$A = \{\text{names for } 1\} = \left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \dots\right\}$$

$$B = \left\{\text{names for } \frac{1}{3}\right\} = \left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \dots\right\}$$

For example,

$$\frac{1}{3} = \frac{1}{3} \times 1 = \frac{1}{3} \times \frac{2}{2} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

EXERCISES

Solve the equations.

$$1. \quad \frac{1}{2} \times \frac{3}{3} = \frac{1 \times 3}{2 \times 3} = a$$

$$5. \quad \frac{2}{3} \times \frac{2}{2} = e$$

$$9. \quad \frac{3}{4} \times \frac{5}{5} = i$$

$$2. \quad \frac{1}{2} \times \frac{5}{5} = \frac{1 \times 5}{2 \times 5} = b$$

$$6. \quad \frac{2}{3} \times \frac{5}{5} = f$$

$$10. \quad \frac{3}{5} \times \frac{2}{2} = j$$

$$3. \quad \frac{1}{4} \times \frac{5}{5} = \frac{1 \times 5}{4 \times 5} = c$$

$$7. \quad \frac{3}{4} \times \frac{2}{2} = g$$

$$11. \quad \frac{4}{5} \times \frac{4}{4} = k$$

$$4. \quad \frac{1}{3} \times \frac{6}{6} = d$$

$$8. \quad \frac{3}{4} \times \frac{3}{3} = h$$

$$12. \quad \frac{4}{5} \times \frac{20}{20} = l$$

We may name $\frac{5}{6}$ and $\frac{7}{9}$ as fractions with the same denominator.

We notice that the LCM of 6 and 9 is 18. $6 \times 3 = 18$, and

$9 \times 2 = 18$, so we multiply $\frac{5}{6}$ by $\frac{3}{3}$ and $\frac{7}{9}$ by $\frac{2}{2}$.

$$\frac{5}{6} = \frac{5}{6} \times \frac{3}{3} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18} \quad \frac{7}{9} = \frac{7}{9} \times \frac{2}{2} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$$

Rename the fractional numbers so that the denominators for each pair are the same.

$$13. \quad \frac{2}{3}, \frac{5}{6}$$

$$15. \quad \frac{2}{3}, \frac{4}{5}$$

$$17. \quad \frac{7}{8}, \frac{4}{5}$$

$$19. \quad \frac{3}{4}, \frac{5}{6}$$

$$21. \quad \frac{4}{5}, \frac{4}{9}$$

$$14. \quad \frac{6}{7}, \frac{4}{5}$$

$$16. \quad \frac{4}{5}, \frac{3}{4}$$

$$18. \quad \frac{1}{2}, \frac{2}{3}$$

$$20. \quad \frac{7}{8}, \frac{7}{10}$$

$$22. \quad \frac{3}{8}, \frac{4}{5}$$

Equivalent fractions

$$A = \{\text{names for } 1\} = \left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \dots\right\}$$

$$B = \left\{\text{names for } \frac{1}{2}\right\} = \left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \dots\right\}$$

$$\frac{2}{4} = \frac{2}{4} \div \frac{2}{2} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2} \qquad \frac{4}{8} = \frac{4}{8} \div \frac{4}{4} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

B is the set of equivalent fractions for the fractional number whose simplest name is $\frac{1}{2}$. We can simplify any member of B by dividing it by 1 in the suitable form from A .

EXERCISES

Copy and complete the equations.

$$1. \quad \frac{2}{6} = \frac{2}{6} \div \frac{a}{2} = \frac{2 \div a}{6 \div 2} = b$$

$$7. \quad \frac{8}{16} = \frac{8 \div m}{16 \div 8} = n$$

$$2. \quad \frac{2}{8} = \frac{2}{8} \div \frac{c}{2} = \frac{2 \div c}{8 \div 2} = d$$

$$8. \quad \frac{6}{8} = \frac{6 \div 2}{8 \div m} = p$$

$$3. \quad \frac{3}{9} = \frac{3}{9} \div \frac{3}{e} = \frac{3 \div 3}{9 \div e} = f$$

$$9. \quad \frac{6}{9} = \frac{6 \div 3}{9 \div s} = r$$

$$4. \quad \frac{3}{12} = \frac{3}{12} \div \frac{n}{3} = \frac{3 \div n}{12 \div 3} = h$$

$$10. \quad \frac{9}{12} = \frac{9 \div 3}{12 \div s} = t$$

$$5. \quad \frac{4}{12} = \frac{4}{12} \div \frac{i}{4} = \frac{4 \div i}{12 \div 4} = j$$

$$11. \quad \frac{8}{12} = \frac{8 \div u}{12 \div 4} = v$$

$$6. \quad \frac{4}{16} = \frac{4}{16} \div \frac{a}{4} = \frac{4 \div a}{16 \div 4} = l$$

$$12. \quad \frac{12}{16} = \frac{12 \div 4}{16 \div w} = x$$

Copy and complete the equations.

$$13. \quad \frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{1 \times 2}{3 \times 2} = \frac{a}{c}$$

$$16. \quad \frac{3}{4} = \frac{3}{4} \times \frac{a}{2} = \frac{3 \times a}{4 \times 2} = \frac{b}{c}$$

$$14. \quad \frac{1}{4} = \frac{1}{4} \times \frac{2}{2} = \frac{1 \times 2}{4 \times 2} = \frac{c}{e}$$

$$17. \quad \frac{2}{3} = \frac{2}{3} \times \frac{v}{4} = \frac{2 \times v}{3 \times 4} = \frac{x}{w}$$

$$15. \quad \frac{1}{5} = \frac{1}{5} \times \frac{e}{3} = \frac{1 \times e}{5 \times 3} = \frac{m}{n}$$

$$18. \quad \frac{2}{5} = \frac{2}{5} \times \frac{2}{m} = \frac{2 \times 2}{5 \times m} = \frac{4}{n}$$

The simplest fraction in $A = \left\{ \frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \dots \right\}$ is $\frac{3}{4}$.

The only common factor of 3 and 4 is 1. Also, $\frac{3}{4}$ is the fraction with the least denominator.

EXERCISES

Name the simplest fraction in the sets of equivalent fractions.

1. $\left\{ \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \dots \right\}$

3. $\left\{ \frac{15}{18}, \frac{5}{6}, \frac{10}{12}, \frac{20}{24}, \dots \right\}$

2. $\left\{ \frac{4}{10}, \frac{6}{15}, \frac{2}{5}, \frac{8}{20}, \dots \right\}$

4. $\left\{ \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{1}{4}, \frac{5}{20}, \dots \right\}$

Study this example for naming a fractional number in simplest form.

$$\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$$

4 is the GCF of
20 and 24.

$\frac{5}{6}$ is in simplest form

because 1 is the only common
factor of 5 and 6.

Name the fractional numbers in simplest form.

5. $\frac{4}{8}$ 9. $\frac{2}{10}$ 13. $\frac{6}{18}$ 17. $\frac{12}{24}$ 21. $\frac{24}{60}$ 25. $\frac{16}{20}$

6. $\frac{8}{12}$ 10. $\frac{6}{9}$ 14. $\frac{18}{24}$ 18. $\frac{12}{15}$ 22. $\frac{15}{60}$ 26. $\frac{42}{48}$

7. $\frac{2}{16}$ 11. $\frac{3}{18}$ 15. $\frac{9}{15}$ 19. $\frac{12}{30}$ 23. $\frac{18}{48}$ 27. $\frac{28}{32}$

8. $\frac{2}{8}$ 12. $\frac{6}{8}$ 16. $\frac{30}{36}$ 20. $\frac{14}{42}$ 24. $\frac{20}{32}$ 28. $\frac{12}{48}$

Complete this sentence.

29. To name a fractional number in its simplest form, we divide the numerator and the denominator by their greatest common ____.

Joining and separating subsets

{ ● ● ○ }

$\frac{1}{3}$ of the objects are red.

$\frac{1}{3}$ of the objects are shaded.

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ of the objects are red or shaded.

{ ● ▲ ○ }

$\frac{2}{3}$ of the objects are red.

$\frac{1}{3}$ of the objects are red triangles.

$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ of the objects are red circles.

EXERCISES

Use the sets to help you name the sums or differences in simplest form.

{ ● ● ▲ △ }

{ ● ● ▲ △ △ }

{ ● ● ▲ ▲ ▲ △ }

{ ● ● ● ● ● ▲ ▲ △ }

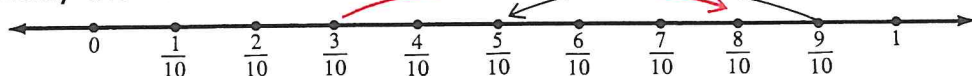
1. $\frac{3}{4} - \frac{1}{4}$

2. $\frac{2}{5} + \frac{1}{5}$

3. $\frac{2}{6} + \frac{3}{6}$

4. $\frac{5}{8} + \frac{2}{8}$

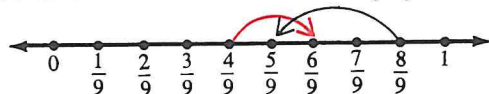
Study the number line.



The red arrow shows the addition $\frac{3}{10} + \frac{5}{10} = \frac{8}{10}$.

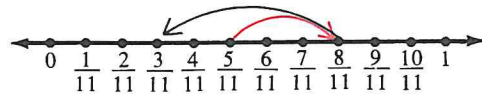
The black arrow shows the subtraction $\frac{9}{10} - \frac{4}{10} = \frac{5}{10}$.

Use the number line to help you name the sums and differences.



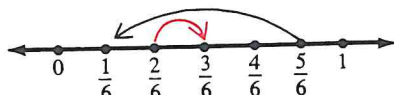
5. $\frac{4}{9} + \frac{2}{9}$

6. $\frac{8}{9} - \frac{3}{9}$



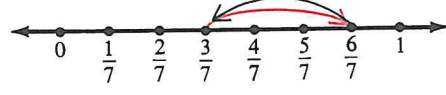
7. $\frac{5}{11} + \frac{4}{11}$

8. $\frac{9}{11} - \frac{7}{11}$



9. $\frac{2}{6} + \frac{1}{6}$

10. $\frac{5}{6} - \frac{4}{6}$



11. $\frac{3}{7} + \frac{3}{7}$

12. $\frac{6}{7} - \frac{3}{7}$

Adding, Subtracting Fractional Numbers

Study the equations and the steps on the right.

$$\frac{3}{8} + \frac{1}{8} = (3 + 1) \times \frac{1}{8} \qquad \frac{3}{8} - \frac{1}{8} = (3 - 1) \times \frac{1}{8}$$

$$= 4 \times \frac{1}{8} \qquad = 2 \times \frac{1}{8}$$

$$= \frac{4}{8} \qquad = \frac{2}{8}$$

$$= \frac{4 \div 4}{8 \div 4} \qquad = \frac{2 \div 2}{8 \div 2}$$

$$= \frac{1}{2} \qquad = \frac{1}{4}$$

1. Use the distributive property.

2. Add or subtract numerators.

3-5. Use the GCF to rename the sum or difference in simplest form.

EXERCISES

Name the sums or differences in simplest form.

1. $\frac{1}{4} + \frac{1}{4} = \frac{a}{4} = b$ 7. $\frac{4}{9} + \frac{2}{9} = \frac{m}{9} = n$ 13. $\frac{7}{8} - \frac{5}{8} = \frac{w}{8} = h$

2. $\frac{1}{6} + \frac{1}{6} = \frac{c}{6} = d$ 8. $\frac{3}{10} + \frac{1}{10} = \frac{e}{10} = p$ 14. $\frac{7}{9} - \frac{2}{9} = \frac{i}{9} = j$

3. $\frac{1}{6} + \frac{2}{6} = \frac{e}{6} = f$ 9. $\frac{5}{8} + \frac{1}{8} = \frac{u}{8} = r$ 15. $\frac{5}{9} - \frac{1}{9} = \frac{k}{9} = l$

4. $\frac{1}{12} + \frac{5}{12} = \frac{c}{12} = h$ 10. $\frac{3}{4} - \frac{1}{4} = \frac{a}{4} = b$ 16. $\frac{7}{8} - \frac{1}{8} = \frac{m}{8} = n$

5. $\frac{1}{9} + \frac{5}{9} = \frac{i}{9} = j$ 11. $\frac{5}{8} - \frac{3}{8} = \frac{c}{8} = d$ 17. $\frac{11}{12} - \frac{5}{12} = \frac{z}{12} = p$

6. $\frac{1}{12} + \frac{7}{12} = \frac{k}{12} = l$ 12. $\frac{5}{6} - \frac{1}{6} = \frac{e}{6} = f$ 18. $\frac{9}{10} - \frac{1}{10} = \frac{u}{10} = r$

Complete the sentence.

19. To add 2 fractional numbers with the same denominator, add the ? and keep the same denominator.

Pair the fractions that name the same number.

20. $\frac{3}{4} - \frac{1}{4}, \frac{2}{9} + \frac{4}{9}, \frac{3}{4}, \frac{2}{5}, \frac{5}{10} + \frac{1}{10}, \frac{1}{8} + \frac{5}{8}, \frac{7}{9} + \frac{2}{9}, \frac{9}{10} + \frac{1}{10}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{4}{10}$



Using sets and numbers

Solve the problems.

- Barry had 21 marbles, Fred had 27 and Bill had 24. They each gave $\frac{1}{3}$ of their marbles to Ben. How many marbles did Ben get?
- Melissa agreed to pay $\frac{1}{4}$ of the cost of the party, Joanne agreed to pay $\frac{1}{3}$, and Brenda agreed to pay $\frac{1}{6}$. If the cost of the party was \$12.36, how much did the girls pay altogether, and how much was left to be paid?
- Each month May's father pays $\frac{1}{12}$ of his annual taxes. If his total annual tax is \$1024.80, how much does he pay each month?
- If the store-keeper allows $\frac{1}{3}$ of his weekly profit for the cost of running his store and $\frac{1}{5}$ of his profit to pay off a loan, how much does he have left from a profit of \$900?
- If a city spends $\frac{2}{5}$ of its income of \$700,000,000 on education, how much does it spend on education?
- If you spend $\frac{1}{3}$ of each day asleep, $\frac{1}{6}$ of each day eating, $\frac{1}{3}$ of each day at school or reading, and the rest at play, how many hours a day do you spend at play?
- One-third of the 36 children have just a sandwich for lunch; one-fourth have just soup. How many do not have a sandwich or soup for lunch?



On the number line we see that $\frac{1}{2}$ is to the left of $\frac{3}{4}$, $\frac{1}{2} < \frac{3}{4}$; $\frac{7}{8}$ is to the right of $\frac{3}{4}$, $\frac{7}{8} > \frac{3}{4}$.

Find the LCM of the denominators in each pair. Write the fractions for each pair with the LCM as the denominator. Then replace each $\bigcirc$ by $<$, $>$, or $=$.

- | | | | |
|---|---|--|--|
| 8. $\frac{2}{3} \bigcirc \frac{3}{4}$ | 11. $\frac{3}{8} \bigcirc \frac{1}{4}$ | 14. $\frac{2}{5} \bigcirc \frac{21}{50}$ | 17. $\frac{2}{3} \bigcirc \frac{11}{15}$ |
| 9. $\frac{1}{3} \bigcirc \frac{5}{6}$ | 12. $\frac{7}{10} \bigcirc \frac{11}{10}$ | 15. $\frac{4}{13} \bigcirc \frac{9}{26}$ | 18. $\frac{5}{7} \bigcirc \frac{13}{21}$ |
| 10. $\frac{1}{2} \bigcirc \frac{5}{12}$ | 13. $\frac{5}{8} \bigcirc \frac{3}{4}$ | 16. $\frac{7}{8} \bigcirc \frac{3}{4}$ | 19. $\frac{11}{16} \bigcirc \frac{5}{8}$ |

Name the sums.

1.
$$\begin{array}{r} 28 \\ + 85 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 76 \\ + 98 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 423 \\ + 252 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 547486 \\ + 238455 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 794486656 \\ + 452209784 \\ \hline \end{array}$$

Name the differences.

6.
$$\begin{array}{r} 53 \\ - 28 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 90 \\ - 37 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 516 \\ - 329 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 400214 \\ - 283276 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 837324344 \\ - 589578272 \\ \hline \end{array}$$

Solve the equations.

11. $2724 + 947 = a$

13. $c - 938 = 6276$

15. $e + 256 = 5216$

12. $r + 341 = 9681$

14. $14,546 + 2378 = d$

16. $27,742 - 4397 = n$

Complete the sentences. Write $<$, $>$, or $=$ to replace each $\bigcirc$.

17. $238 \bigcirc 153 + 79$

23. $144 \div 8 \bigcirc 9 \times 16$

18. $328 + 417 \bigcirc 984 - 236$

24. $171 \div 9 \bigcirc 126 \div 7$

19. $3 \times 65 \bigcirc 2 \times 84$

25. $3 \times 444 \bigcirc 4 \times 333$

20. $28 \times 43 \bigcirc 43 \times 28$

26. $3 \times (20 + 4) \bigcirc 60 + 12$

21. $9 \times 284 \bigcirc 2606$

27. $800 - 72 \bigcirc 900 - 728$

22. $100 \div 4 \bigcirc 50 \div 2$

28. $207 \div 9 \bigcirc 192 \div 8$

Name the products.

29.
$$\begin{array}{r} 3609 \\ \times 4 \\ \hline \end{array}$$

30.
$$\begin{array}{r} 128 \\ \times 14 \\ \hline \end{array}$$

31.
$$\begin{array}{r} 6839 \\ \times 62 \\ \hline \end{array}$$

32.
$$\begin{array}{r} 28529 \\ \times 167 \\ \hline \end{array}$$

Name the quotients and remainders.

33. $8 \overline{)9646}$

36. $6 \overline{)8004}$

39. $34 \overline{)926}$

42. $38 \overline{)2117}$

34. $7 \overline{)1058}$

37. $9 \overline{)13069}$

40. $56 \overline{)150}$

43. $49 \overline{)2486}$

35. $4 \overline{)7462}$

38. $29 \overline{)1987}$

41. $65 \overline{)12367}$

44. $76 \overline{)200068}$

Use the function rule on the left to complete each ordered pair.

45. $(3 \times m) + 4 = f(m): \quad \{(1, 7), (2, 10), (3, a), (4, b), (6, c), (7, d), (8, e)\}$

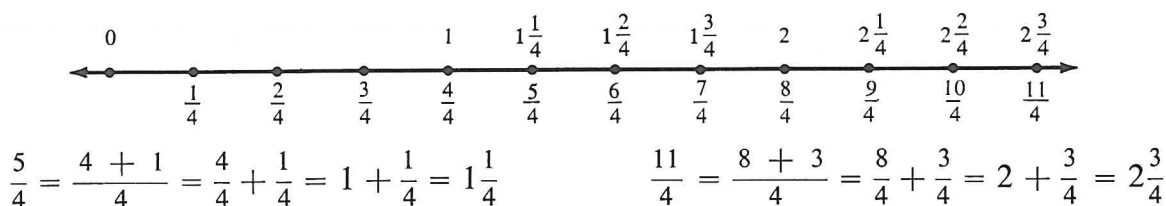
46. $(6 \times m) - 5 = f(m): \quad \{(1, 1), (2, 7), (3, f), (4, g), (5, h), (8, i), (10, j)\}$

47. $(8 \times m) + 6 = f(m): \quad \{(2, 22), (3, 30), (4, k), (5, l), (6, m), (10, n), (12, o)\}$

48. $(4 \times m) - 3 = f(m): \quad \{(1, 1), (2, 5), (3, p), (4, q), (5, r), (6, s), (8, t)\}$



Fractional numbers greater than 1

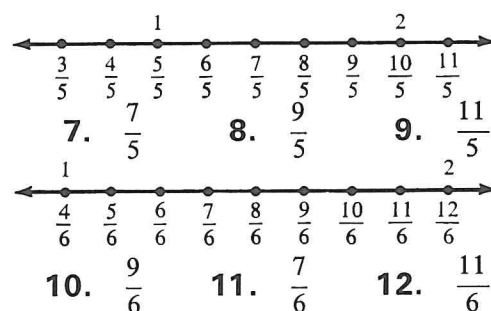
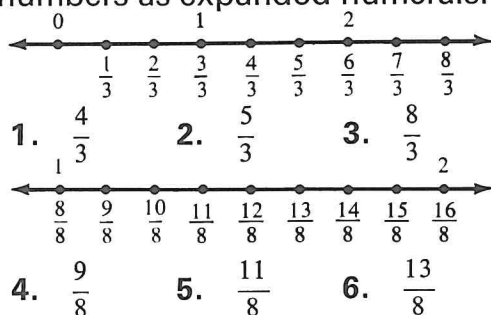


A numeral like $1 + \frac{1}{4}$ or $2 + \frac{3}{4}$ is an **expanded numeral**. A

numeral like $1\frac{1}{4}$ or $2\frac{3}{4}$ is a **mixed numeral** — one part names a whole number, the other part names a fractional number.

EXERCISES

Use the number lines to help you name the fractional numbers as expanded numerals.



13–24. Name the fractional numbers in exercises 1–12 as mixed numerals.

We may write
a mixed numeral
as a fraction.

$$1\frac{1}{4} = 1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{4+1}{4} = \frac{5}{4}$$

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$$

Write the mixed numerals as fractions.

25. $2\frac{3}{5}$ 27. $2\frac{1}{6}$ 29. $5\frac{1}{6}$ 31. $8\frac{1}{3}$ 33. $3\frac{1}{5}$ 35. $9\frac{1}{2}$
26. $3\frac{1}{4}$ 28. $4\frac{1}{4}$ 30. $6\frac{1}{2}$ 32. $3\frac{1}{3}$ 34. $4\frac{1}{3}$ 36. $12\frac{1}{2}$

Fractional numbers greater than 1

We may use equations or the working form to name a fractional number greater than 1 as a mixed numeral.

$$\begin{aligned}\frac{13}{5} &= 13 \div 5 \\ &= (10 + 3) \div 5 \\ &= (10 \div 5) + (3 \div 5) \\ &= 2 + \frac{3}{5} \\ &= 2\frac{3}{5}\end{aligned}$$

$$\begin{array}{r} 2 \\ 5 \overline{)13} \\ \underline{10} \\ 3 \end{array}$$

EXERCISES

Use equations to name the fractional numbers as mixed numerals.

- | | | | | | | |
|------------------|--------------------|-------------------|-------------------|-------------------|--------------------|---------------------|
| 1. $\frac{7}{4}$ | 3. $\frac{23}{10}$ | 5. $\frac{15}{4}$ | 7. $\frac{14}{9}$ | 9. $\frac{5}{3}$ | 11. $\frac{16}{3}$ | 13. $\frac{15}{11}$ |
| 2. $\frac{9}{6}$ | 4. $\frac{12}{7}$ | 6. $\frac{20}{8}$ | 8. $\frac{18}{5}$ | 10. $\frac{9}{2}$ | 12. $\frac{36}{5}$ | 14. $\frac{13}{5}$ |

Use the working form to name the fractional numbers as mixed numerals.

- | | | | | | | |
|--------------------|--------------------|--------------------|--------------------|---------------------|---------------------|---------------------|
| 15. $\frac{8}{3}$ | 17. $\frac{31}{5}$ | 19. $\frac{42}{8}$ | 21. $\frac{26}{5}$ | 23. $\frac{15}{8}$ | 25. $\frac{24}{11}$ | 27. $\frac{29}{14}$ |
| 16. $\frac{17}{4}$ | 18. $\frac{19}{2}$ | 20. $\frac{13}{3}$ | 22. $\frac{22}{7}$ | 24. $\frac{46}{10}$ | 26. $\frac{15}{13}$ | 28. $\frac{35}{9}$ |

$$3\frac{3}{4} = \left(3 \times \frac{4}{4}\right) + \left(3 \times \frac{1}{4}\right) = \left(12 \times \frac{1}{4}\right) + \left(3 \times \frac{1}{4}\right) = \frac{15}{4}$$

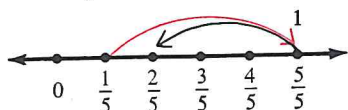
We may write mixed numerals as fractions.

Write the mixed numerals as fractions.

- | | | | | | |
|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------|
| 29. $2\frac{3}{8}$ | 31. $2\frac{7}{8}$ | 33. $3\frac{3}{5}$ | 35. $5\frac{2}{3}$ | 37. $6\frac{1}{6}$ | 39. $4\frac{5}{6}$ |
| 30. $4\frac{5}{8}$ | 32. $6\frac{5}{8}$ | 34. $7\frac{1}{3}$ | 36. $4\frac{3}{4}$ | 38. $5\frac{3}{7}$ | 40. $2\frac{9}{10}$ |

● Sums equal to or greater than 1

Study the number lines.

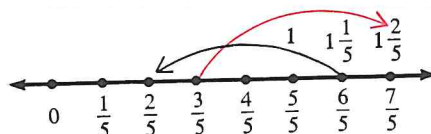


The red arrow shows

$$\frac{1}{5} + \frac{4}{5} = \frac{1+4}{5} = \frac{5}{5} = 1$$

The black arrow shows

$$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$$



The red arrow shows

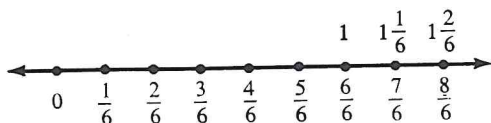
$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

The black arrow shows

$$1\frac{1}{5} - \frac{4}{5} = \frac{6}{5} - \frac{4}{5} = \frac{6-4}{5} = \frac{2}{5}$$

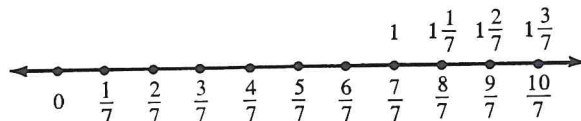
EXERCISES

Study the number lines and name the sums or differences.



1. $\frac{5}{6} + \frac{1}{6} = \frac{a}{6} = b$

2. $\frac{5}{6} + \frac{2}{6} = \frac{e}{6} = f$



3. $1 - \frac{3}{7} = \frac{c}{7} - \frac{3}{7} = d$

4. $1\frac{3}{7} - \frac{4}{7} = \frac{g}{7} - \frac{4}{7} = h$

Draw number lines, if necessary, to help you name the sums or differences.

5. $\frac{2}{3} + \frac{2}{3} = \frac{a}{3} = b$

6. $1 - \frac{5}{8} = \frac{c}{8} - \frac{5}{8} = d$

7. $\frac{3}{4} + \frac{2}{4} = \frac{e}{4} = f$

8. $1\frac{1}{8} - \frac{4}{8} = \frac{h}{8} - \frac{4}{8} = g$

9. $1\frac{2}{7} - \frac{5}{7} = \frac{i}{7} - \frac{5}{7} = j$

10. $1\frac{7}{9} - \frac{8}{9} = \frac{k}{9} - \frac{8}{9} = l$

11. $1\frac{1}{3} - \frac{2}{3} = d$

12. $\frac{9}{10} + \frac{7}{10} = h$

13. $1\frac{3}{10} - \frac{9}{10} = i$

Study the examples.

We named the sum in its simplest form as a mixed numeral.

$$\begin{array}{r} \frac{3}{4} \\ + \frac{3}{4} \\ \hline \end{array}$$

$$\frac{6}{4} = 1 + \frac{2}{4} = 1\frac{1}{2}$$

$$1\frac{1}{4} = \frac{5}{4}$$

$$\begin{array}{r} \frac{3}{4} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$\frac{2}{4} = \frac{1}{2}$$

We also named the difference in its simplest form.

EXERCISES

Name the sums or differences in simplest form.

1. $\frac{5}{8} + \frac{6}{8}$

3. $\frac{3}{4} + \frac{5}{4}$

5. $1\frac{1}{6} - \frac{2}{6}$

7. $1\frac{1}{4} - \frac{3}{4}$

2. $\frac{9}{10} + \frac{4}{10}$

4. $\frac{3}{8} + \frac{7}{8}$

6. $1\frac{2}{5} - \frac{3}{5}$

8. $1\frac{1}{10} - \frac{6}{10}$

9. $\begin{array}{r} 1\frac{4}{5} \\ - \frac{3}{5} \\ \hline \end{array}$

10. $\begin{array}{r} 1\frac{6}{7} \\ - \frac{3}{7} \\ \hline \end{array}$

11. $\begin{array}{r} \frac{9}{10} \\ + \frac{8}{10} \\ \hline \end{array}$

12. $\begin{array}{r} 1\frac{1}{8} \\ - \frac{3}{8} \\ \hline \end{array}$

13. $\begin{array}{r} 1\frac{3}{8} \\ + \frac{5}{8} \\ \hline \end{array}$

14. $\begin{array}{r} 1\frac{4}{5} \\ - \frac{4}{5} \\ \hline \end{array}$

In this subtraction we need to rename $7\frac{1}{4}$ as $6\frac{5}{4}$.

$$7\frac{1}{4} = 6\frac{5}{4}$$

To do this we think: $7\frac{1}{4} = 6 + 1\frac{1}{4}$

$$\begin{array}{r} \frac{3}{4} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$= 6\frac{5}{4}$$

$$6\frac{2}{4} = 6\frac{1}{2}$$

Name the differences in simplest form.

15. $\begin{array}{r} 6\frac{4}{5} \\ - \frac{3}{5} \\ \hline \end{array}$

16. $\begin{array}{r} 6\frac{6}{7} \\ - \frac{4}{7} \\ \hline \end{array}$

17. $\begin{array}{r} 2\frac{9}{10} \\ - \frac{7}{10} \\ \hline \end{array}$

18. $\begin{array}{r} 6\frac{2}{5} \\ - \frac{4}{5} \\ \hline \end{array}$

19. $\begin{array}{r} 5\frac{3}{8} \\ - \frac{5}{8} \\ \hline \end{array}$

20. $\begin{array}{r} 2\frac{1}{4} \\ - \frac{3}{4} \\ \hline \end{array}$

21. $\begin{array}{r} 6\frac{5}{9} \\ - \frac{4}{9} \\ \hline \end{array}$

22. $\begin{array}{r} 4\frac{3}{5} \\ - \frac{3}{5} \\ \hline \end{array}$

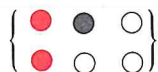
23. $\begin{array}{r} 3\frac{5}{8} \\ - \frac{4}{8} \\ \hline \end{array}$

24. $\begin{array}{r} 6\frac{5}{9} \\ - \frac{8}{9} \\ \hline \end{array}$

25. $\begin{array}{r} 4\frac{3}{5} \\ - \frac{4}{5} \\ \hline \end{array}$

26. $\begin{array}{r} 3\frac{5}{8} \\ - \frac{7}{8} \\ \hline \end{array}$

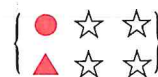
Joining and separating subsets



$\frac{1}{3}$ of the objects are red.

$\frac{1}{6}$ of the objects are black.

$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$
of the objects are red or black.



$\frac{1}{3}$ of the objects are red.

$\frac{1}{6}$ of the objects are red triangles.

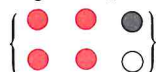
$\frac{1}{3} - \frac{1}{6} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
of the objects are red circles.

EXERCISES

Use the sets to help you name the sums in simplest form.



1. $\frac{1}{4} + \frac{1}{8} = \frac{a}{8} + \frac{1}{8} = b$



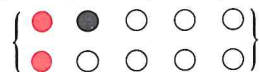
2. $\frac{2}{3} + \frac{1}{6} = \frac{e}{6} + \frac{1}{6} = f$



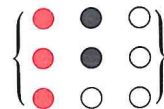
3. $\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{i}{8} = j$



4. $\frac{1}{2} + \frac{1}{4} = \frac{c}{4} + \frac{1}{4} = d$

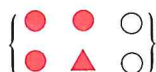


5. $\frac{1}{5} + \frac{1}{10} = \frac{h}{10} + \frac{1}{10} = g$

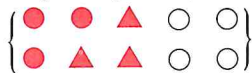


6. $\frac{1}{3} + \frac{2}{9} = \frac{k}{9} + \frac{2}{9} = l$

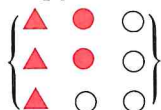
Use the sets to help you name the differences in simplest form.



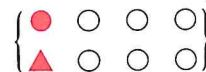
7. $\frac{2}{3} - \frac{1}{6} = \frac{a}{6} - \frac{1}{6} = b$



8. $\frac{3}{5} - \frac{3}{10} = \frac{e}{10} - \frac{3}{10} = f$



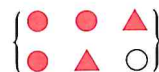
9. $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - \frac{i}{9} = j$



10. $\frac{1}{4} - \frac{1}{8} = \frac{c}{8} - \frac{1}{8} = d$



11. $\frac{1}{2} - \frac{5}{12} = \frac{h}{12} - \frac{5}{12} = g$



12. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{k}{6} = l$

Study the equations.

$(3 \times 5) + (4 \times 5) = (3 + 4) \times 5$ $= 7 \times 5$ $= 35$	$(3 \times 5) + (2 \times 10) = (3 \times 5) + (4 \times 5)$ $= (3 + 4) \times 5$ $= 7 \times 5$ $= 35$
$\frac{2}{9} + \frac{5}{9} = (2 + 5) \times \frac{1}{9}$ $= \frac{7}{9}$	$\frac{1}{4} + \frac{5}{8} = \frac{2}{8} + \frac{5}{8}$ $= (2 + 5) \times \frac{1}{8}$ $= \frac{7}{8}$
$\frac{7}{9} - \frac{2}{9} = (7 - 2) \times \frac{1}{9}$ $= \frac{5}{9}$	$\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8}$ $= (7 - 2) \times \frac{1}{8}$ $= \frac{5}{8}$
In these equations we can apply the distributive property because we have a common factor.	In these equations we must rename one of the addends before we can apply the distributive property.

EXERCISES

Apply the distributive property to solve the equations.

- $(3 \times 4) + (2 \times 4) = a$
- $(5 \times 6) + (2 \times 6) = b$
- $(3 \times 7) + (4 \times 7) = c$
- $\frac{3}{8} + \frac{1}{8} = g$
- $\frac{7}{3} - \frac{5}{3} = h$
- $\frac{5}{8} + \frac{2}{8} = i$
- $\frac{3}{8} + \frac{4}{8} = j$
- $(8 \times 9) - (3 \times 9) = d$
- $(6 \times 8) - (3 \times 8) = e$
- $(5 \times 12) - (3 \times 12) = f$
- $\frac{4}{10} - \frac{3}{10} = k$
- $\frac{11}{16} - \frac{6}{16} = l$
- $\frac{3}{5} - \frac{1}{2} = m$
- $\frac{1}{2} - \frac{1}{5} = n$
- $\frac{2}{5} - \frac{3}{10} = p$
- $\frac{3}{8} - \frac{5}{16} = q$
- $\frac{3}{4} + \frac{1}{8} = r$
- $\frac{2}{3} - \frac{1}{6} = s$



Renaming before adding or subtracting

To name the sum $\frac{1}{4} + \frac{5}{12}$, we notice that 12 is a multiple of 4 and that the LCM of 4 and 12 is 12. We must rename $\frac{1}{4}$ as $\frac{3}{12}$ before we can add.

$$\begin{aligned}\frac{1}{4} + \frac{5}{12} &= \frac{3}{12} + \frac{5}{12} \\ &= \frac{8}{12} = \frac{2}{3}\end{aligned}$$

To name the sum $\frac{2}{3} + \frac{1}{4}$, we notice that 12 is the LCM of 3 and 4. We must rename $\frac{2}{3}$ as $\frac{8}{12}$ and $\frac{1}{4}$ as $\frac{3}{12}$ before we can add.

$$\begin{aligned}\frac{2}{3} + \frac{1}{4} &= \frac{8}{12} + \frac{3}{12} \\ &= \frac{11}{12}\end{aligned}$$

EXERCISES

Name the sums or differences.

$$\begin{aligned}1. \quad \frac{1}{2} + \frac{1}{4} &= \frac{a}{8} + \frac{b}{8} \\ &= \frac{c}{8} = \frac{d}{4}\end{aligned}$$

$$\begin{aligned}5. \quad \frac{1}{5} + \frac{1}{2} &= \frac{h}{10} + \frac{i}{10} \\ &= \frac{c}{10}\end{aligned}$$

$$\begin{aligned}9. \quad \frac{1}{3} - \frac{1}{5} &= \frac{d}{15} - \frac{e}{15} \\ &= \frac{f}{15}\end{aligned}$$

$$\begin{aligned}2. \quad \frac{1}{3} + \frac{1}{2} &= \frac{k}{6} + \frac{l}{6} \\ &= \frac{m}{6}\end{aligned}$$

$$\begin{aligned}6. \quad \frac{1}{2} + \frac{2}{3} &= \frac{r}{6} + \frac{s}{6} \\ &= \frac{t}{6} = 1\frac{u}{6}\end{aligned}$$

$$\begin{aligned}10. \quad \frac{1}{2} - \frac{2}{5} &= \frac{m}{10} - \frac{n}{10} \\ &= \frac{v}{10}\end{aligned}$$

$$\begin{aligned}3. \quad \frac{1}{4} + \frac{1}{6} &= \frac{e}{12} + \frac{f}{12} \\ &= \frac{m}{12}\end{aligned}$$

$$\begin{aligned}7. \quad \frac{1}{2} - \frac{1}{3} &= \frac{a}{6} - \frac{b}{6} \\ &= \frac{c}{6}\end{aligned}$$

$$\begin{aligned}11. \quad \frac{1}{4} - \frac{1}{6} &= \frac{x}{12} - \frac{h}{12} \\ &= \frac{i}{12}\end{aligned}$$

$$\begin{aligned}4. \quad \frac{2}{3} + \frac{3}{4} &= \frac{n}{12} + \frac{a}{12} \\ &= \frac{c}{12} = 1\frac{e}{12}\end{aligned}$$

$$\begin{aligned}8. \quad \frac{3}{4} - \frac{1}{3} &= \frac{u}{12} - \frac{k}{12} \\ &= \frac{l}{12}\end{aligned}$$

$$\begin{aligned}12. \quad \frac{2}{5} - \frac{1}{4} &= \frac{w}{20} - \frac{z}{20} \\ &= \frac{r}{20}\end{aligned}$$

Name the sums or differences.

$$13. \quad \frac{3}{4} - \frac{1}{2}$$

$$16. \quad \frac{1}{2} + \frac{3}{5}$$

$$19. \quad \frac{5}{6} - \frac{7}{18}$$

$$22. \quad \frac{1}{5} + \frac{1}{6}$$

$$25. \quad \frac{3}{11} + \frac{1}{2}$$

$$14. \quad \frac{7}{8} - \frac{1}{4}$$

$$17. \quad \frac{3}{7} + \frac{5}{14}$$

$$20. \quad \frac{5}{9} + \frac{2}{3}$$

$$23. \quad \frac{4}{5} - \frac{1}{4}$$

$$26. \quad \frac{5}{6} - \frac{2}{3}$$

$$15. \quad \frac{1}{5} + \frac{3}{4}$$

$$18. \quad \frac{2}{3} - \frac{5}{12}$$

$$21. \quad \frac{3}{8} + \frac{11}{24}$$

$$24. \quad \frac{9}{10} - \frac{2}{5}$$

$$27. \quad \frac{3}{4} + \frac{15}{6}$$

Study these methods
for naming the sum
and difference.

$$\begin{array}{r} \frac{1}{3} = \frac{4}{12} \\ + \frac{1}{4} = + \frac{3}{12} \\ \hline \frac{7}{12} \end{array}$$

$$\begin{array}{r} \frac{1}{3} = \frac{4}{12} \\ - \frac{1}{4} = - \frac{3}{12} \\ \hline \frac{1}{12} \end{array}$$

EXERCISES

Name the sums or differences in simplest form.

$$\begin{array}{r} 1. \quad \frac{2}{5} = \frac{a}{10} \\ + \frac{1}{2} = + \frac{b}{10} \\ \hline \frac{c}{10} \end{array}$$

$$\begin{array}{r} 4. \quad \frac{1}{3} = \frac{a}{6} \\ + \frac{1}{2} = + \frac{c}{6} \\ \hline \frac{e}{6} \end{array}$$

$$\begin{array}{r} 7. \quad \frac{1}{3} = \frac{a}{15} \\ - \frac{1}{5} = - \frac{b}{15} \\ \hline \frac{c}{15} \end{array}$$

$$\begin{array}{r} 10. \quad \frac{2}{3} = \frac{r}{12} \\ - \frac{1}{4} = - \frac{u}{12} \\ \hline \frac{n}{12} \end{array}$$

$$\begin{array}{r} 2. \quad \frac{2}{5} = \frac{m}{20} \\ + \frac{1}{4} = + \frac{n}{20} \\ \hline \frac{u}{20} \end{array}$$

$$\begin{array}{r} 5. \quad \frac{1}{4} = \frac{a}{12} \\ + \frac{1}{6} = + \frac{c}{12} \\ \hline \frac{e}{12} \end{array}$$

$$\begin{array}{r} 8. \quad \frac{3}{4} = \frac{m}{8} \\ - \frac{1}{8} = - \frac{n}{8} \\ \hline \frac{a}{8} \end{array}$$

$$\begin{array}{r} 11. \quad \frac{1}{2} = \frac{e}{6} \\ - \frac{1}{3} = - \frac{a}{6} \\ \hline \frac{r}{6} \end{array}$$

$$\begin{array}{r} 3. \quad \frac{3}{4} = \frac{d}{12} \\ + \frac{1}{3} = + \frac{e}{12} \\ \hline \frac{f}{12} \end{array}$$

$$\begin{array}{r} 6. \quad \frac{1}{2} = \frac{s}{14} \\ + \frac{3}{7} = + \frac{t}{14} \\ \hline \frac{u}{14} \end{array}$$

$$\begin{array}{r} 9. \quad \frac{1}{4} = \frac{n}{12} \\ - \frac{1}{6} = - \frac{r}{12} \\ \hline \frac{s}{12} \end{array}$$

$$\begin{array}{r} 12. \quad \frac{5}{6} = \frac{s}{18} \\ - \frac{2}{9} = - \frac{t}{18} \\ \hline \frac{u}{18} \end{array}$$

Name the sums or differences in simplest form.

$$\begin{array}{r} 13. \quad \frac{2}{3} \\ + \frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad \frac{2}{3} \\ - \frac{2}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad \frac{2}{5} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad \frac{2}{5} \\ - \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad \frac{1}{2} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad \frac{1}{3} \\ - \frac{1}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad \frac{1}{3} \\ - \frac{4}{15} \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad \frac{1}{6} \\ + \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad \frac{3}{4} \\ - \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad \frac{2}{3} \\ + \frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad \frac{1}{4} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad \frac{1}{4} \\ - \frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad \frac{2}{3} \\ - \frac{1}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad \frac{1}{6} \\ + \frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad \frac{2}{5} \\ - \frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad \frac{3}{5} \\ + \frac{4}{6} \\ \hline \end{array}$$

Both addends greater than 1

Study these methods for naming the sum $1\frac{3}{4} + 2\frac{3}{4}$.

Equations	Expanded numerals	Short form
$1\frac{3}{4} + 2\frac{3}{4} = \left(1 + \frac{3}{4}\right) + \left(2 + \frac{3}{4}\right)$ $= (1 + 2) + \left(\frac{3}{4} + \frac{3}{4}\right)$ $= 3 + \frac{6}{4}$ $= 3 + 1\frac{2}{4} = 4\frac{1}{2}$	$1\frac{3}{4} = 1 + \frac{3}{4}$ $+ 2\frac{3}{4} = + 2 + \frac{3}{4}$ $3 + \frac{6}{4} = 4\frac{1}{2}$	$1\frac{3}{4}$ $+ 2\frac{3}{4}$ $3\frac{6}{4} = 4\frac{1}{2}$

EXERCISES

Use equations to solve the equations.

$$\begin{array}{ll}
 1. \quad 2\frac{4}{7} + 5\frac{1}{7} = (2 + a) + \left(5 + \frac{1}{7}\right) & 2. \quad 3\frac{2}{8} + 4\frac{7}{8} = (3 + f) + \left(4 + \frac{7}{8}\right) \\
 = (2 + 5) + \left(b + \frac{1}{7}\right) & = (3 + 4) + \left(\frac{2}{8} + g\right) \\
 = c + d & = h + i \\
 = e & = j \\
 3. \quad 3\frac{5}{9} + 7\frac{2}{9} = \left(3 + \frac{5}{9}\right) + \left(k + \frac{2}{9}\right) & 4. \quad 6\frac{1}{5} + 2\frac{4}{5} = \left(6 + \frac{1}{5}\right) + (2 + p) \\
 = (3 + 7) + \left(l + \frac{2}{9}\right) & = (6 + 2) + \left(q + \frac{4}{5}\right) \\
 = m + n & = r + s \\
 = a & = t
 \end{array}$$

Use expanded numerals to name the sums.

$$\begin{array}{llllll}
 5. \quad 5\frac{2}{7} & 7. \quad 3\frac{2}{6} & 9. \quad 2\frac{1}{4} & 11. \quad 7\frac{3}{4} & 13. \quad 8\frac{7}{9} & 15. \quad 4\frac{2}{5} & 17. \quad 1\frac{5}{8} \\
 + 4\frac{3}{7} & + 1\frac{5}{6} & + 6\frac{2}{2} & + 6\frac{1}{8} & + 6\frac{8}{9} & + 2\frac{9}{10} & + 2\frac{7}{8} \\
 \hline
 6. \quad 3\frac{3}{11} & 8. \quad 5\frac{3}{5} & 10. \quad 7\frac{6}{7} & 12. \quad 6\frac{8}{10} & 14. \quad 4\frac{3}{4} & 16. \quad 7\frac{2}{3} & 18. \quad 6\frac{4}{9} \\
 + 4\frac{9}{11} & + 2\frac{4}{5} & + 8\frac{1}{14} & + 1\frac{3}{10} & + 2\frac{3}{4} & + 1\frac{1}{6} & + 5\frac{7}{9} \\
 \hline
 \end{array}$$

19–32. Use the short form to name the sums in exercises 5–18.

Study these equations.

$$\begin{aligned} 1\frac{3}{4} + 2\frac{3}{4} &= 3 + \frac{3+3}{4} \\ &= 3 + \frac{6}{4} \\ &= 3 + 1\frac{2}{4} \\ &= 4\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 1\frac{3}{4} + 2\frac{7}{8} &= 1\frac{6}{8} + 2\frac{7}{8} \\ &= 3 + \frac{6+7}{8} \\ &= 3 + \frac{13}{8} \\ &= 3 + 1\frac{5}{8} = 4\frac{5}{8} \end{aligned}$$

EXERCISES

Use equations to name the sums in simplest form.

1. $2\frac{3}{7} + 3\frac{2}{7}$

6. $2\frac{5}{6} + 3\frac{3}{6}$

11. $5\frac{1}{3} + 2\frac{3}{4}$

2. $4\frac{4}{9} + 2\frac{3}{9}$

7. $4\frac{1}{2} + 2\frac{3}{8}$

12. $4\frac{2}{3} + 5\frac{1}{4}$

3. $5\frac{3}{8} + 2\frac{4}{8}$

8. $2\frac{7}{8} + 1\frac{1}{4}$

13. $3\frac{1}{2} + 4\frac{2}{3}$

4. $3\frac{9}{10} + 2\frac{6}{10}$

9. $5\frac{2}{3} + 3\frac{5}{6}$

14. $6\frac{1}{2} + 3\frac{3}{5}$

5. $4\frac{5}{6} + 3\frac{4}{6}$

10. $4\frac{2}{3} + 2\frac{3}{4}$

15. $6\frac{2}{3} + 5\frac{2}{5}$

Name the sums in simplest form.

16. $2\frac{3}{4}$

18. $2\frac{7}{9}$

20. $5\frac{1}{2}$

22. $9\frac{1}{4}$

24. $2\frac{1}{3}$

26. $4\frac{3}{4}$

$+ 1\frac{3}{4}$

$+ 4\frac{7}{9}$

$+ 3\frac{3}{4}$

$+ 2\frac{7}{8}$

$+ 4\frac{1}{2}$

$+ 1\frac{2}{3}$

17. $4\frac{5}{8}$

19. $3\frac{6}{10}$

21. $6\frac{3}{8}$

23. $4\frac{1}{2}$

25. $3\frac{2}{3}$

27. $3\frac{1}{4}$

$+ 3\frac{5}{8}$

$+ 4\frac{7}{10}$

$+ 5\frac{3}{4}$

$+ 6\frac{5}{8}$

$+ 6\frac{1}{2}$

$+ 2\frac{5}{6}$

Solve this problem.

28. If Mary needs $2\frac{1}{3}$ yards of material to make a red dress and $3\frac{3}{4}$ yards of material to make a blue dress, how many yards does she need all together?

The sum and given addend both greater than 1

Study these methods for naming the difference $4\frac{1}{4} - 2\frac{3}{4}$.

$$\begin{aligned}
 \text{A} \quad 4\frac{1}{4} - 2\frac{3}{4} &= (4 - 2) + \left(\frac{1}{4} - \frac{3}{4}\right) \\
 &= 2 + \left(\frac{1}{4} - \frac{3}{4}\right) \\
 &= 1 + \left(1 + \frac{1}{4} - \frac{3}{4}\right) \\
 &= 1 + \frac{5 - 3}{4} \\
 &= 1 + \frac{2}{4} = 1\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{B} \quad 4\frac{1}{4} &= \cancel{4} + \cancel{\frac{1}{4}} \\
 - 2\frac{3}{4} &= -2 - \frac{3}{4} \\
 \hline
 1 + \frac{2}{4} &= 1\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{C} \quad 4\frac{1}{4} &= \cancel{4} + \cancel{\frac{1}{4}} \\
 - 2\frac{3}{4} &= -2 - \frac{3}{4} \\
 \hline
 1\frac{2}{4} &= 1\frac{1}{2}
 \end{aligned}$$

Since we do not know how to subtract $\frac{3}{4}$ from $\frac{1}{4}$, we rename 1 as $\frac{4}{4}$, and then subtract $\frac{3}{4}$ from $\frac{5}{4}$.

EXERCISES

Use method A to name the differences. Remember that you may have to rename before you subtract.

1. $5\frac{1}{3} - 3\frac{2}{3}$

3. $5\frac{1}{5} - 2\frac{3}{5}$

5. $3\frac{1}{6} - 1\frac{1}{4}$

7. $8\frac{1}{4} - 3\frac{3}{8}$

2. $6\frac{1}{4} - 2\frac{3}{4}$

4. $8\frac{1}{7} - 4\frac{6}{7}$

6. $6\frac{1}{10} - 2\frac{4}{5}$

8. $9\frac{3}{8} - 6\frac{1}{2}$

Use method B to name the differences.

9. $6\frac{1}{3} - 3\frac{2}{3}$

11. $8\frac{2}{7} - 3\frac{5}{7}$

13. $11\frac{3}{10} - 4\frac{9}{10}$

15. $13\frac{3}{8} - 9\frac{7}{8}$

17. $8\frac{3}{10} - 2\frac{1}{5}$

19. $6\frac{1}{4} - 3\frac{5}{6}$

10. $2\frac{1}{4} - 1\frac{3}{4}$

12. $7\frac{5}{9} - 2\frac{8}{9}$

14. $9\frac{3}{5} - 5\frac{4}{5}$

16. $10\frac{1}{3} - 5\frac{3}{4}$

18. $9\frac{1}{6} - 6\frac{2}{3}$

20. $7\frac{1}{3} - 5\frac{4}{9}$

21–32. Use method C to name the differences in exercises 9–20.

Subtracting fractional numbers

Study these methods for naming the difference $4\frac{1}{4} - 2\frac{3}{4}$.

A

$$\begin{aligned} 4\frac{1}{4} - 2\frac{3}{4} &= 2 + \frac{1-3}{4} \\ &= 1 + \frac{5-3}{4} \\ &= 1 + \frac{2}{4} \\ &= 1\frac{1}{2} \end{aligned}$$

B

$$\begin{aligned} 4\frac{1}{4} - 2\frac{3}{4} &= 2\frac{1-3}{4} \\ &= 1\frac{5-3}{4} \\ &= 1\frac{2}{4} \\ &= 1\frac{1}{2} \end{aligned}$$

Since we do not know how to subtract 3 from 1 in

the fraction $\frac{1-3}{4}$,

we rename 2 as $1 + \frac{4}{4}$

$$2 + \frac{1-3}{4} = 1\frac{5-3}{4}$$

EXERCISES

Solve the equations to name the differences.

$$\begin{aligned} 1. \quad 5\frac{1}{3} - 3\frac{2}{3} &= 2 + \frac{a-2}{3} \\ &= 1 + \frac{b-2}{3} \\ &= 1 + c \\ &= d \end{aligned}$$

$$\begin{aligned} 2. \quad 7\frac{2}{5} - 4\frac{3}{5} &= 3 + \frac{e-3}{5} \\ &= 2 + \frac{f-3}{5} \\ &= 2 + g \\ &= h \end{aligned}$$

$$\begin{aligned} 3. \quad 9\frac{1}{4} - 2\frac{3}{4} &= 7 + \frac{i-3}{4} \\ &= 6 + \frac{k-3}{4} \\ &= 6 + j \\ &= l \end{aligned}$$

$$\begin{aligned} 4. \quad 8\frac{4}{7} - 1\frac{6}{7} &= 7 + \frac{m-6}{7} \\ &= 6 + \frac{n-6}{7} \\ &= 6 + r \\ &= p \end{aligned}$$

$$\begin{aligned} 5. \quad 9\frac{3}{5} - 6\frac{4}{5} &= 3\frac{a-4}{5} \\ &= 2\frac{b-4}{5} \\ &= c \end{aligned}$$

$$\begin{aligned} 6. \quad 4\frac{4}{6} - 1\frac{5}{6} &= 3\frac{d-5}{6} \\ &= 2\frac{e-5}{6} \\ &= f \end{aligned}$$

Name the differences.

$$7. \quad 5\frac{3}{8} - 2\frac{7}{8}$$

$$9. \quad 8\frac{3}{5} - 6\frac{4}{5}$$

$$11. \quad 11\frac{2}{9} - 7\frac{5}{9}$$

$$13. \quad 6\frac{7}{11} - 3\frac{9}{11}$$

$$8. \quad 4\frac{1}{7} - 2\frac{5}{7}$$

$$10. \quad 9\frac{7}{10} - 7\frac{9}{10}$$

$$12. \quad 13\frac{1}{4} - 4\frac{3}{4}$$

$$14. \quad 15\frac{3}{7} - 8\frac{5}{7}$$



Another method of subtracting

Study the equations.

To subtract 28 from 41

$$\begin{aligned} 41 - 28 &= 41 + 0 - 28 \\ &= 41 + 2 - 2 - 28 \\ &= 43 - 30 \\ &= 13 \end{aligned}$$

To subtract $2\frac{3}{4}$ from $4\frac{1}{4}$

$$\begin{aligned} 4\frac{1}{4} - 2\frac{3}{4} &= 4\frac{1}{4} + 0 - 2\frac{3}{4} \\ &= 4\frac{1}{4} + \left(\frac{1}{4} - \frac{1}{4}\right) - 2\frac{3}{4} \\ &= 4\frac{1}{2} - 3 \\ &= 1\frac{1}{2} \end{aligned}$$

EXERCISES

Name the differences.

1. $\begin{array}{r} 8 \\ - 6 \\ \hline \end{array}$

2. $\begin{array}{r} 10 \\ - 8 \\ \hline \end{array}$

3. $\begin{array}{r} 12 \\ - 10 \\ \hline \end{array}$

4. $\begin{array}{r} 18 \\ - 16 \\ \hline \end{array}$

5. $\begin{array}{r} 20 \\ - 18 \\ \hline \end{array}$

6. $\begin{array}{r} 22 \\ - 20 \\ \hline \end{array}$

7. $\begin{array}{r} \frac{4}{8} \\ - \frac{2}{8} \\ \hline \end{array}$

8. $\begin{array}{r} \frac{5}{8} \\ - \frac{3}{8} \\ \hline \end{array}$

9. $\begin{array}{r} \frac{6}{8} \\ - \frac{4}{8} \\ \hline \end{array}$

10. $\begin{array}{r} 2\frac{4}{8} \\ - 1\frac{2}{8} \\ \hline \end{array}$

11. $\begin{array}{r} 2\frac{5}{8} \\ - 1\frac{3}{8} \\ \hline \end{array}$

12. $\begin{array}{r} 2\frac{6}{8} \\ - 1\frac{4}{8} \\ \hline \end{array}$

13. $\begin{array}{r} 52 \\ - 28 \\ \hline \end{array}$

14. $\begin{array}{r} 54 \\ - 30 \\ \hline \end{array}$

15. $\begin{array}{r} 73 \\ - 39 \\ \hline \end{array}$

16. $\begin{array}{r} 74 \\ - 40 \\ \hline \end{array}$

17. $\begin{array}{r} 82 \\ - 58 \\ \hline \end{array}$

18. $\begin{array}{r} 84 \\ - 60 \\ \hline \end{array}$

19. $\begin{array}{r} 5\frac{1}{4} \\ - 2\frac{3}{4} \\ \hline \end{array}$

20. $\begin{array}{r} 5\frac{1}{2} \\ - 3 \\ \hline \end{array}$

21. $\begin{array}{r} 6\frac{1}{3} \\ - 2\frac{2}{3} \\ \hline \end{array}$

22. $\begin{array}{r} 6\frac{2}{3} \\ - 3 \\ \hline \end{array}$

23. $\begin{array}{r} 7\frac{2}{5} \\ - 2\frac{4}{5} \\ \hline \end{array}$

24. $\begin{array}{r} 7\frac{3}{5} \\ - 3 \\ \hline \end{array}$

25. $\begin{array}{r} 3\frac{1}{6} \\ - 1\frac{5}{6} \\ \hline \end{array}$

26. $\begin{array}{r} 4\frac{1}{8} \\ - 2\frac{7}{8} \\ \hline \end{array}$

27. $\begin{array}{r} 6\frac{1}{8} \\ - 2\frac{5}{8} \\ \hline \end{array}$

28. $\begin{array}{r} 3\frac{1}{5} \\ - 1\frac{3}{5} \\ \hline \end{array}$

29. $\begin{array}{r} 3\frac{2}{5} \\ - 1\frac{3}{5} \\ \hline \end{array}$

30. $\begin{array}{r} 3\frac{1}{5} \\ - 1\frac{4}{5} \\ \hline \end{array}$

31. $\begin{array}{r} 5\frac{1}{8} \\ - 2\frac{5}{8} \\ \hline \end{array}$

32. $\begin{array}{r} 6\frac{3}{8} \\ - 2\frac{5}{8} \\ \hline \end{array}$

33. $\begin{array}{r} 6\frac{3}{8} \\ - 2\frac{7}{8} \\ \hline \end{array}$

34. $\begin{array}{r} 6\frac{1}{7} \\ - 2\frac{3}{7} \\ \hline \end{array}$

35. $\begin{array}{r} 6\frac{5}{7} \\ - 2\frac{6}{7} \\ \hline \end{array}$

36. $\begin{array}{r} 3\frac{1}{10} \\ - 1\frac{9}{10} \\ \hline \end{array}$

Solve the problems.

1. Sara bought $3\frac{3}{4}$ yards of blue material and $2\frac{1}{3}$ yards of green material. How many yards of material did she buy all together?

2. Mary bought $5\frac{1}{4}$ yards of red material and $3\frac{2}{3}$ yards of yellow material. How many yards of material did she buy all together?

3. Bob bought $2\frac{3}{4}$ lb. of top quality grass seed and $4\frac{1}{2}$ lb. of second quality grass seed. How much grass seed did he buy all together?

4. Don noticed that one can of peaches contained $1\frac{3}{4}$ lb., another contained $1\frac{1}{2}$ lb. How much more was there in the first can than the second?

5. In problem 1 how many more yards of blue material than green material did Sara buy?

6. In problem 2 how many more yards of red material than yellow material did Mary buy?

7. There are 18 girls in Mary's class. One half of the girls prefer pop music to folk music, one third prefer folk music, the rest have no preference. Name a fractional number to answer the question: How many more of the girls prefer pop music to folk music?

8. Name a whole number to answer the question in problem 7.

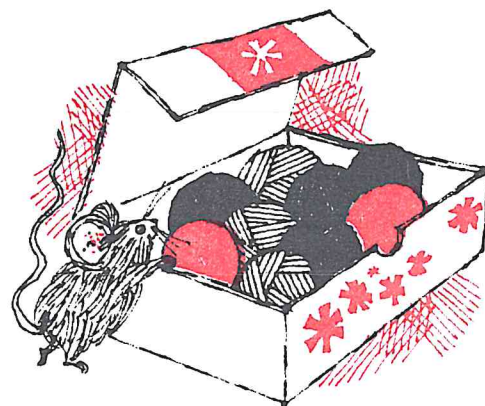
9. In problem 7 how many girls have no preference?

10. Joan owns 25 records. Two-fifths are folk music, the rest are pop music. How many pop music records does she own?



● Probability

There are 2 red balls, 3 gray balls, and 4 black balls in the box. That is, $\frac{2}{9}$ of the balls are red, $\frac{3}{9} = \frac{1}{3}$ are gray, and $\frac{4}{9}$ are black. If you reach in the box without looking, the chance (probability) that you will get a red ball is $\frac{2}{9}$.



We use a fractional number to compare a subset with a set, so we also use a fractional number to state a **probability**. We say that the probability of getting a red ball is $\frac{2}{9}$ and we write $P(\text{red ball}) = \frac{2}{9}$.

EXERCISES

Solve the problems.

1. In the box above, what is the probability of getting a gray ball? a black ball?
2. If you had 3 pennies and 7 dimes in your pocket, and 1 coin fell out, what is the probability that the coin that fell out was a dime? a penny?
3. If there are 4 red ribbons, 2 yellow ribbons, and 3 blue ribbons in a drawer, what is the probability of getting a red ribbon if you reach in the drawer without looking? What is the probability of getting a yellow ribbon?
4. If you throw a dart at a calendar for the month of March and the dart lands on a numeral, what is the probability that it lands on 10? on a multiple of 5? on a number greater than 28?
5. Imagine a street with 15 houses on it. Imagine you have friends living in 3 of the houses on that street but you don't know where. If you go to any house on the street, what is the probability that it is the house of one of your friends? is not the house of one of your friends?
6. If there are lawyers' offices on the fifth, tenth, and fifteenth floors of a 15-story office building and you get off the elevator at any floor, what is the probability of getting off at a floor on which there are lawyer's offices?



If you choose any number in $A = \{3, 5, 7, 10, 12, 14, 16, 18\}$, the probability that the number you choose is an even number is 5 out of 8, that is $\frac{5}{8}$. We shall write this as $P(\text{even number}) = \frac{5}{8}$.

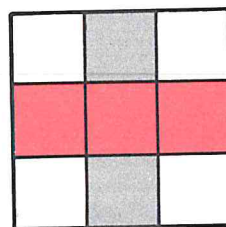
EXERCISES

Solve the problems.

1. For set A above, what is the probability that the number you choose is an odd number? a multiple of 3? a multiple of 7?
2. What is the probability that a whole number from 11 to 20 is a prime? a multiple of 3? a multiple of 4?
3. What is the probability that a whole number from 61 to 70 is a multiple of 7? a multiple of 5?
4. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. What is the probability that a whole number from 1 to 24 is a factor of 24? is not a factor of 24?
5. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30. What is the probability that a whole number from 1 to 30 is a factor of 30? is not a factor of 30?
6. There are 25 primes from 1 to 100. What is the probability that a whole number from 1 to 100 is a prime? is not a prime? is an even number? is a multiple of 4? is a multiple of 5?
7. What is the probability that the ones' digit in a numeral is a multiple of 3? a multiple of 4?
- *8. If you have to divide any whole number by 29, what is the probability that there will be a remainder?
9. Eight of the 36 multiplication facts are squares, like $3 \times 3 = 9$. What is the probability that if you use a multiplication fact, the product will be a square? will not be a square?
10. On one assembly line in the factory where Bill's father works, one out of every 250 parts is likely to be faulty. Using this probability, how many faulty parts can be expected in 2000 parts?

● Probability

If you throw a dart at this board, the probability that you will hit a red square is $\frac{3}{9} = \frac{1}{3}$; that you will hit a gray square is $\frac{2}{9}$.



$$\frac{3}{9} > \frac{2}{9}, \text{ so } P(\text{red square}) > P(\text{gray square}).$$

$$\frac{3}{9} + \frac{2}{9} = \frac{5}{9}, \text{ so } P(\text{red or gray square}) = \frac{5}{9}.$$

EXERCISES

Solve the problems.

There are 6 black, 5 red, and 4 green balls in a basket, and you choose one without looking.

1. Are you more likely to choose a black ball or a red ball?
a red ball or a green ball?
2. What is the probability of choosing either a black ball or a red ball? either a black ball or a green ball?

If you choose any member of the set $A = \{\bullet\bullet\bullet\bullet\bullet\bullet \color{red}\bullet\bullet\bullet\bullet \blacktriangle\blacktriangle\}$, name

3. $P(\bullet)$
5. $P(\blacktriangle)$
7. $P(\bullet \text{ or } \blacktriangle)$
9. $P(\bullet, \color{red}\bullet \text{ or } \blacktriangle)$
4. $P(\color{red}\bullet)$
6. $P(\bullet \text{ or } \color{red}\bullet)$
8. $P(\color{red}\bullet \text{ or } \blacktriangle)$
10. $P(\blacksquare)$

There are 24 children in Madeline's class, 8 of whom like ice cream better than pop corn or candy, 4 like pop corn best, 10 like candy best, and 2 do not like any of these. If you speak to any member of Madeline's class, what is the probability that the food he likes best is

11. ice cream?
12. pop corn?
13. candy?
14. none of these?
15. ice cream or pop corn?
17. ice cream or none of these?
16. ice cream or candy?
18. pop corn or none of these?

Is the member of Madeline's class to whom you speak more likely to like

19. ice cream rather than pop corn?
21. ice cream or pop corn rather than candy?
20. pop corn rather than candy?



In the set $A = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ the probability of choosing a prime number is $\frac{4}{10} = \frac{2}{5}$. The probability of choosing a number which is not a prime is $\frac{6}{10} = \frac{3}{5}$. $\frac{3}{5} > \frac{2}{5}$, so $P(\text{not prime}) > P(\text{prime})$.
 $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$, so $P(\text{prime or not prime}) = 1$.

EXERCISES

Solve the problems.

For the set $B = \{2, 3, 4, 5, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 21\}$ name the probability of choosing

1. a prime.
2. a multiple of 3.
3. a multiple of 4.
4. a prime or a multiple of 3.
5. a prime or a multiple of 4.
6. an even number.
7. a multiple of 2 or 3.
8. a multiple of 2 or 3 or a prime.
9. a number greater than 21.

Jane said to Jim, "Let's play a game. We will put in this box pieces of paper with the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 on them. You pick a numeral and, if it is a multiple of 3, you win. If not, I win."

10. Who has the better chance of winning, Jane or Jim?
11. If they change the game so that Jim wins if he picks a multiple of 4 and Jane if she picks a multiple of 3, who has the better chance of winning?
12. Can you change the rules so that Jane and Jim have the same chance of winning?

Complete the sentences.

13. If something is certain to happen, its probability is ____?
14. If something cannot happen, its probability is ____?
15. Any probability is greater than or equal to ____? and less than or equal to ____?
16. If A and B can both happen but not at the same time, then $P(A \text{ or } B) = P(A) +$ ____?

CHECKING UP

Complete the equivalent fractions in the sets.

1. $\left\{\frac{1}{2}, \frac{a}{4}, \frac{b}{6}, \frac{c}{8}, \frac{d}{10}, \frac{e}{12}, \frac{f}{14}\right\}$

3. $\left\{1, \frac{m}{2}, \frac{n}{3}, \frac{b}{4}, \frac{v}{5}, \frac{w}{6}, \frac{r}{7}\right\}$

2. $\left\{\frac{2}{3}, \frac{a}{6}, \frac{h}{9}, \frac{i}{12}, \frac{c}{15}, \frac{k}{18}, \frac{e}{21}\right\}$

4. $\left\{\frac{1}{4}, \frac{s}{8}, \frac{t}{12}, \frac{u}{16}, \frac{v}{20}, \frac{w}{24}, \frac{x}{28}\right\}$

Solve the equations.

5. $\frac{1}{5} \times 25 = a$ and $3 \times \left(\frac{1}{5} \times 25\right) = b$, so $\frac{3}{5} \times 25 = c$

6. $\frac{1}{7} \times 14 = d$ and $5 \times \left(\frac{1}{7} \times 14\right) = e$, so $\frac{5}{7} \times 14 = f$

7. $\frac{1}{3} \times 12 = g$ and $2 \times \left(\frac{1}{3} \times 12\right) = h$, so $\frac{2}{3} \times 12 = i$

8. $\frac{1}{6} \times 24 = j$ and $4 \times \left(\frac{1}{6} \times 24\right) = k$, so $\frac{4}{6} \times 24 = l$

Complete this chart.

9.

dividend	36	3	18	5	3	<i>d</i>	3	<i>f</i>	<i>h</i>
divisor	4	4	9	6	7	11	<i>e</i>	<i>g</i>	<i>i</i>
quotient	9	$\frac{3}{4}$	<i>a</i>	<i>b</i>	<i>c</i>	$\frac{6}{11}$	$\frac{3}{13}$	$\frac{4}{9}$	$\frac{5}{12}$

Name the sums or differences in simplest form.

10. $\frac{2}{3} + \frac{1}{3}$

11. $\frac{7}{8} - \frac{5}{8}$

12. $5\frac{7}{10} + 2\frac{5}{10}$

13. $2\frac{3}{4} + 5\frac{3}{5}$

14. $\frac{2}{3}$

16. $\frac{3}{4}$

18. $\frac{2}{9}$

20. $\frac{4}{5}$

22. $11\frac{1}{8}$

24. $13\frac{1}{4}$

26. $9\frac{3}{7}$

$+\frac{1}{4}$

$+\frac{3}{8}$

$+\frac{1}{2}$

$-\frac{1}{3}$

$-7\frac{3}{4}$

$-6\frac{5}{6}$

$-3\frac{1}{2}$

15. $\frac{5}{6}$

17. $\frac{3}{4}$

19. $\frac{4}{6}$

21. $3\frac{3}{4}$

23. $5\frac{2}{5}$

25. $4\frac{2}{3}$

27. $10\frac{3}{5}$

$-\frac{2}{3}$

$-\frac{1}{5}$

$-\frac{2}{7}$

$+2\frac{2}{3}$

$+4\frac{3}{10}$

$+4\frac{1}{5}$

$-5\frac{5}{6}$



To name a fractional number in its simplest form, we divide the numerator and denominator by their greatest common factor.

Solve the equations.

$$\begin{array}{lll} 1. \quad \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{a}{c} & 3. \quad \frac{18}{21} = \frac{18 \div 3}{21 \div 3} = \frac{a}{e} & 5. \quad \frac{8}{10} = \frac{8 \div x}{10 \div z} = \frac{m}{n} \\ 2. \quad \frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{u}{v} & 4. \quad \frac{12}{16} = \frac{12 \div e}{16 \div r} = \frac{n}{s} & 6. \quad \frac{32}{40} = \frac{32 \div r}{40 \div s} = \frac{u}{n} \end{array}$$

To name the sum (or difference) of 2 fractional numbers, the denominators must be the same. We use the distributive property and add or subtract the numerators, keeping the denominator.

Solve the equations.

$$\begin{array}{ll} 7. \quad \frac{2}{5} + \frac{1}{5} = \left(2 \times \frac{1}{5}\right) + \left(1 \times \frac{1}{5}\right) & 8. \quad \frac{8}{9} - \frac{5}{9} = \left(8 \times \frac{1}{9}\right) - \left(5 \times \frac{1}{9}\right) \\ & = a \times \frac{1}{5} & = c \times \frac{1}{9} \\ & = b & = d \end{array}$$

Name the sums or differences.

$$\begin{array}{lllllll} 9. \quad \frac{2}{5} & 10. \quad \frac{3}{7} & 11. \quad \frac{5}{9} & 12. \quad \frac{8}{10} & 13. \quad 1\frac{5}{11} & 14. \quad 6\frac{3}{4} & 15. \quad 10\frac{11}{13} \\ + \frac{2}{5} & + \frac{2}{7} & - \frac{3}{9} & - \frac{6}{10} & + 2\frac{3}{11} & - 2\frac{1}{4} & - 8\frac{9}{13} \end{array}$$

To name the sum (or difference) of 2 fractional numbers with different denominators, we must first rename the numbers so they have the same denominator.

Name the sums or differences.

$$\begin{array}{lllllll} 16. \quad \frac{2}{3} & 17. \quad \frac{1}{4} & 18. \quad \frac{3}{4} & 19. \quad \frac{2}{5} & 20. \quad 5\frac{3}{8} & 21. \quad 9\frac{2}{3} & 22. \quad 11\frac{3}{8} \\ + \frac{3}{4} & - \frac{1}{6} & - \frac{1}{2} & - \frac{1}{4} & + 2\frac{7}{16} & + 2\frac{4}{5} & - 3\frac{2}{3} \end{array}$$



LOOKING WAY BACK

Name the sums or differences.

$$\begin{array}{r} 1. \quad 367 \\ 482 \\ 576 \\ 491 \\ + 507 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 5831 \\ 6782 \\ 5961 \\ 7054 \\ + 8103 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 91110 \\ 27654 \\ 32541 \\ 81607 \\ + 59402 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 216735 \\ + 359482 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 678514 \\ - 327104 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 476914 \\ - 288784 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 5961754 \\ - 3898537 \\ \hline \end{array}$$

Name the products, quotients or remainders.

$$\begin{array}{r} 8. \quad 376 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 892 \\ \times 76 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 5435 \\ \times 27 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 38245 \\ \times 62 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 671 \\ \times 537 \\ \hline \end{array}$$

$$13. \quad 4 \overline{)736}$$

$$15. \quad 6 \overline{)8254}$$

$$17. \quad 65 \overline{)9762}$$

$$19. \quad 352 \overline{)679}$$

$$21. \quad 276 \overline{)3817652}$$

$$14. \quad 3 \overline{)927}$$

$$16. \quad 27 \overline{)348}$$

$$18. \quad 84 \overline{)5327}$$

$$20. \quad 584 \overline{)6731}$$

$$22. \quad 217 \overline{)94360}$$

Use the prime factorizations to name the greatest common factor and least common multiple.

$$20 = 2 \times 2 \times 5$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$27 = 3 \times 3 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$54 = 2 \times 3 \times 3 \times 3$$

$$23. \quad \text{GCF} = a$$

$$25. \quad \text{GCF} = c$$

$$27. \quad \text{GCF} = e$$

$$24. \quad \text{LCM} = b$$

$$26. \quad \text{LCM} = d$$

$$28. \quad \text{LCM} = f$$

Use the equations on the left to complete each number pair.

$$29. \quad m + 3 = f(m): \quad \{(1, 4), (3, a), (7, b), (9, c), (10, d), (13, e), (15, f)\}$$

$$30. \quad m - 5 = f(m): \quad \{(20, 15), (17, g), (9, h), (15, i), (7, j), (11, k), (5, l)\}$$

$$31. \quad (3 \times m) + 4 = f(m): \quad \{(2, 10), (5, 19), (4, m), (7, n), (5, e), (9, p), (11, q)\}$$

$$32. \quad m \div 6 = f(m): \quad \{(18, r), (12, s), (4, t), (6, u), (8, v), (2, w), (5, x)\}$$

$$33. \quad m + \frac{1}{2} = f(m): \quad \left\{ \left(\frac{1}{4}, y \right), \left(\frac{3}{4}, z \right), \left(\frac{5}{8}, a \right), \left(\frac{7}{8}, b \right), \left(1\frac{1}{4}, c \right), \left(1\frac{1}{8}, d \right) \right\}$$

Complete the magic squares.

34.

2	$\frac{1}{4}$	a
n	$1\frac{1}{4}$	c
m	$2\frac{1}{4}$	e

35.

$2\frac{2}{3}$	$\frac{1}{3}$	2
1	r	s
u	3	v

Any fractional number may be expressed as the sum of 2 or more fractional numbers with numerator 1. For example,

$$\begin{aligned}\frac{7}{8} &= \left(\frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) + \frac{1}{8} \\ &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} \\ &= \left(\frac{1}{4} + \frac{1}{4}\right) + \frac{1}{4} + \frac{1}{8} \\ &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\end{aligned}$$

Rename the fractional numbers in this way.

1. $\frac{5}{6}$ 2. $\frac{5}{8}$ 3. $\frac{7}{12}$ 4. $\frac{11}{12}$ 5. $\frac{7}{10}$ 6. $\frac{9}{10}$

You may add $3\frac{5}{8}$ and $4\frac{7}{8}$ in your head, like this:

Step 1: $3\frac{5}{8} + 4 = 7\frac{5}{8}$,

Notice how we renamed

Step 2: $7\frac{5}{8} + \frac{3}{8} = 8$,

$\frac{7}{8}$ as $\frac{3}{8} + \frac{4}{8}$ to add the $\frac{3}{8}$

Step 3: $8 + \frac{4}{8} = 8\frac{1}{2}$,

to $7\frac{5}{8}$ in Step 2.

Use this method to name the sums.

7. $4\frac{4}{5} + 3\frac{3}{5}$

9. $4\frac{3}{4} + 2\frac{3}{4}$

11. $8\frac{5}{6} + 2\frac{2}{3}$

13. $8\frac{5}{6} + 4\frac{1}{2}$

8. $6\frac{5}{8} + 4\frac{5}{8}$

10. $9\frac{5}{7} + 4\frac{3}{7}$

12. $4\frac{3}{4} + 5\frac{5}{8}$

14. $2\frac{1}{2} + 5\frac{7}{8}$

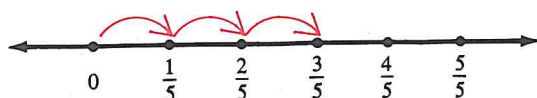
Just For Fun

15. Milly said, "In the school sports today I ran three fourths." Her mother said, "Three fourths of you cannot run. Either all of you ran or you didn't run at all." Explain this story and find others like it.

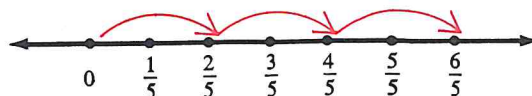
CHAPTER 8 • Multiplication-Division of Fractional Numbers

● One factor less than one

Study the number lines.



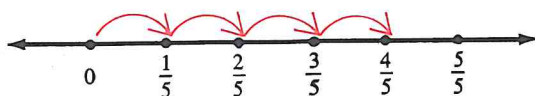
$$3 \times \frac{1}{5} = \frac{3}{5}$$



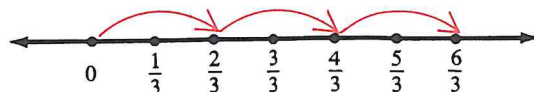
$$3 \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

EXERCISES

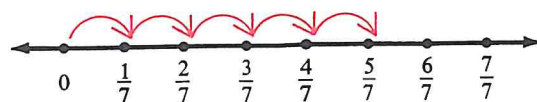
Use the number lines to help you solve the equations.



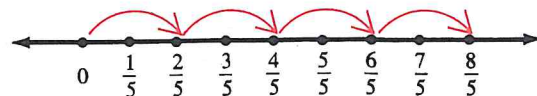
$$1. \quad 4 \times \frac{1}{5} = a$$



$$4. \quad 3 \times \frac{2}{3} = \frac{e}{3} = f$$



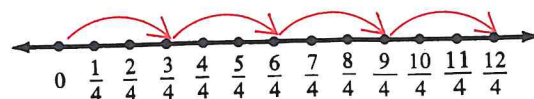
$$2. \quad 5 \times \frac{1}{7} = b$$



$$5. \quad 4 \times \frac{2}{5} = \frac{h}{5} = p$$



$$3. \quad 2 \times \frac{1}{3} = c$$



$$6. \quad 4 \times \frac{3}{4} = \frac{m}{4} = n$$

Use the commutative property of multiplication to help you solve the equations. Draw a number line if necessary.

$$7. \quad \frac{2}{5} \times 5 = \frac{a}{5} = b$$

$$9. \quad \frac{2}{3} \times 9 = \frac{e}{3} = f$$

$$11. \quad \frac{3}{5} \times 5 = \frac{i}{5} = j$$

$$8. \quad \frac{3}{4} \times 8 = \frac{c}{4} = d$$

$$10. \quad \frac{2}{3} \times 4 = \frac{g}{3} = h$$

$$12. \quad \frac{2}{5} \times 3 = \frac{k}{5} = l$$

Study the methods for naming the product $7 \times \frac{3}{4}$.

$$\begin{aligned} 7 \times \frac{3}{4} &= 7 \times \left(3 \times \frac{1}{4}\right) \\ &= (7 \times 3) \times \frac{1}{4} \\ &= 21 \times \frac{1}{4} = 5\frac{1}{4} \end{aligned}$$

$$\begin{array}{r} \frac{3}{4} \\ \times 7 \\ \hline 21 \\ \frac{21}{4} = 5\frac{1}{4} \end{array}$$

EXERCISES

Solve the equations.

$$\begin{aligned} 1. \quad 8 \times \frac{3}{4} &= 8 \times \left(a \times \frac{1}{4}\right) \\ &= (8 \times 3) \times \frac{1}{4} \\ &= b \times \frac{1}{4} = c \end{aligned}$$

$$\begin{aligned} 3. \quad 14 \times \frac{5}{7} &= 14 \times \left(g \times \frac{1}{7}\right) \\ &= (14 \times 5) \times \frac{1}{7} \\ &= h \times \frac{1}{7} = i \end{aligned}$$

$$\begin{aligned} 2. \quad 10 \times \frac{2}{5} &= 10 \times \left(d \times \frac{1}{5}\right) \\ &= (10 \times 2) \times \frac{1}{5} \\ &= e \times \frac{1}{5} = f \end{aligned}$$

$$\begin{aligned} 4. \quad 8 \times \frac{7}{8} &= 8 \times \left(j \times \frac{1}{8}\right) \\ &= (8 \times j) \times \frac{1}{8} \\ &= k \times \frac{1}{8} = m \end{aligned}$$

Solve the equations.

$$\begin{array}{llll} 5. \quad 16 \times \frac{3}{8} = a & 9. \quad 9 \times \frac{2}{3} = e & 13. \quad \frac{4}{9} \times 10 = i & 17. \quad \frac{5}{6} \times 30 = m \\ 6. \quad 12 \times \frac{2}{3} = b & 10. \quad \frac{2}{3} \times 8 = f & 14. \quad \frac{3}{4} \times 12 = j & 18. \quad \frac{7}{9} \times 18 = n \\ 7. \quad 10 \times \frac{4}{5} = c & 11. \quad \frac{1}{4} \times 11 = g & 15. \quad \frac{2}{3} \times 9 = k & 19. \quad \frac{5}{8} \times 16 = p \\ 8. \quad 18 \times \frac{5}{6} = d & 12. \quad \frac{3}{8} \times 6 = h & 16. \quad \frac{2}{7} \times 14 = l & 20. \quad \frac{2}{3} \times 15 = q \end{array}$$

Name the products.

$$\begin{array}{llllllll} 21. \quad \frac{3}{5} & 22. \quad \frac{4}{5} & 23. \quad \frac{5}{8} & 24. \quad \frac{3}{4} & 25. \quad 4 & 26. \quad 5 & 27. \quad 6 & 28. \quad 8 \\ \times 4 & \times 2 & \times 3 & \times 6 & \times \frac{2}{3} & \times \frac{3}{4} & \times \frac{4}{5} & \times \frac{2}{3} \end{array}$$

● One factor less than one

Study the examples.

$$\begin{aligned} 5 \times \frac{4}{7} &= \frac{5 \times 4}{7} \\ &= \frac{20}{7} \\ &= 2\frac{6}{7} \end{aligned}$$

$$\begin{array}{r} 6 \\ \times \frac{3}{8} \\ \hline \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4} \end{array}$$

To name the product of a whole number and a fraction, multiply the numerator of the fraction by the whole number.

EXERCISES

Name the products in simplest form.

- | | | | | |
|----------------------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1. $2 \times \frac{3}{7}$ | 6. $\frac{2}{3} \times 5$ | 11. $\frac{5}{11} \times 2$ | 16. $6 \times \frac{3}{10}$ | 21. $\frac{1}{4} \times 17$ |
| 2. $3 \times \frac{4}{5}$ | 7. $10 \times \frac{2}{5}$ | 12. $\frac{3}{8} \times 4$ | 17. $\frac{6}{7} \times 4$ | 22. $10 \times \frac{2}{3}$ |
| 3. $2 \times \frac{4}{11}$ | 8. $\frac{4}{7} \times 3$ | 13. $2 \times \frac{3}{13}$ | 18. $16 \times \frac{4}{5}$ | 23. $\frac{5}{4} \times 2$ |
| 4. $4 \times \frac{3}{4}$ | 9. $\frac{3}{4} \times 9$ | 14. $16 \times \frac{5}{8}$ | 19. $\frac{3}{4} \times 28$ | 24. $7 \times \frac{4}{5}$ |
| 5. $6 \times \frac{7}{8}$ | 10. $\frac{1}{5} \times 6$ | 15. $\frac{3}{7} \times 3$ | 20. $6 \times \frac{7}{8}$ | 25. $\frac{4}{17} \times 8$ |

Name the products in simplest form.

- | | | | | | |
|---------------------------------|---------------------------------|---------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 26. $\frac{5}{6}$
$\times 4$ | 27. $\frac{7}{8}$
$\times 6$ | 28. $\frac{5}{9}$
$\times 3$ | 29. $\frac{7}{11}$
$\times 4$ | 30. $\frac{9}{13}$
$\times 5$ | 31. $\frac{7}{10}$
$\times 7$ |
| 32. 5
$\times \frac{6}{7}$ | 33. 8
$\times \frac{3}{4}$ | 34. 9
$\times \frac{2}{9}$ | 35. 7
$\times \frac{2}{3}$ | 36. 3
$\times \frac{1}{2}$ | 37. 13
$\times \frac{5}{6}$ |

Complete the sentence and the equation.

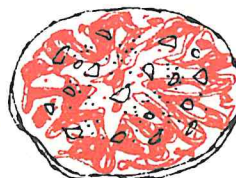
38. **The commutative property of multiplication:** The order of the factors does not affect the _____. For example,

$$\frac{3}{8} \times 6 = 6 \times a$$

1. There are 28 students in Mrs. Ellen's class. If half of the class are boys, how many boys are there?
2. Mary has \$12 to spend for her birthday. If she spends $\frac{1}{3}$ of her money for a purse, how much money does she have left?
3. There is a sale at Sammy's Sporting Goods Store. Every item is reduced in price by $\frac{2}{5}$.
 - a. If a basketball usually costs \$6, how much will you save if you buy it during Sammy's sale?
 - b. A tennis racket usually costs \$10. What will be its price during Sammy's sale?
 - c. What will be the sale price of a ping pong table if it usually costs \$20?



4. The Jones family is taking a 24-day vacation. Mr. Jones said that they will spend $\frac{1}{2}$ of the time in Colorado and $\frac{1}{4}$ of the time in Wyoming. How many days will they spend in Colorado and Wyoming?



5. There are 32 students in Miss Reynolds' class. If $\frac{3}{4}$ of the students like pizza, how many students do not like pizza?
- *6. There are 120 people in a room. One-third of the people are blonds, $\frac{1}{4}$ of the people are redheads, and $\frac{1}{6}$ of the people are gray-haired. If the people left are all brunettes, how many brunettes are there?

A decorating project

Jane and Bob plan to work together on redecorating Bob's room. Jane will make curtains and a bedspread. Bob will paint the walls and put up 2 shelves. Jane needs $12\frac{1}{2}$ yards of material at \$4 a yard for the curtains and $10\frac{1}{2}$ yards of material at \$7 a yard for the bedspread. Bob needs $2\frac{1}{2}$ gallons of paint at \$5 a gallon and $18\frac{3}{4}$ feet of lumber at 12¢ a foot. If they also plan to buy 2 rugs at \$8.25 each, how much money will they need to complete the redecorating? If it takes them 60 hours each to do the work and they allow 75¢ an hour for their time, what will be the total cost?





Both factors less than one

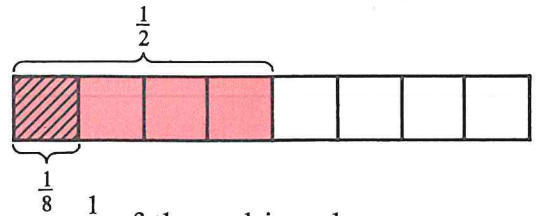
{       }

$\frac{1}{2}$ of the objects are red.

$\frac{1}{3}$ of the red objects is a square.

$\frac{1}{6}$ of all the objects is a red square.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$



$\frac{1}{2}$ of the rod is red.

$\frac{1}{4}$ of the red part is shaded.

$\frac{1}{8}$ of the rod is red and shaded.

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

EXERCISES

Use the sets to help you complete the sentences.

{     }

1. $\frac{?}{?}$ of the objects are red.
2. $\frac{?}{?}$ of the red objects is a triangle.
3. $\frac{?}{?}$ of all the objects is a red triangle.
4. $\frac{1}{2} \times \frac{1}{2} = \frac{?}{?}$

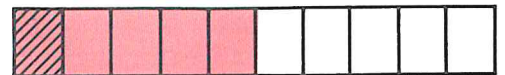
{             }

5. $\frac{?}{?}$ of the objects are red.
6. $\frac{?}{?}$ of the red objects is a square.
7. $\frac{?}{?}$ of all the objects is a red square.
8. $\frac{1}{3} \times \frac{1}{4} = \frac{?}{?}$

Use the rods to help you complete the sentences.



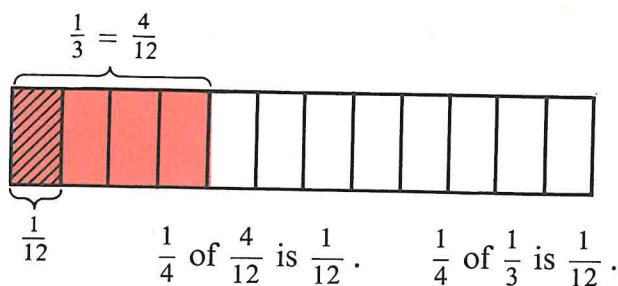
9. $\frac{?}{?}$ of the rod is red.
10. $\frac{?}{?}$ of the red part is shaded.
11. $\frac{?}{?}$ of the rod is red and shaded.
12. $\frac{1}{3} \times \frac{1}{5} = \frac{?}{?}$



13. $\frac{?}{?}$ of the rod is red.
14. $\frac{?}{?}$ of the red part is shaded.
15. $\frac{?}{?}$ of the rod is red and shaded.
16. $\frac{1}{5} \times \frac{1}{2} = \frac{?}{?}$

Both factors less than one

Study the rods and the equations for naming the product $\frac{1}{4} \times \frac{1}{3}$.



$$\begin{aligned}\frac{1}{4} \times \frac{1}{3} &= \frac{1}{\textcolor{red}{4}} \times \frac{\textcolor{red}{4}}{12} \\ &= \frac{1}{4} \times \left(4 \times \frac{1}{12}\right) \\ &= \left(\frac{1}{4} \times 4\right) \times \frac{1}{12} = \frac{1}{12}\end{aligned}$$

EXERCISES

Solve the equations.

1. $\frac{1}{4} \times 4 = a$
2. $\frac{1}{3} \times 3 = b$
3. $\frac{1}{4} \times 5 = c$
4. $\frac{1}{4} \times 6 = d$
5. $8 \times \frac{1}{8} = e$
6. $7 \times \frac{1}{7} = f$
7. $9 \times \frac{1}{9} = g$
8. $10 \times \frac{1}{10} = h$

Copy and complete the equations.

9. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{\textcolor{red}{2}} \times \frac{\textcolor{red}{2}}{8} = a$
10. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{2} \times \frac{2}{c} = b$
11. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \times \frac{3}{m} = e$
12. $\frac{1}{4} \times \frac{1}{5} = \frac{1}{4} \times \frac{4}{r} = i$
13. $\frac{1}{4} \times \frac{1}{8} = \frac{1}{4} \times \frac{4}{s} = q$
14. $\frac{1}{5} \times \frac{1}{2} = \frac{1}{5} \times \frac{5}{u} = n$

Solve the equations.

15. $\frac{1}{3} \times \frac{1}{6} = a$
16. $\frac{1}{2} \times \frac{1}{7} = b$
17. $\frac{1}{5} \times \frac{1}{4} = c$
18. $\frac{1}{10} \times \frac{1}{2} = d$
19. $\frac{1}{4} \times \frac{1}{6} = e$
20. $\frac{1}{9} \times \frac{1}{3} = f$
21. $\frac{1}{3} \times \frac{1}{4} = g$
22. $\frac{1}{6} \times \frac{1}{5} = h$

Solve the problems.

23. If $\frac{1}{3}$ of the children in the 6th grade are girls and $\frac{1}{3}$ of the girls are over 11 years old, what part of the 6th graders are girls over 11 years old?
24. If $\frac{1}{2}$ of the children in the 5th grade are boys and $\frac{1}{4}$ of the boys are left-handed, what part of the 5th graders are left-handed boys?



Number patterns

Study this number pattern.

$$\frac{1}{4} \times 16 = \frac{1}{4} \times 4 \times 4 = 4$$

$$\frac{1}{4} \times 8 = \frac{1}{4} \times 4 \times 2 = 2$$

$$\frac{1}{4} \times 4 = \frac{1}{4} \times 4 \times 1 = 1$$

$$\frac{1}{4} \times 2 = \frac{1}{4} \times 4 \times \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{4} \times 1 = \frac{1}{4} \times 4 \times \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{4} \times 4 \times \frac{1}{8} = \frac{1}{8}$$

EXERCISES

Copy and complete the number patterns.

1. $\frac{1}{2} \times 8 = \frac{1}{2} \times 2 \times 4 = 4$

7. $27 \times \frac{1}{3} = 9 \times 3 \times \frac{1}{3} = m$

2. $\frac{1}{2} \times 4 = \frac{1}{2} \times 2 \times a = b$

8. $9 \times \frac{1}{3} = 3 \times 3 \times n = p$

3. $\frac{1}{2} \times 2 = \frac{1}{2} \times 2 \times c = d$

9. $3 \times \frac{1}{3} = 1 \times q \times \frac{1}{3} = r$

4. $\frac{1}{2} \times 1 = \frac{1}{2} \times e \times \frac{1}{2} = f$

10. $1 \times \frac{1}{3} = \frac{1}{3} \times 3 \times s = t$

5. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \times 2 \times g = h$

11. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \times 3 \times u = v$

6. $\frac{1}{2} \times \frac{1}{4} = i \times j \times k = l$

12. $\frac{1}{9} \times \frac{1}{3} = w \times x \times y = z$

Complete the sentence.

13. To name the product of two fractional numbers like $\frac{1}{2}$ and $\frac{1}{3}$, we multiply the ____.

Name the products.

14. $\frac{1}{2} \times \frac{1}{3}$ 17. $\frac{1}{7} \times \frac{1}{6}$ 20. $\frac{1}{4} \times \frac{1}{10}$ 23. $\frac{1}{20} \times \frac{1}{6}$ 26. $\frac{1}{5} \times \frac{1}{8} \times \frac{1}{9}$
 15. $\frac{1}{8} \times \frac{1}{4}$ 18. $\frac{1}{9} \times \frac{1}{3}$ 21. $\frac{1}{7} \times \frac{1}{12}$ 24. $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8}$ 27. $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{8}$
 16. $\frac{1}{9} \times \frac{1}{5}$ 19. $\frac{1}{6} \times \frac{1}{6}$ 22. $\frac{1}{4} \times \frac{1}{12}$ 25. $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{3}$ 28. $\frac{1}{4} \times \frac{1}{20} \times \frac{1}{5}$

Both factors less than one

Study the methods
for naming the products.

$$\frac{2}{3} \times \frac{1}{4} = \left(2 \times \frac{1}{3}\right) \times \frac{1}{4}$$

$$= 2 \times \left(\frac{1}{3} \times \frac{1}{4}\right)$$

$$= 2 \times \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3}$$

$$= \frac{2}{12} = \frac{1}{6}$$

EXERCISES

Solve the equations.

$$1. \quad \frac{3}{4} \times \frac{1}{3} = \left(a \times \frac{1}{4}\right) \times \frac{1}{3}$$

$$= 3 \times \left(\frac{1}{4} \times b\right)$$

$$= 3 \times c = \frac{d}{12} = \frac{e}{4}$$

$$2. \quad \frac{2}{3} \times \frac{1}{2} = \left(f \times \frac{1}{3}\right) \times \frac{1}{2}$$

$$= 2 \times \left(\frac{1}{3} \times g\right)$$

$$= 2 \times h = \frac{i}{6} = \frac{k}{3}$$

$$3. \quad \frac{4}{5} \times \frac{1}{4} = \left(l \times \frac{1}{5}\right) \times \frac{1}{4}$$

$$= 4 \times \left(\frac{1}{5} \times m\right)$$

$$= 4 \times n = \frac{r}{20} = \frac{s}{5}$$

$$4. \quad \frac{7}{8} \times \frac{1}{7} = \left(t \times \frac{1}{8}\right) \times \frac{1}{7}$$

$$= u \times \left(\frac{1}{8} \times v\right)$$

$$= 7 \times \frac{w}{56} = \frac{x}{8}$$

Name the products in simplest form.

$$5. \quad \frac{2}{5} \times \frac{1}{2}$$

$$12. \quad \frac{7}{10} \times \frac{1}{2}$$

$$19. \quad \frac{4}{5} \times \frac{1}{4}$$

$$26. \quad \frac{1}{6} \times \frac{3}{4}$$

$$6. \quad \frac{2}{3} \times \frac{1}{4}$$

$$13. \quad \frac{1}{7} \times \frac{3}{5}$$

$$20. \quad \frac{3}{8} \times \frac{1}{9}$$

$$27. \quad \frac{1}{7} \times \frac{2}{3}$$

$$7. \quad \frac{3}{4} \times \frac{1}{5}$$

$$14. \quad \frac{1}{4} \times \frac{5}{6}$$

$$21. \quad \frac{1}{2} \times \frac{3}{5}$$

$$28. \quad \frac{1}{10} \times \frac{5}{8}$$

$$8. \quad \frac{5}{6} \times \frac{1}{2}$$

$$15. \quad \frac{1}{3} \times \frac{7}{8}$$

$$22. \quad \frac{1}{3} \times \frac{3}{8}$$

$$29. \quad \frac{4}{11} \times \frac{1}{2}$$

$$9. \quad \frac{4}{9} \times \frac{1}{4}$$

$$16. \quad \frac{1}{6} \times \frac{3}{8}$$

$$23. \quad \frac{1}{4} \times \frac{5}{6}$$

$$30. \quad \frac{1}{18} \times \frac{12}{13}$$

$$10. \quad \frac{4}{5} \times \frac{1}{5}$$

$$17. \quad \frac{5}{8} \times \frac{1}{10}$$

$$24. \quad \frac{1}{2} \times \frac{7}{9}$$

$$31. \quad \frac{1}{9} \times \frac{6}{7}$$

$$11. \quad \frac{5}{8} \times \frac{1}{3}$$

$$18. \quad \frac{7}{10} \times \frac{1}{7}$$

$$25. \quad \frac{1}{5} \times \frac{2}{3}$$

$$32. \quad \frac{15}{16} \times \frac{1}{20}$$

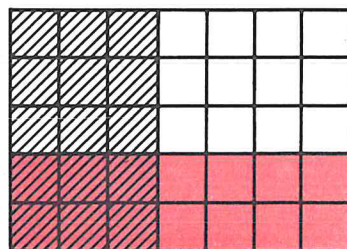
Both factors less than one

Study the diagram.

$\frac{2}{5}$ of the rows are *red*.

$\frac{3}{7}$ of the columns are *shaded*.

$\frac{6}{35}$ of the small square regions are *shaded*
and *red*.



$$\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$$

EXERCISES

Use the diagrams to help you solve the equations.



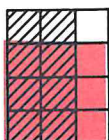
1. $\frac{2}{3} \times \frac{3}{4} = a$



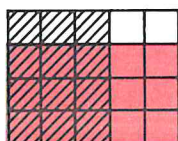
2. $\frac{2}{3} \times \frac{4}{5} = b$



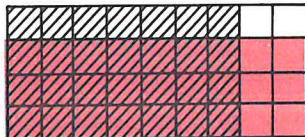
3. $\frac{2}{3} \times \frac{5}{6} = c$



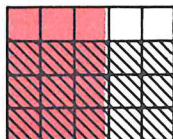
4. $\frac{3}{4} \times \frac{2}{3} = d$



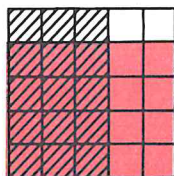
5. $\frac{3}{4} \times \frac{3}{5} = e$



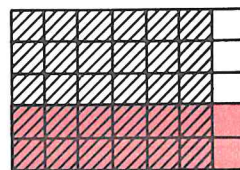
6. $\frac{3}{4} \times \frac{7}{9} = f$



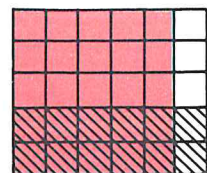
7. $\frac{3}{5} \times \frac{3}{4} = g$



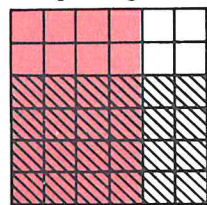
8. $\frac{4}{5} \times \frac{3}{5} = h$



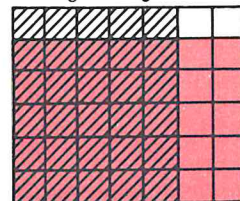
9. $\frac{2}{5} \times \frac{6}{7} = i$



10. $\frac{5}{6} \times \frac{2}{5} = j$



11. $\frac{4}{6} \times \frac{4}{6} = k$



12. $\frac{5}{6} \times \frac{5}{7} = l$

Both factors less than one

Study the methods
for naming the product

$$\frac{2}{5} \times \frac{3}{7}$$

Using the associative and
commutative properties

$$\begin{aligned}\frac{2}{5} \times \frac{3}{7} &= \left(2 \times \frac{1}{5}\right) \times \left(3 \times \frac{1}{7}\right) \\ &= (2 \times 3) \times \left(\frac{1}{5} \times \frac{1}{7}\right) \\ &= 6 \times \frac{1}{35} = \frac{6}{35}\end{aligned}$$

Short form

$$\begin{aligned}\frac{2}{5} \times \frac{3}{7} &= \frac{2 \times 3}{5 \times 7} \\ &= \frac{6}{35}\end{aligned}$$

EXERCISES

Use the associative and commutative properties to name the products.

1. $\frac{3}{4} \times \frac{2}{5}$

5. $\frac{7}{10} \times \frac{2}{3}$

9. $\frac{4}{7} \times \frac{3}{8}$

13. $\frac{2}{3} \times \frac{5}{7}$

17. $\frac{4}{10} \times \frac{2}{3}$

2. $\frac{2}{3} \times \frac{3}{7}$

6. $\frac{5}{6} \times \frac{3}{4}$

10. $\frac{3}{4} \times \frac{9}{10}$

14. $\frac{2}{5} \times \frac{5}{6}$

18. $\frac{6}{7} \times \frac{4}{5}$

3. $\frac{4}{5} \times \frac{2}{9}$

7. $\frac{3}{10} \times \frac{5}{7}$

11. $\frac{4}{5} \times \frac{4}{7}$

15. $\frac{7}{11} \times \frac{2}{3}$

19. $\frac{7}{9} \times \frac{5}{6}$

4. $\frac{3}{7} \times \frac{3}{8}$

8. $\frac{5}{9} \times \frac{2}{3}$

12. $\frac{5}{9} \times \frac{2}{3}$

16. $\frac{5}{8} \times \frac{3}{7}$

20. $\frac{3}{4} \times \frac{5}{11}$

Use the short form to name the products.

21. $\frac{5}{6} \times \frac{2}{5}$

25. $\frac{3}{10} \times \frac{5}{7}$

29. $\frac{4}{7} \times \frac{3}{8}$

33. $\frac{2}{3} \times \frac{7}{13}$

37. $\frac{3}{4} \times \frac{3}{4}$

22. $\frac{3}{7} \times \frac{2}{3}$

26. $\frac{4}{5} \times \frac{4}{7}$

30. $\frac{5}{9} \times \frac{2}{5}$

34. $\frac{3}{5} \times \frac{2}{5}$

38. $\frac{7}{8} \times \frac{2}{9}$

23. $\frac{3}{4} \times \frac{3}{5}$

27. $\frac{3}{8} \times \frac{3}{4}$

31. $\frac{5}{6} \times \frac{3}{11}$

35. $\frac{2}{3} \times \frac{2}{3}$

39. $\frac{5}{11} \times \frac{2}{3}$

24. $\frac{5}{9} \times \frac{2}{3}$

28. $\frac{4}{10} \times \frac{2}{7}$

32. $\frac{5}{7} \times \frac{5}{8}$

36. $\frac{3}{4} \times \frac{5}{6}$

40. $\frac{4}{13} \times \frac{2}{4}$

Solve the problems.

41. Susan bought $\frac{3}{4}$ yard of material. If she used $\frac{4}{5}$ of it to make a doll's blouse, how much of a yard did she use?

42. Lois bought $\frac{7}{8}$ yard of material. If she used $\frac{2}{3}$ of it to make a blouse, how much of a yard did she use?



Multiplication with fractional numbers

Study the equations.

$$\begin{aligned}\frac{3}{4} \times \frac{2}{5} &= \frac{3 \times 2}{4 \times 5} \\ &= \frac{6}{20} = \frac{3}{10}\end{aligned}$$

$$\begin{aligned}\frac{3}{2} \times \frac{5}{4} &= \frac{3 \times 5}{2 \times 4} \\ &= \frac{15}{8} = 1\frac{7}{8}\end{aligned}$$

EXERCISES

Name the products in simplest form.

- | | | | | |
|-------------------------------------|--------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| 1. $\frac{2}{3} \times \frac{3}{4}$ | 5. $\frac{3}{4} \times \frac{6}{7}$ | 9. $\frac{3}{7} \times \frac{5}{8}$ | 13. $\frac{3}{8} \times \frac{5}{11}$ | 17. $\frac{5}{6} \times \frac{3}{10}$ |
| 2. $\frac{5}{7} \times \frac{2}{3}$ | 6. $\frac{5}{9} \times \frac{4}{7}$ | 10. $\frac{3}{10} \times \frac{2}{7}$ | 14. $\frac{4}{13} \times \frac{2}{3}$ | 18. $\frac{2}{3} \times \frac{2}{17}$ |
| 3. $\frac{1}{2} \times \frac{5}{6}$ | 7. $\frac{9}{10} \times \frac{1}{4}$ | 11. $\frac{1}{5} \times \frac{6}{7}$ | 15. $\frac{7}{8} \times \frac{1}{9}$ | 19. $\frac{3}{14} \times \frac{4}{5}$ |
| 4. $\frac{3}{8} \times \frac{2}{5}$ | 8. $\frac{3}{11} \times \frac{7}{9}$ | 12. $\frac{3}{4} \times \frac{2}{9}$ | 16. $\frac{7}{10} \times \frac{3}{4}$ | 20. $\frac{5}{6} \times \frac{2}{7}$ |

Name the products in simplest form.

- | | | | |
|--------------------------------------|--|---------------------------------------|---------------------------------------|
| 21. $\frac{3}{2} \times \frac{4}{5}$ | 24. $\frac{6}{5} \times \frac{2}{3}$ | 27. $\frac{5}{4} \times \frac{3}{4}$ | 30. $\frac{8}{5} \times \frac{3}{5}$ |
| 22. $\frac{5}{7} \times \frac{7}{6}$ | 25. $\frac{9}{10} \times \frac{3}{5}$ | 28. $\frac{10}{9} \times \frac{2}{3}$ | 31. $\frac{3}{4} \times \frac{12}{6}$ |
| 23. $\frac{4}{3} \times \frac{2}{7}$ | 26. $\frac{7}{8} \times \frac{11}{10}$ | 29. $\frac{4}{3} \times \frac{5}{6}$ | 32. $\frac{5}{8} \times \frac{11}{7}$ |

Name the products in simplest form.

- | | | | |
|--------------------------------------|---------------------------------------|---------------------------------------|---|
| 33. $\frac{5}{4} \times \frac{3}{2}$ | 36. $\frac{7}{5} \times \frac{5}{3}$ | 39. $\frac{7}{6} \times \frac{11}{8}$ | 42. $\frac{14}{3} \times \frac{8}{7}$ |
| 34. $\frac{6}{5} \times \frac{7}{4}$ | 37. $\frac{11}{3} \times \frac{5}{3}$ | 40. $\frac{8}{3} \times \frac{5}{2}$ | 43. $\frac{13}{7} \times \frac{7}{3}$ |
| 35. $\frac{3}{2} \times \frac{9}{8}$ | 38. $\frac{9}{4} \times \frac{3}{2}$ | 41. $\frac{11}{5} \times \frac{5}{4}$ | 44. $\frac{20}{11} \times \frac{22}{5}$ |

Complete the sentence.

45. To name the product of 2 fractional numbers, multiply the numerators and the ____?

Multiplication with fractional numbers

Study the equations.

$$\frac{6}{21} \times \frac{3}{4} = \frac{6 \times 3}{21 \times 4}$$

$$= \frac{3 \times 2 \times 3}{7 \times 3 \times 2 \times 2}$$

$$= \frac{3}{7 \times 2} \times \frac{3 \times 2}{3 \times 2}$$

$$= \frac{3}{7 \times 2} = \frac{3}{14}$$

$$\frac{6}{21} \times \frac{3}{4} = \frac{3 \times 2}{7 \times 3} \times \frac{3}{2 \times 2}$$

$$= \frac{3}{7 \times 2} \times \frac{3 \times 2}{3 \times 2}$$

$$= \frac{3}{14} \times 1$$

$$= \frac{3}{14}$$

3×2 is the GCF of the numerator and denominator. We divide the numerator and denominator by the GCF.

EXERCISES

Copy and complete the equations.

$$\begin{aligned} 1. \quad \frac{2}{3} \times \frac{6}{7} &= \frac{2 \times 3 \times a}{3 \times b} \\ &= \frac{2 \times 2 \times c}{7 \times 3} \\ &= \frac{2 \times 2}{7} \times \frac{d}{3} = e \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{2}{9} \times \frac{3}{5} &= \frac{2 \times 3}{3 \times f \times 5} \\ &= \frac{2 \times 3}{3 \times g \times 3} \\ &= \frac{2}{3 \times 5} \times \frac{h}{3} = i \end{aligned}$$

Name the products in simplest form.

$$3. \quad \frac{2}{3} \times \frac{3}{4}$$

$$7. \quad \frac{8}{11} \times \frac{3}{4}$$

$$11. \quad \frac{1}{2} \times \frac{4}{11}$$

$$15. \quad \frac{5}{8} \times \frac{7}{25}$$

$$4. \quad \frac{5}{6} \times \frac{3}{8}$$

$$8. \quad \frac{3}{10} \times \frac{5}{9}$$

$$12. \quad \frac{5}{8} \times \frac{2}{3}$$

$$16. \quad \frac{8}{13} \times \frac{3}{4}$$

$$5. \quad \frac{4}{7} \times \frac{1}{2}$$

$$9. \quad \frac{2}{11} \times \frac{3}{4}$$

$$13. \quad \frac{2}{3} \times \frac{9}{11}$$

$$17. \quad \frac{5}{6} \times \frac{12}{17}$$

$$6. \quad \frac{3}{8} \times \frac{2}{9}$$

$$10. \quad \frac{12}{13} \times \frac{5}{6}$$

$$14. \quad \frac{3}{4} \times \frac{6}{21}$$

$$18. \quad \frac{3}{11} \times \frac{22}{23}$$

Complete this sentence.

19. To name the fractional number $\frac{3 \times 2}{4 \times 5}$ in simplest form, we divide the numerator and denominator by the ? ? factor.

● Multiplying fractional numbers greater than one

Study this method for naming the product $1\frac{1}{2} \times 1\frac{3}{4}$.

$$1\frac{1}{2} \times 1\frac{3}{4} = \frac{3}{2} \times \frac{7}{4} = \frac{3 \times 7}{2 \times 4} = \frac{21}{8} = 2\frac{5}{8}$$

EXERCISES

Name the products as mixed fractions.

1. $1\frac{1}{3} \times 1\frac{1}{2}$

4. $1\frac{2}{7} \times 3\frac{1}{5}$

7. $6\frac{1}{2} \times 1\frac{5}{7}$

10. $3\frac{1}{7} \times 1\frac{4}{5}$

2. $1\frac{5}{6} \times 2\frac{1}{3}$

5. $3\frac{1}{2} \times 2\frac{1}{4}$

8. $4\frac{1}{2} \times 1\frac{2}{3}$

11. $2\frac{3}{4} \times 9\frac{1}{2}$

3. $2\frac{3}{5} \times 1\frac{1}{4}$

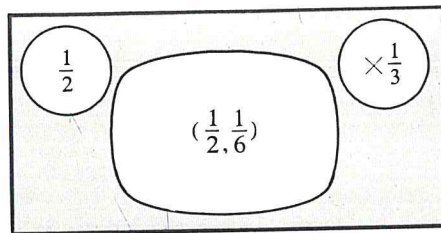
6. $1\frac{7}{8} \times 2\frac{1}{3}$

9. $5\frac{1}{3} \times 2\frac{1}{8}$

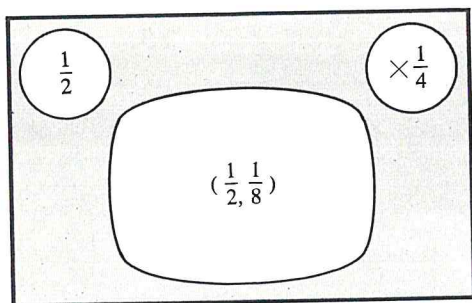
12. $5\frac{1}{10} \times 4\frac{1}{8}$

We may use the function TV to show multiplication with fractional numbers. In our example, the first number in the pair is $\frac{1}{2}$, the rule is multiply by $\frac{1}{3}$, the pair is $(\frac{1}{2}, \frac{1}{6})$.

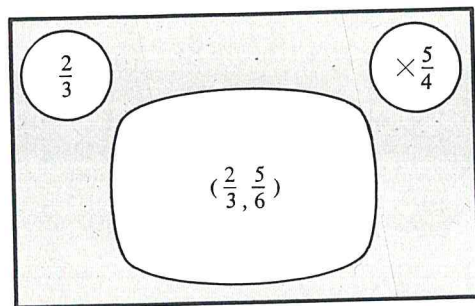
Other pairs in the function would be $\{(\frac{1}{3}, \frac{1}{9}), (\frac{2}{3}, \frac{2}{9}), (\frac{1}{4}, \frac{1}{12}), (\frac{3}{4}, \frac{1}{4})\}$.



Use the function TV to find the rule and complete the ordered pairs.



13. $\{(\frac{1}{2}, \frac{1}{8}), (\frac{1}{3}, a), (\frac{2}{3}, b), (\frac{3}{4}, c), (\frac{5}{6}, d), (\frac{1}{5}, e)\}$



**14. $\{(\frac{2}{3}, \frac{5}{6}), (\frac{2}{7}, g), (\frac{5}{6}, h), (\frac{7}{4}, i), (\frac{5}{8}, j), (\frac{1}{2}, k)\}$

Practice with fractional numbers

Name the sums, differences, or products in simplest form.

1. $\frac{3}{4} + \frac{5}{8}$

3. $\frac{2}{3} + \frac{5}{6}$

5. $1\frac{1}{2} + 2\frac{3}{4}$

7. $1\frac{5}{7} - \frac{8}{21}$

2. $\frac{3}{4} - \frac{5}{8}$

4. $\frac{4}{5} - \frac{7}{15}$

6. $1\frac{2}{3} - \frac{1}{2}$

8. $2\frac{1}{3} + 1\frac{2}{5}$

9. $\frac{3}{4}$

11. $\frac{3}{4}$

13. $\frac{7}{9}$

15. $\frac{7}{8}$

17. $1\frac{3}{4}$

19. $4\frac{3}{7}$

21. $5\frac{2}{5}$

$+\frac{3}{8}$

$-\frac{1}{8}$

$-\frac{1}{2}$

$+\frac{3}{4}$

$+\frac{2\frac{1}{8}}$

$+\frac{2\frac{1}{2}}$

$+\frac{1\frac{2}{3}}$

10. $\frac{2}{3}$

12. $\frac{2}{3}$

14. $\frac{4}{5}$

16. $\frac{4}{5}$

18. $3\frac{1}{3}$

20. $6\frac{1}{9}$

22. $3\frac{1}{2}$

$+\frac{1}{4}$

$-\frac{1}{4}$

$+\frac{5}{6}$

$+\frac{9}{10}$

$-\frac{2\frac{1}{2}}$

$-\frac{3\frac{2}{3}}$

$-\frac{1\frac{5}{6}}$

23. $\frac{1}{3} \times \frac{1}{5}$

27. $\frac{3}{4} \times \frac{2}{5}$

31. $\frac{5}{8} \times \frac{2}{3}$

35. $\frac{5}{4} \times \frac{2}{3}$

39. $\frac{13}{8} \times \frac{5}{2}$

24. $\frac{2}{3} \times \frac{1}{5}$

28. $\frac{5}{6} \times \frac{3}{5}$

32. $\frac{3}{10} \times \frac{5}{3}$

36. $\frac{6}{7} \times \frac{9}{5}$

40. $\frac{11}{7} \times \frac{12}{5}$

25. $\frac{2}{3} \times \frac{2}{5}$

29. $\frac{7}{8} \times \frac{2}{3}$

33. $\frac{9}{7} \times \frac{3}{8}$

37. $\frac{7}{3} \times \frac{6}{5}$

41. $\frac{16}{9} \times \frac{10}{4}$

26. $\frac{4}{3} \times \frac{4}{5}$

30. $\frac{6}{7} \times \frac{1}{2}$

34. $\frac{7}{13} \times \frac{3}{5}$

38. $\frac{8}{5} \times \frac{9}{4}$

42. $\frac{15}{14} \times \frac{7}{3}$

Solve the problems

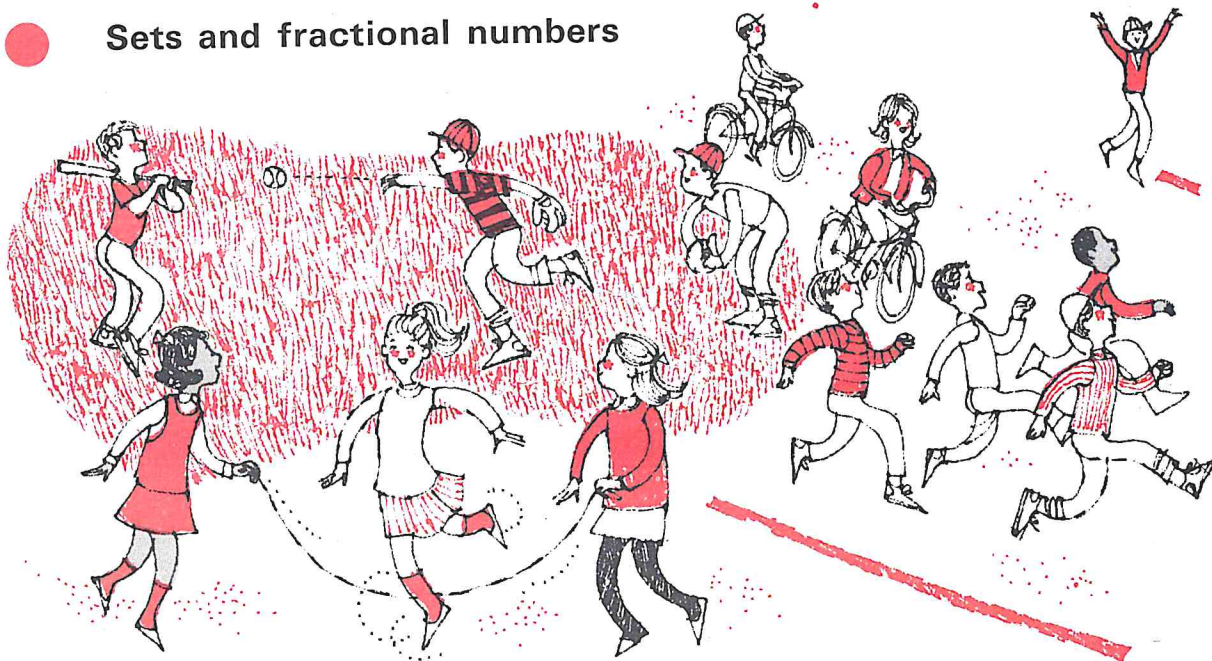
43. If Betty uses $\frac{1}{2}$ cup of flour for each dozen cookies, how many cups will she use for $5\frac{1}{2}$ dozen cookies?

44. If Laura uses $\frac{1}{3}$ tablespoons of vanilla for each pound of candy, how many tablespoons of vanilla will she use to make 7 pounds?

45. If $\frac{3}{4}$ of the books on the shelf have blue covers and $\frac{2}{3}$ of the blue books are about history, how many blue history books are there?

46. If $\frac{2}{5}$ of the books on the shelf are yellow and $\frac{5}{6}$ of the yellow books are about science, how many yellow science books are there?

Sets and fractional numbers



1. Two-thirds of the girls are riding bicycles. If one-fourth of the girls riding bikes are wearing ribbons, how many of the girls are riding bikes and wearing ribbons?
2. If two-fifths of the girls riding bicycles are wearing pink dresses, how many of the girls are riding bicycles and wearing pink dresses?
3. Three-eighths of the boys are racing. Two-sevenths of the boys racing are wearing hats. How many of the boys are racing with their hats on?
4. Two-fifths of the boys are playing baseball. Three-fourths of the boys playing baseball are wearing green socks. How many of the boys are wearing green socks and playing baseball?
5. One-third of the girls are jumping rope. Nine-tenths of the girls jumping rope are wearing striped dresses. How many of the girls are jumping rope and wearing striped dresses?
6. One-half of the children are boys. One-fifth of the boys have brown hair. How many of the children are boys with brown hair?
7. Three-fourths of the children must soon go home. One-half of the children going home soon will ride bicycles. How many of the children will soon ride bicycles home?

Solve the equations.

$$\begin{array}{rcl} 1. & 60 \times (30 + 4) = a + c = e \\ & 9 \times (30 + 4) = n + r = s \\ \hline & 69 \times 34 = u + v = w \end{array}$$

$$\begin{array}{rcl} 2. & 50 \times (70 + 3) = a + b = c \\ & 17 \times (70 + 3) = d + e = f \\ \hline & 67 \times 73 = g + h = i \end{array}$$

$$\begin{array}{rcl} 3. & 40 \times (80 + 5) = j + k = l \\ & 7 \times (80 + 5) = m + n = o \\ \hline & 47 \times 85 = p + q = r \end{array}$$

$$\begin{array}{rcl} 4. & 40 \times (50 + 7) = s + t = u \\ & 16 \times (50 + 7) = v + w = x \\ \hline & 56 \times 57 = y + z = a \end{array}$$

Complete the number patterns.

5.

×	60	5	65
70	<i>a</i>	350	<i>b</i>
13	<i>c</i>	65	<i>d</i>
83	<i>e</i>	415	<i>f</i>

7.

×	40	7	47
30	<i>g</i>	210	<i>h</i>
19	<i>i</i>	133	<i>j</i>
49	<i>k</i>	343	<i>l</i>

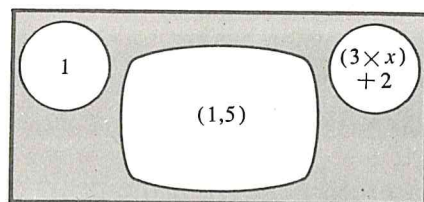
6.

×	20	18	38
40	<i>m</i>	720	<i>n</i>
12	240	<i>o</i>	<i>p</i>
52	<i>q</i>	<i>r</i>	<i>s</i>

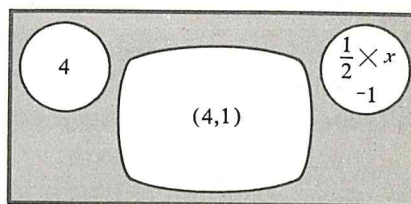
8.

×	50	16	66
60	3000	<i>t</i>	<i>u</i>
15	<i>v</i>	<i>w</i>	<i>x</i>
75	3750	<i>y</i>	<i>z</i>

Use the function TV to complete the number pairs.



9. $\{(1, 5), (2, 8), (3, a), (4, b), (5, c), (6, d), (7, e), (8, f), (10, g)\}$



10. $\{(4, 1), (6, 2), (8, h), (10, i), (12, j), (14, k), (16, l), (18, m), (20, n)\}$

Name the quotients and remainders.

11. $35 \overline{)9768}$

13. $65 \overline{)37842}$

*15. $627 \overline{)754321}$

*17. $716 \overline{)521684}$

12. $47 \overline{)5426}$

14. $87 \overline{)95603}$

*16. $589 \overline{)713542}$

*18. $836 \overline{)469218}$



Probability

$$A = \{\text{red circle}, \text{red square}\} \quad B = \{\text{black triangle}, \text{black circle}, \text{white circle}\}$$

$$A \otimes B = \{(\text{red circle}, \text{black triangle}), (\text{red circle}, \text{black circle}), (\text{red circle}, \text{white circle}), (\text{red square}, \text{black triangle}), (\text{red square}, \text{black circle}), (\text{red square}, \text{white circle})\}$$

The **probability** of choosing  from A is $\frac{1}{2}$, of choosing  from B is $\frac{1}{3}$, so the probability of choosing $(\text{red circle}, \text{black triangle})$ from $A \otimes B$ is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

$$C = \{\text{chocolate ice cream, vanilla ice cream}\}$$

$$D = \{\text{cherry pie, apple pie, peach pie}\}$$

If you choose ice cream and pie without seeing what you get, the **probability** of getting chocolate ice cream is $\frac{1}{2}$, of getting cherry pie is $\frac{1}{3}$. So the probability of getting chocolate ice cream with cherry pie is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

EXERCISES

Solve the problems.

- Sarah has 4 skirts and 5 sweaters. How many pairings can she make of skirts and sweaters?
- If one of the skirts and one of the sweaters are red and, if Sarah chooses a skirt and a sweater without looking, what is the probability that she will get:
 - a red skirt when she chooses a skirt?
 - a red sweater when she chooses a sweater?
 - a red skirt **and** a red sweater if she chooses one of each at the same time?
- There are 3 apples in a basket and 4 pears in another basket. If one of the apples and one of the pears are green and you choose an apple and a pear without looking, what is the probability of getting:
 - a green apple from the apples?
 - a green pear from the pears?
 - a green apple **and** a green pear?
- In problem 2, what is the probability of getting:
 - a skirt that is not red?
 - a sweater that is not red?
 - a skirt **and** a sweater that are not red?
- In problem 3, what is the probability of getting:
 - a ripe apple?
 - a ripe pear?
 - a ripe apple **and** a ripe pear?



$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B = \{2, 4, 6, 8\} \quad C = \{6\}$$

If you choose any whole number between 0 and 10, the probability of choosing an even number is $\frac{n(B)}{n(A)} = \frac{4}{9}$. If you choose a number from B , the probability of choosing a multiple of 3 is $\frac{n(C)}{n(B)} = \frac{1}{4}$. The probability of choosing from A a number that is both a multiple of 3 **and** an even number is $\frac{1}{4} \times \frac{4}{9} = \frac{1}{9}$.

EXERCISES

Solve the problems.

1.
 - a. If you choose any whole number between 0 and 20, what is the probability of choosing an even number?
 - b. If you choose any number from the even numbers between 0 and 20, what is the probability of choosing a multiple of 3?
 - c. If you choose any whole number between 0 and 20, what is the probability of choosing a number that is a multiple of 3 **and** an even number?
 - d. If you choose any whole number between 0 and 20, what is the probability of choosing a number that is a multiple of 3 **and** of 4?
2. If there are 3 apples and 4 pears in a basket and one piece of fruit falls out, what is the probability that the piece that falls out is
 - a. an apple?
 - b. a pear?
3. If you choose any whole number between 1 and 10, what is the probability that it is
 - a. prime?
 - b. even?
 - c. prime **and** even?
4. If the probability of rain on Monday is $\frac{1}{3}$, of rain on Tuesday is $\frac{1}{4}$, what is the probability of rain on both Monday and Tuesday?



Multiplication and division

Study the number patterns.

$$8 \times 8 = 64 \quad 1 \times 8 = 8$$

$$16 \div 8 = 2 \quad 16 \div 1 = 16$$

$$4 \times 8 = 32 \quad \frac{1}{2} \times 8 = 4$$

$$16 \div 4 = 4 \quad 16 \div \frac{1}{2} = 32$$

$$2 \times 8 = 16 \quad \frac{1}{4} \times 8 = 2$$

$$16 \div 2 = 8 \quad 16 \div \frac{1}{4} = 64$$

EXERCISES

Complete the number patterns.

1. $1 \times 8 = a$ $1 \div 8 = h$

$$1 \times 4 = b \quad 1 \div 4 = i$$

$$1 \times 2 = c \quad 1 \div 2 = j$$

$$1 \times 1 = d \quad 1 \div 1 = k$$

$$1 \times \frac{1}{2} = e \quad 1 \div \frac{1}{2} = l$$

$$1 \times \frac{1}{4} = f \quad 1 \div \frac{1}{4} = m$$

2. $9 \div 27 = o$ $9 \times 27 = u$

$$9 \div 9 = p \quad 9 \times 9 = v$$

$$9 \div 3 = q \quad 9 \times 3 = w$$

$$9 \div 1 = r \quad 9 \times 1 = x$$

$$9 \div \frac{1}{3} = s \quad 9 \times \frac{1}{3} = y$$

$$9 \div \frac{1}{9} = t \quad 9 \times \frac{1}{9} = z$$

Complete the sentences.

3. In a multiplication pattern, if one factor remains the same, the less the other factor, the _____ the product.
4. In a division pattern, if the dividend remains the same, the less the divisor, the _____ the quotient.
5. Division is the opposite of _____.

Use the first equation to solve the others in the patterns.

$$1 \div \frac{1}{3} = 3$$

$$1 \div \frac{1}{4} = 4$$

$$3 \div \frac{1}{4} = 12$$

6. $2 \div \frac{1}{3} = a$

9. $2 \div \frac{1}{4} = d$

12. $6 \div \frac{1}{4} = j$

7. $3 \div \frac{1}{3} = b$

10. $3 \div \frac{1}{4} = e$

13. $9 \div \frac{1}{4} = k$

8. $4 \div \frac{1}{3} = c$

11. $4 \div \frac{1}{4} = f$

14. $12 \div \frac{1}{4} = e$

Multiplication and division are opposites

Study the examples.

A. Dividing by 4 is the same as multiplying by $\frac{1}{4}$.

$$12 \div 4 = 3; 12 \times \frac{1}{4} = 3$$

$$\text{Thus, } 12 \div 4 = 12 \times \frac{1}{4}$$

B. Dividing by $\frac{1}{4}$ is the same as multiplying by 4.

$$3 \div \frac{1}{4} = 12; 3 \times 4 = 12$$

$$\text{Thus, } 3 \div \frac{1}{4} = 3 \times 4$$

C. Multiplying by 4, then dividing by 4, is the same as multiplying by 1.

$$(3 \times 4) \div 4 = 12 \div 4 = 3$$

$$3 \times (4 \div 4) = 3 \times 1 = 3$$

$$\text{Thus, } (3 \times 4) \div 4 = 3 \times 1$$

D. Multiplying first by 4, then by $\frac{1}{4}$, is the same as multiplying by 1.

$$\begin{aligned}(3 \times 4) \times \frac{1}{4} &= 3 \times \left(4 \times \frac{1}{4}\right) \\ &= 3 \times 1 = 3\end{aligned}$$

EXERCISES

Solve the equations.

1. $\frac{1}{3} \times 9 = 9 \div a = b$

3. $\frac{1}{7} \times 21 = 21 \div e = f$

2. $\frac{1}{6} \times 12 = 12 \div c = d$

4. $\frac{1}{5} \times 10 = 10 \div g = h$

5. $2 \div \frac{1}{5} = 2 \times i = j$

7. $7 \div \frac{1}{9} = 7 \times m = n$

6. $4 \div \frac{1}{4} = 4 \times k = l$

8. $4 \div \frac{1}{2} = 4 \times p = q$

9. $(2 \times 3) \div 3 = 2 \times 1 = t$

11. $(4 \times 5) \div 5 = 4 \times x = z$

10. $(3 \times 6) \div 6 = 3 \times u = w$

12. $(8 \times 7) \div 7 = 8 \times a = b$

13. $\begin{aligned}10 \times \frac{1}{2} &= (5 \times 2) \times \frac{1}{2} \\ &= 5 \times (2 \times \frac{1}{2}) \\ &= 5 \times c \\ &= d\end{aligned}$

14. $\begin{aligned}12 \times \frac{1}{3} &= (4 \times 3) \times \frac{1}{3} \\ &= 4 \times (e \times \frac{1}{3}) \\ &= 4 \times f \\ &= g\end{aligned}$



Reciprocals

Study the equations.

A. $4 \times \frac{1}{4} = 1$

The product of 4 and $\frac{1}{4}$ is 1.

B. $\frac{4}{3} \times \frac{3}{4} = \frac{4 \times 3}{3 \times 4} = \frac{12}{12} = 1$

The product of $\frac{4}{3}$ and $\frac{3}{4}$ is 1.

Two numbers whose product is 1 are called **reciprocals**.

EXERCISES

Solve the equations, then name the reciprocal of the given factor.

1. $a \times \frac{1}{3} = 1$ 3. $c \times \frac{1}{5} = 1$ 5. $7 \times e = 1$ 7. $\frac{2}{3} \times g = 1$
 2. $b \times \frac{1}{4} = 1$ 4. $2 \times d = 1$ 6. $11 \times f = 1$ 8. $h \times \frac{6}{5} = 1$

Solve the equations.

9. $\frac{7}{8} \times \frac{8}{7} = a$ 11. $\frac{6}{7} \times \frac{7}{6} = c$ 13. $\frac{5}{9} \times e = 1$ 15. $\frac{11}{13} \times \frac{13}{11} = g$
 10. $b \times \frac{3}{5} = 1$ 12. $\frac{9}{2} \times d = 1$ 14. $\frac{10}{3} \times f = 1$ 16. $h \times \frac{8}{5} = 1$

Complete the ordered pairs if the rule is, "Name the reciprocal."

17. $\left\{ (1, 1), \left(\frac{1}{2}, 2\right), \left(\frac{1}{3}, c\right), \left(\frac{1}{4}, d\right), \left(\frac{1}{5}, e\right), \left(\frac{1}{9}, f\right), \left(\frac{1}{13}, g\right), \left(\frac{1}{23}, h\right) \right\}$
 18. $\left\{ \left(\frac{2}{3}, a\right), \left(\frac{5}{6}, b\right), \left(\frac{7}{8}, c\right), \left(\frac{4}{5}, d\right), \left(\frac{6}{7}, e\right), \left(\frac{5}{9}, f\right), \left(\frac{9}{10}, g\right), \left(\frac{5}{17}, h\right) \right\}$

Complete the sentence.

19. The reciprocal of a fractional number of the form $\frac{1}{n}$ is $\frac{1}{\frac{1}{n}}$,
 but n cannot be 0.

Solve the problem.

20. There are 30 students in Mrs. Henderson's sixth grade class.
 $\frac{1}{30}$ of the class had a perfect spelling test. How many of the students had spelled all of the words correctly?



Study the equations.

$$12 \div 3 = 4$$

Dividing by 3 is the same as multiplying

$$12 \times \frac{1}{3} = 4$$

by its reciprocal, $\frac{1}{3}$.

$$8 \div \frac{1}{2} = 16$$

$$8 \times 2 = 16$$

Dividing by $\frac{1}{2}$ is the same as multiplying by its reciprocal, 2.

EXERCISES

Use reciprocals to solve the equations.

1. $8 \div 4 = 8 \times a = b$

8. $7 \div \frac{1}{2} = 7 \times p = q$

2. $9 \div 3 = 9 \times c = d$

9. $9 \div \frac{1}{5} = 9 \times r = s$

3. $10 \div 5 = 10 \times e = f$

10. $6 \div \frac{1}{9} = 6 \times t = u$

4. $12 \div 6 = 12 \times g = h$

11. $5 \div \frac{1}{11} = 5 \times v = w$

5. $18 \div 9 = 18 \times i = j$

12. $22 \div \frac{1}{3} = 22 \times x = y$

6. $21 \div 7 = 21 \times k = l$

13. $8 \div \frac{2}{3} = 8 \times z = a$

7. $24 \div 8 = 24 \times m = n$

14. $9 \div \frac{4}{3} = 9 \times b = c$

Use reciprocals to name the quotients.

15. $11 \div \frac{1}{2}$

17. $7 \div \frac{1}{9}$

19. $9 \div \frac{4}{3}$

21. $48 \div \frac{6}{7}$

23. $40 \div \frac{8}{5}$

16. $16 \div \frac{1}{3}$

18. $6 \div \frac{3}{5}$

20. $15 \div \frac{3}{5}$

22. $10 \div \frac{1}{5}$

24. $50 \div \frac{2}{3}$

Solve the problems.

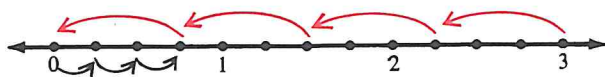
25. One pound of butter is often sold in 4 sticks of the same weight. How much does each stick weigh?

26. A toothpaste commercial said that $\frac{1}{3}$ of all people tested had fewer cavities by brushing with Brand X. Three people were tested. How many had fewer cavities?

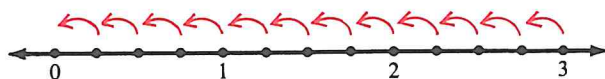


Division with fractional numbers

Study the number lines and equations.



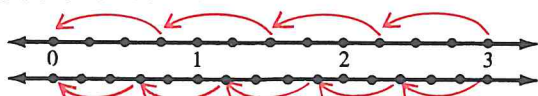
$$3 \div 4 = 3 \times \frac{1}{4} = \frac{3}{4}$$



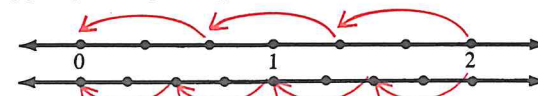
$$3 \div \frac{1}{4} = 3 \times 4 = 12$$

EXERCISES

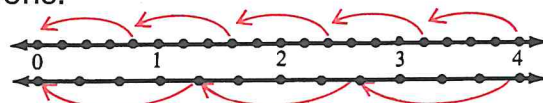
Use the number lines to solve the equations.



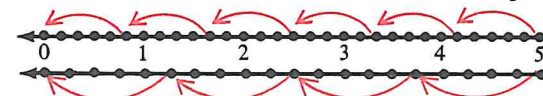
1. $3 \div 4 = a$ 2. $3 \div 5 = b$



3. $2 \div 3 = c$ 4. $2 \div 5 = d$



5. $4 \div 5 = e$ 6. $4 \div 3 = f$



7. $5 \div 6 = g$ 8. $5 \div 4 = h$

Answer the questions, then solve the equations.

9. How many fourths in 1? in 2? $1 \div 4 = m$, $2 \div \frac{1}{4} = n$
10. How many eighths in 1? in 3? $1 \div \frac{1}{8} = m$, $3 \div \frac{1}{8} = n$
11. How many fifths in 1? in 4? $1 \div \frac{1}{5} = m$, $4 \div \frac{1}{5} = n$
12. How many thirds in 1? in 2? $1 \div \frac{1}{3} = m$, $2 \div \frac{1}{3} = n$
13. How many ninths in 1? in 3? $1 \div \frac{1}{9} = m$, $3 \div \frac{1}{9} = n$

Draw number lines if necessary to help you solve the equations.

14. $4 \div \frac{1}{2} = a$ 17. $2 \div \frac{1}{7} = d$ 20. $6 \div \frac{1}{2} = g$ 23. $10 \div \frac{1}{4} = j$
15. $2 \div \frac{1}{3} = b$ 18. $2 \div \frac{1}{8} = e$ 21. $6 \div \frac{1}{3} = h$ 24. $10 \div \frac{1}{5} = k$
16. $2 \div \frac{1}{4} = c$ 19. $2 \div \frac{1}{3} = f$ 22. $6 \div \frac{1}{4} = i$ 25. $10 \div \frac{1}{6} = l$

Study the equations.

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times 4$$

$$= \left(3 \times \frac{1}{4}\right) \times 4$$

$$= 3 \times \left(\frac{1}{4} \times 4\right)$$

$$= 3 \times 1 = 3$$

$$\frac{3}{5} \div \frac{2}{5} = \frac{3}{5} \times \frac{5}{2}$$

$$= \left(3 \times \frac{1}{5}\right) \times \left(5 \times \frac{1}{2}\right)$$

$$= 3 \times \left(\frac{1}{5} \times 5\right) \times \frac{1}{2}$$

$$= 3 \times 1 \times \frac{1}{2} = \frac{3}{2}$$

EXERCISES

Answer the questions, then solve the equations.

1. How many fourths in three-fourths?

$$\frac{3}{4} \div \frac{1}{4} = n$$

2. How many eighths in five-eighths?

$$\frac{5}{8} \div \frac{1}{8} = n$$

3. How many fifths in four-fifths?

$$\frac{4}{5} \div \frac{1}{5} = n$$

Use equations to name the quotients.

4. $\frac{3}{5} \div \frac{1}{5} = a$ 10. $\frac{4}{9} \div \frac{2}{9} = g$ 16. $\frac{5}{8} \div \frac{3}{8} = m$ 22. $\frac{8}{3} \div \frac{2}{3} = s$

5. $\frac{3}{8} \div \frac{1}{8} = b$ 11. $\frac{6}{8} \div \frac{3}{8} = h$ 17. $\frac{2}{3} \div \frac{2}{3} = n$ 23. $\frac{7}{5} \div \frac{9}{5} = t$

6. $\frac{4}{5} \div \frac{1}{5} = c$ 12. $\frac{3}{7} \div \frac{2}{7} = i$ 18. $\frac{5}{11} \div \frac{4}{11} = o$ 24. $\frac{8}{3} \div \frac{4}{3} = u$

7. $\frac{6}{8} \div \frac{1}{8} = d$ 13. $\frac{5}{8} \div \frac{1}{8} = j$ 19. $\frac{1}{2} \div \frac{3}{2} = p$ 25. $\frac{11}{6} \div \frac{4}{6} = v$

8. $\frac{4}{5} \div \frac{3}{5} = e$ 14. $\frac{3}{5} \div \frac{2}{5} = k$ 20. $\frac{5}{13} \div \frac{2}{13} = q$ 26. $\frac{7}{5} \div \frac{16}{5} = w$

9. $\frac{4}{5} \div \frac{2}{5} = f$ 15. $\frac{4}{9} \div \frac{2}{9} = l$ 21. $\frac{7}{4} \div \frac{5}{4} = r$ 27. $\frac{10}{7} \div \frac{10}{7} = x$

Solve the problems.

28. Mustard is sometimes packed in $\frac{1}{8}$ oz. packets. How many of these packets are needed for $\frac{7}{8}$ oz. of mustard?

29. If a power lawnmower uses $\frac{2}{5}$ gal. of gas in an hour, how many hours will it take to use $\frac{3}{5}$ gal. of gas?



Using reciprocals

Study the equations.

$$\begin{aligned}\frac{1}{2} \div 3 &= \frac{1}{2} \times \frac{1}{3} \\ &= \frac{1}{6}\end{aligned}$$

$$\begin{aligned}\frac{3}{4} \div 2 &= \frac{3}{4} \times \frac{1}{2} = \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \\ &= 3 \times \left(\frac{1}{4} \times \frac{1}{2}\right) = \frac{3}{8}\end{aligned}$$

EXERCISES

Solve the equations.

$$\begin{aligned}1. \quad \frac{1}{4} \div 5 &= \frac{1}{4} \times a \\ &= b\end{aligned}$$

$$\begin{aligned}2. \quad \frac{1}{7} \div 8 &= \frac{1}{7} \times a \\ &= b\end{aligned}$$

$$\begin{aligned}3. \quad \frac{1}{9} \div 5 &= \frac{1}{9} \times a \\ &= b\end{aligned}$$

$$\begin{aligned}4. \quad \frac{2}{3} \div 4 &= \frac{2}{3} \times a \\ &= \left(2 \times \frac{1}{3}\right) \times b \\ &= 2 \times \left(c \times \frac{1}{4}\right) = d\end{aligned}$$

$$\begin{aligned}5. \quad \frac{5}{6} \div 3 &= \frac{5}{6} \times a \\ &= \left(5 \times \frac{1}{6}\right) \times b \\ &= 5 \times \left(c \times \frac{1}{3}\right) = d\end{aligned}$$

Name these quotients in simplest form.

$$6. \quad \frac{2}{5} \div 2$$

$$11. \quad \frac{7}{13} \div 7$$

$$16. \quad \frac{5}{8} \div 2$$

$$21. \quad \frac{5}{9} \div 2$$

$$7. \quad \frac{5}{6} \div 5$$

$$12. \quad \frac{10}{11} \div 5$$

$$17. \quad \frac{2}{3} \div 6$$

$$22. \quad \frac{7}{13} \div 2$$

$$8. \quad \frac{8}{11} \div 4$$

$$13. \quad \frac{8}{13} \div 2$$

$$18. \quad \frac{5}{6} \div 3$$

$$23. \quad \frac{7}{8} \div 4$$

$$9. \quad \frac{6}{9} \div 3$$

$$14. \quad \frac{15}{17} \div 3$$

$$19. \quad \frac{4}{7} \div 3$$

$$24. \quad \frac{3}{4} \div 4$$

$$10. \quad \frac{12}{13} \div 6$$

$$15. \quad \frac{14}{17} \div 7$$

$$20. \quad \frac{4}{5} \div 4$$

$$25. \quad \frac{4}{11} \div 2$$

Solve the problems.

26. If a cookie recipe takes $\frac{3}{4}$ lb. of sugar, how much sugar is needed for half the cookie recipe?

27. If a power lawnmower uses $\frac{4}{5}$ gallon of gas in half an hour, how much gas does it use in half that time?

Study the equations
for naming the quotient

$$\frac{2}{3} \div \frac{3}{5}$$

Step 1: Change the division
to multiplication and
 $\frac{3}{5}$ to its reciprocal, $\frac{5}{3}$.

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3}$$

Step 2: Name the product
as on page 244.

$$\frac{2}{3} \times \frac{5}{3} = \frac{10}{9} = 1\frac{1}{9}$$

EXERCISES

Solve the equations.

$$1. \quad \frac{3}{4} \div \frac{2}{5} = \frac{a}{4} \times \frac{5}{b}$$

$$4. \quad \frac{5}{9} \div \frac{7}{8} = \frac{5}{g} \times \frac{h}{7}$$

$$7. \quad \frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times m$$

$$2. \quad \frac{4}{7} \div \frac{2}{3} = \frac{c}{7} \times \frac{3}{d}$$

$$5. \quad \frac{3}{10} \div \frac{4}{7} = \frac{3}{i} \times \frac{j}{4}$$

$$8. \quad \frac{3}{7} \div \frac{1}{5} = \frac{3}{7} \times n$$

$$3. \quad \frac{3}{8} \div \frac{5}{6} = \frac{e}{8} \times \frac{6}{f}$$

$$6. \quad \frac{4}{5} \div \frac{3}{4} = \frac{4}{k} \times \frac{l}{3}$$

$$9. \quad \frac{3}{4} \div \frac{7}{9} = \frac{3}{4} \times p$$

Name the quotients.

$$10. \quad \frac{3}{7} \div \frac{1}{2}$$

$$15. \quad \frac{3}{8} \div \frac{5}{6}$$

$$20. \quad \frac{7}{10} \div \frac{2}{3}$$

$$25. \quad \frac{6}{9} \div \frac{6}{7}$$

$$11. \quad \frac{5}{6} \div \frac{2}{3}$$

$$16. \quad \frac{2}{3} \div \frac{2}{9}$$

$$21. \quad \frac{2}{3} \div \frac{5}{11}$$

$$26. \quad \frac{5}{11} \div \frac{4}{5}$$

$$12. \quad \frac{3}{4} \div \frac{2}{5}$$

$$17. \quad \frac{3}{7} \div \frac{1}{5}$$

$$22. \quad \frac{4}{7} \div \frac{2}{3}$$

$$27. \quad \frac{3}{4} \div \frac{3}{10}$$

$$13. \quad \frac{3}{8} \div \frac{3}{4}$$

$$18. \quad \frac{3}{4} \div \frac{7}{9}$$

$$23. \quad \frac{4}{5} \div \frac{3}{4}$$

$$28. \quad \frac{3}{5} \div \frac{7}{11}$$

$$14. \quad \frac{4}{9} \div \frac{3}{5}$$

$$19. \quad \frac{3}{10} \div \frac{4}{7}$$

$$24. \quad \frac{4}{13} \div \frac{1}{3}$$

$$29. \quad \frac{5}{6} \div \frac{9}{14}$$

Name the quotients as mixed numerals. The first one is begun for you.

30. How many two-thirds are there in three-fourths?

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{a}{8} = b$$

31. How many one-eighths are there in seven-ninths?

32. How many two-sevenths are there in five-sixths?

33. How many three-fifths are there in nine-tenths?

34. How many two-fifths are there in three-fourths?

Both factors greater than one

Study these methods
for naming the product

$$4 \times 4\frac{2}{3}$$

A

$$\begin{array}{r} 4\frac{2}{3} = 4 + \frac{2}{3} \\ \times 4 \quad \quad \times 4 \\ \hline 16 + 2\frac{2}{3} = 18\frac{2}{3} \end{array}$$

B

$$\begin{array}{r} 4 \times 4\frac{2}{3} = 4 \times \frac{14}{3} \\ = \frac{56}{3} \\ = 18\frac{2}{3} \end{array}$$

EXERCISES

Use method A to name the products.

$$\begin{array}{r} 1. \quad 3\frac{1}{5} \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 3\frac{1}{6} \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 2\frac{3}{5} \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 5\frac{5}{6} \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 5\frac{3}{11} \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 9\frac{3}{10} \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 4\frac{3}{4} \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 4\frac{2}{7} \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 3\frac{4}{7} \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 4\frac{5}{8} \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 4\frac{2}{5} \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 2\frac{7}{8} \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 2\frac{2}{3} \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 1\frac{4}{5} \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 6\frac{5}{9} \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 11\frac{1}{2} \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 6\frac{6}{7} \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 5\frac{2}{3} \\ \times 7 \\ \hline \end{array}$$

Use method B to name the products.

$$19. \quad 3 \times 4\frac{1}{5}$$

$$23. \quad 7 \times 3\frac{5}{8}$$

$$27. \quad 9 \times 7\frac{3}{8}$$

$$31. \quad 5 \times 9\frac{3}{4}$$

$$20. \quad 2 \times 5\frac{3}{7}$$

$$24. \quad 6 \times 4\frac{5}{9}$$

$$28. \quad 5 \times 8\frac{1}{4}$$

$$32. \quad 9 \times 6\frac{2}{3}$$

$$21. \quad 5 \times 6\frac{3}{4}$$

$$25. \quad 8 \times 3\frac{4}{5}$$

$$29. \quad 10 \times 2\frac{7}{9}$$

$$33. \quad 5 \times 11\frac{5}{7}$$

$$22. \quad 4 \times 2\frac{2}{3}$$

$$26. \quad 2 \times 9\frac{2}{5}$$

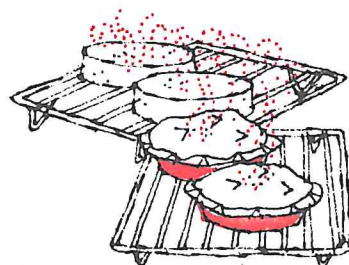
$$30. \quad 6 \times 7\frac{5}{6}$$

$$34. \quad 8 \times 9\frac{6}{7}$$

Solve the problems.

35. If a cake recipe takes $2\frac{2}{3}$ cups of flour, how many cups of flour would you need for 3 cakes?

36. If a pie recipe takes $1\frac{3}{4}$ cups of flour, how many cups of flour would you need for 5 pies?



Both factors greater than one

Study the equations.

$$\begin{aligned} 4\frac{2}{3} \times 1\frac{1}{4} &= \frac{14}{3} \times \frac{5}{4} \\ &= \frac{70}{12} \\ &= 5\frac{10}{12} = 5\frac{5}{6} \end{aligned}$$

$$\begin{aligned} 4\frac{2}{3} \div 1\frac{1}{4} &= \frac{14}{3} \div \frac{5}{4} \\ &= \frac{14}{3} \times \frac{4}{5} \\ &= \frac{56}{15} = 3\frac{11}{15} \end{aligned}$$

EXERCISES

Solve the equations to name the products.

$$\begin{aligned} 1. \quad 2\frac{3}{4} \times 1\frac{1}{3} &= \frac{a}{4} \times \frac{b}{3} \\ &= \frac{c}{12} \\ &= 3\frac{d}{12} = e \end{aligned}$$

$$\begin{aligned} 2. \quad 1\frac{2}{5} \times 3\frac{1}{2} &= \frac{f}{5} \times \frac{g}{2} \\ &= \frac{h}{10} \\ &= 4\frac{i}{10} = j \end{aligned}$$

Name the products.

$$3. \quad 2\frac{3}{4} \times 1\frac{1}{3}$$

$$6. \quad 2\frac{5}{6} \times 2\frac{1}{3}$$

$$9. \quad 6\frac{1}{2} \times 3\frac{1}{3}$$

$$12. \quad 7\frac{1}{2} \times 2\frac{3}{4}$$

$$4. \quad 5\frac{2}{3} \times 2\frac{1}{2}$$

$$7. \quad 1\frac{3}{8} \times 4\frac{1}{2}$$

$$10. \quad 4\frac{2}{3} \times 3\frac{1}{2}$$

$$13. \quad 2\frac{2}{5} \times 3\frac{1}{4}$$

$$5. \quad 3\frac{2}{5} \times 1\frac{2}{3}$$

$$8. \quad 2\frac{3}{5} \times 3\frac{2}{5}$$

$$11. \quad 2\frac{4}{5} \times 1\frac{4}{7}$$

$$14. \quad 5\frac{3}{4} \times 2\frac{5}{6}$$

Solve the equations to name the quotients.

$$\begin{aligned} 15. \quad 3\frac{2}{3} \div 2\frac{1}{2} &= \frac{a}{3} \div \frac{b}{2} \\ &= \frac{11}{3} \times c \\ &= \frac{d}{15} = 1\frac{e}{15} \end{aligned}$$

$$\begin{aligned} 16. \quad 2\frac{5}{6} \div 1\frac{1}{5} &= \frac{f}{6} \div \frac{g}{5} \\ &= \frac{17}{6} \times h \\ &= \frac{i}{36} = 2\frac{j}{36} \end{aligned}$$

Name the quotients.

$$17. \quad 2\frac{3}{4} \div 5\frac{1}{2}$$

$$19. \quad 2\frac{1}{2} \div 3\frac{2}{3}$$

$$21. \quad 3\frac{3}{5} \div 2\frac{2}{5}$$

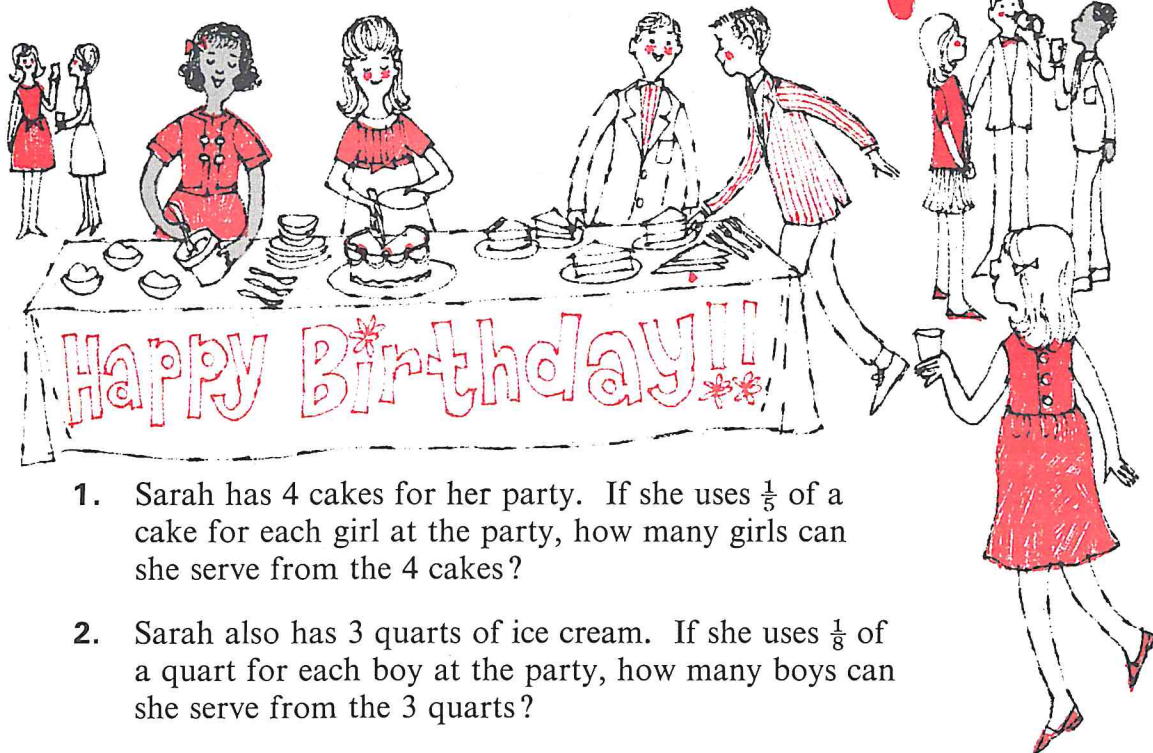
$$23. \quad 1\frac{3}{8} \div 5\frac{1}{2}$$

$$18. \quad 3\frac{1}{3} \div 1\frac{3}{4}$$

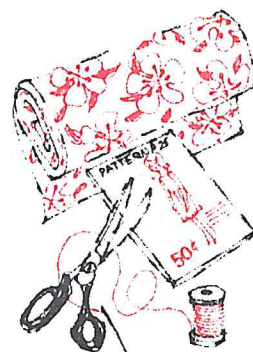
$$20. \quad 1\frac{2}{7} \div 4\frac{1}{3}$$

$$22. \quad 5\frac{3}{4} \div 3\frac{3}{4}$$

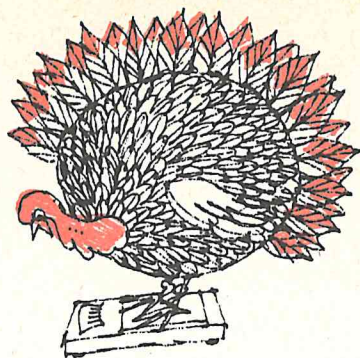
$$24. \quad 2\frac{7}{9} \div 2\frac{3}{4}$$



1. Sarah has 4 cakes for her party. If she uses $\frac{1}{5}$ of a cake for each girl at the party, how many girls can she serve from the 4 cakes?
2. Sarah also has 3 quarts of ice cream. If she uses $\frac{1}{8}$ of a quart for each boy at the party, how many boys can she serve from the 3 quarts?
- *3. How many pieces, each $\frac{1}{4}$ foot long, can Bob cut from a board $\frac{7}{8}$ feet long?
4. How many pieces of ribbon, each $\frac{1}{8}$ yard long, can Betty cut from a piece of ribbon $\frac{3}{4}$ yard long?
5. How many $\frac{1}{8}$ pint containers can Jerry fill from a container holding $1\frac{1}{2}$ pints?
6. If Sally needs $\frac{2}{5}$ of a yard for each doll's dress, how many dresses can she make from $1\frac{1}{5}$ yards?
- *7. Bill wants to know how many $\frac{2}{5}$ inches there are in $\frac{7}{10}$ inches. Can you help him solve this problem?
8. How many $\frac{3}{4}$ ounce cups can Lu-Ann fill from a bag containing 12 ounces of mints?
9. Jimmy has $\frac{3}{2}$ yards of screening. How many $\frac{3}{4}$ yard pieces can he cut from this screening?
10. Jennifer spends 4 hours in all practicing her music lessons. How many $\frac{1}{2}$ hour practices must she have for these lessons?



Interesting number facts about birds



The ostrich is considered the largest bird in the world. Some ostriches found in Africa weigh as much as 290 lbs. and stand $7\frac{3}{4}$ feet high. Some can run at a speed of more than 30 m.p.h.

Here are some more interesting number facts about birds.

1. Some South American albatrosses have wingspans that measure $11\frac{1}{3}$ feet.
2. Some pigeons have been recorded to fly as fast as $91\frac{1}{2}$ m.p.h.
3. The ostrich egg is considered the largest in the world. Some have been found that are $6\frac{1}{2}$ inches long and have a diameter of $4\frac{3}{4}$ inches.
4. The smallest birds in the world are hummingbirds. One was found that was only $2\frac{1}{2}$ inches long and weighed $\frac{1}{13}$ oz.
5. The smallest eggs found are laid by the Jamaican hummingbird. They are about the size of a pea with a total length of about $\frac{1}{8}$ inch.
6. The largest hummingbird is $8\frac{1}{2}$ inches long and weighs $\frac{2}{3}$ of an ounce.
7. The largest turkey ever weighed was recorded to be $64\frac{3}{4}$ lbs.
8. Probably the fastest moving living creature is the spine-tailed swift which has flown at speeds ranging between $171\frac{4}{5}$ m.p.h. and $219\frac{1}{2}$ m.p.h.

CHECKING UP

Name the products in simplest form.

1. $\frac{1}{3} \times 4$

3. $\frac{3}{4} \times 12$

5. $\frac{5}{9} \times 6$

7. $\frac{2}{3} \times 9$

2. $\frac{3}{5} \times 10$

4. $\frac{9}{10} \times 10$

6. $\frac{2}{7} \times 8$

8. $\frac{1}{9} \times 8$

Name the products in simplest form.

9. $\frac{1}{2} \times \frac{1}{2}$

11. $\frac{3}{7} \times \frac{4}{5}$

13. $\frac{7}{10} \times \frac{5}{6}$

15. $\frac{2}{5} \times \frac{4}{5}$

10. $\frac{2}{3} \times \frac{3}{4}$

12. $\frac{4}{9} \times \frac{5}{8}$

14. $\frac{2}{11} \times \frac{3}{7}$

16. $\frac{5}{12} \times \frac{8}{9}$

Name the quotients in simplest form.

17. $6 \div \frac{1}{6}$

19. $10 \div \frac{3}{4}$

21. $4 \div \frac{7}{8}$

23. $9 \div \frac{2}{5}$

18. $8 \div \frac{2}{3}$

20. $12 \div \frac{1}{2}$

22. $3 \div \frac{5}{9}$

24. $1 \div \frac{3}{11}$

Name the quotients in simplest form.

25. $\frac{1}{2} \div \frac{2}{3}$

28. $\frac{4}{5} \div \frac{2}{9}$

31. $\frac{7}{10} \div \frac{5}{6}$

34. $\frac{14}{19} \div \frac{5}{7}$

26. $\frac{3}{4} \div \frac{1}{4}$

29. $\frac{7}{8} \div \frac{1}{3}$

32. $\frac{5}{17} \div \frac{2}{3}$

35. $\frac{2}{3} \div \frac{5}{9}$

27. $\frac{6}{7} \div \frac{5}{8}$

30. $\frac{4}{11} \div \frac{11}{13}$

33. $\frac{4}{9} \div \frac{3}{4}$

36. $\frac{9}{20} \div \frac{7}{10}$

Solve the problems.

37. Two-fifths of the girls in the sixth grade class have brown hair. If $\frac{3}{4}$ of these girls also have blue eyes, what part of the girls have blue eyes and brown hair?

38. One-fourth of the boys in the sixth grade class are 5 feet tall. If $\frac{2}{3}$ of them are also 12 years of age, what part of the boys are 5 feet tall and 12 years old?

39. Sally bought $\frac{3}{4}$ yard of material to make aprons. If each apron requires $\frac{1}{3}$ yard, how many aprons can she make?



To name the product of a whole number and a fractional number, multiply the whole number by the numerator, then divide this product by the denominator.

Name the products.

1. $8 \times \frac{3}{4}$ 2. $16 \times \frac{7}{8}$ 3. $\frac{5}{6} \times 18$ 4. $\frac{2}{3} \times 12$ 5. $\frac{3}{10} \times 20$

To name the product of 2 fractional numbers, multiply the numerators and the denominators.

Name the products.

6. $\frac{3}{7} \times \frac{2}{3}$ 7. $\frac{4}{13} \times \frac{2}{4}$ 8. $\frac{4}{5} \times \frac{4}{7}$ 9. $\frac{3}{8} \times \frac{3}{4}$ 10. $\frac{3}{4} \times \frac{3}{4}$

Dividing by a fractional number of the form $\frac{1}{n}$ ($n \neq 0$) is the same as multiplying by n . For example, $4 \div \frac{1}{2} = 4 \times 2 = 8$.

Name the quotients.

11. $2 \div \frac{1}{5}$ 12. $10 \div \frac{1}{5}$ 13. $3 \div \frac{1}{7}$ 14. $8 \div \frac{1}{10}$ 15. $7 \div \frac{1}{11}$

To name the quotient $\frac{2}{3} \div \frac{3}{5}$, change the division to multiplication and $\frac{3}{5}$ to $\frac{5}{3}$. In general, $\frac{a}{b} \div \frac{m}{n} = \frac{a}{b} \times \frac{n}{m}$. (b and $n \neq 0$)

Name the quotients.

16. $\frac{5}{6} \div \frac{2}{3}$ 18. $\frac{4}{7} \div \frac{2}{3}$ 20. $\frac{5}{9} \div \frac{7}{8}$ 22. $\frac{3}{11} \div \frac{1}{2}$ 24. $\frac{6}{9} \div \frac{6}{7}$
 17. $\frac{3}{8} \div \frac{3}{4}$ 19. $\frac{3}{7} \div \frac{1}{5}$ 21. $\frac{4}{9} \div \frac{3}{5}$ 23. $\frac{3}{4} \div \frac{2}{5}$ 25. $\frac{3}{4} \div \frac{7}{9}$



LOOKING WAY BACK

Complete the number charts.

1.

exponent numeral	decimal numeral
3^2	<i>a</i>
2^3	<i>b</i>
5^c	25
4^2	<i>d</i>
2^e	32

2.

exponent numeral	decimal numeral
10^1	<i>f</i>
10^2	<i>g</i>
10^h	1000
10^4	<i>i</i>
10^j	100,000

3.

base 10 numeral	base 5 numeral
3	<i>k</i>
10	<i>l</i>
<i>m</i>	11
<i>n</i>	23
15	<i>o</i>

4.

base 10 numeral	base 2 numeral
3	<i>p</i>
5	<i>q</i>
<i>r</i>	10
<i>s</i>	101
9	<i>t</i>

Complete the number charts.

5.

+	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{2}$	1	<i>a</i>	<i>b</i>	<i>c</i>
$\frac{2}{3}$	<i>d</i>	1	<i>e</i>	<i>f</i>
$\frac{3}{4}$	<i>g</i>	<i>h</i>	1	<i>i</i>
$\frac{2}{5}$	<i>j</i>	<i>k</i>	<i>l</i>	$\frac{3}{5}$

6.

-	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{5}$
$\frac{6}{7}$	$\frac{5}{14}$	<i>m</i>	<i>n</i>	<i>o</i>
$\frac{7}{8}$	<i>p</i>	$\frac{5}{24}$	<i>q</i>	<i>r</i>
$\frac{8}{9}$	<i>s</i>	<i>t</i>	$\frac{5}{36}$	<i>u</i>
$\frac{9}{10}$	<i>v</i>	<i>w</i>	<i>x</i>	$\frac{5}{10}$

Name the sums or differences.

7.
$$\begin{array}{r} 5487632 \\ + 9618037 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 9417638 \\ - 259745 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 3596842 \\ + 1171659 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 4501347 \\ - 2693072 \\ \hline \end{array}$$

Study this method for renaming the product $\frac{2}{3} \times \frac{4}{5}$.

$$\begin{aligned}\frac{2}{3} \times \frac{4}{5} &= \left(1 - \frac{1}{3}\right) \times \frac{4}{5} \\ &= \left(1 \times \frac{4}{5}\right) - \left(\frac{1}{3} \times \frac{4}{5}\right) \\ &= \frac{4}{5} - \frac{4}{15} \\ &= \frac{12}{15} - \frac{4}{15} \\ &= \frac{8}{15}\end{aligned}$$

Step 1: Rename $\frac{2}{3}$ as $1 - \frac{1}{3}$

Step 2: Use the Distributive Property.

Step 3: $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.

Step 4: Rename $\frac{4}{5}$ as $\frac{12}{15}$.

Step 5: Name the difference.

EXERCISES

Solve the equations.

$$\begin{aligned}1. \quad \frac{3}{4} \times \frac{2}{3} &= (1 - a) \times \frac{2}{3} \\ &= \left(b - \frac{1}{4}\right) \times \frac{2}{3} \\ &= \frac{2}{3} - c \\ &= d - \frac{2}{12} \\ &= e\end{aligned}$$

$$\begin{aligned}3. \quad \frac{5}{6} \times \frac{1}{2} &= (1 - k) \times \frac{1}{2} \\ &= \left(l - \frac{1}{6}\right) \times \frac{1}{2} \\ &= \frac{1}{2} - m \\ &= n - \frac{1}{12} \\ &= o\end{aligned}$$

$$\begin{aligned}2. \quad \frac{4}{5} \times \frac{3}{4} &= (1 - g) \times \frac{3}{4} \\ &= \left(h - \frac{1}{5}\right) \times \frac{3}{4} \\ &= \frac{3}{4} - i \\ &= j - \frac{3}{20} \\ &= k\end{aligned}$$

$$\begin{aligned}4. \quad \frac{6}{7} \times \frac{2}{3} &= (1 - p) \times \frac{2}{3} \\ &= \left(q - \frac{1}{7}\right) \times \frac{2}{3} \\ &= \frac{2}{3} - r \\ &= s - \frac{2}{21} \\ &= t\end{aligned}$$

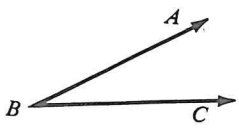
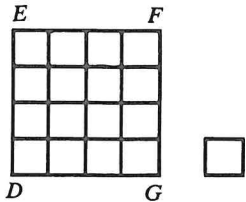
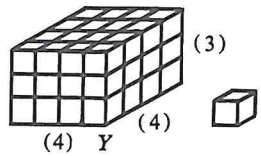
Use the method above to name the products.

$$5. \quad \frac{5}{6} \times \frac{3}{4} = a \qquad 6. \quad \frac{3}{4} \times \frac{1}{2} = b \qquad 7. \quad \frac{7}{8} \times \frac{2}{3} = c \qquad 8. \quad \frac{6}{7} \times \frac{5}{8} = d$$

CHAPTER 9 • Geometry and Graphing

Measurement

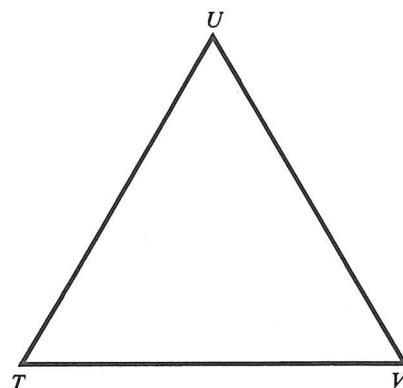
In measurement, we compare a **unit** with the object being measured. The **measure** tells *how many units* match the object.

object	unit	statement
line segments 	length: inch, foot, yard, mile, etc.	$m(\overline{AB}) = 4$ The length of $\overline{AB}$ is 4 units.
angles 	angle: degree	$m(\angle ABC) = 25$ The size of angle ABC is 25°
regions 	square: square inch, square foot, square yard, square mile, etc.	$m(DEFG) = 16$ The area of the square region $DEFG$ is 16 square units.
space 	cube: cubic inch, cubic foot, cubic yard, cubic mile, etc.	$m(Y) = 48$ The volume in the rectangular prism Y is 48 cubic units.

EXERCISES

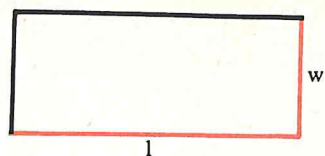
Name the measures for triangle TUV in inches or degrees.

- $m(\overline{UV}) = a$
- $m(\overline{TU}) = b$
- $m(\overline{VT}) = c$
- $m(\angle UVT) = d$
- $m(\angle TUV) = e$
- $m(\angle VTU) = f$

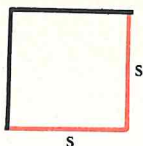


Write equations for the measures of the regions.

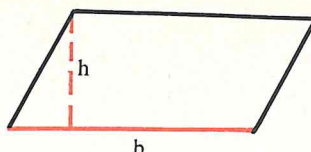
Let A name the *number* of units of area.



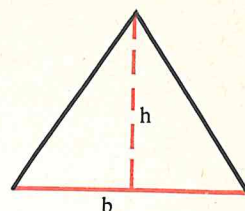
7. rectangle



8. square



9. parallelogram



10. triangle

Name the area of the regions formed by

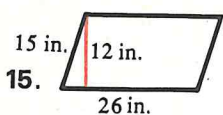
11. a rectangle with sides of 12'' and 17''.

12. a square with sides of 23 yards.

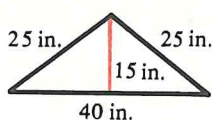
13. a parallelogram with a height of 14' and a base of 19'.

14. a triangle with a height of 24'' and a base of 33''.

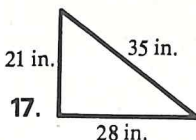
Use the measures shown to name the areas of the regions.



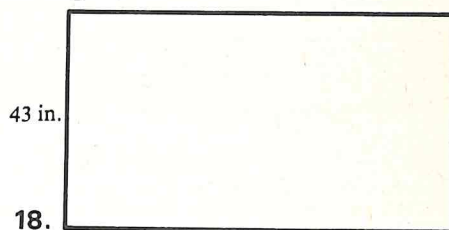
15.



16.



17.



18.

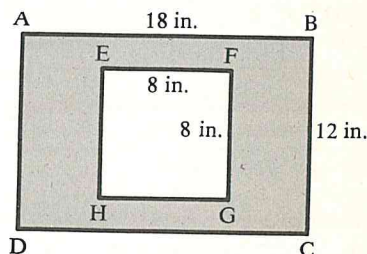
19–22. Name the perimeter of each polygon in exercises 15 to 18.

Use the figure on the right to help you

23. name the area of the rectangular region $ABCD$.

24. name the area of the square region $EFGH$.

25. name the area of the shaded region.



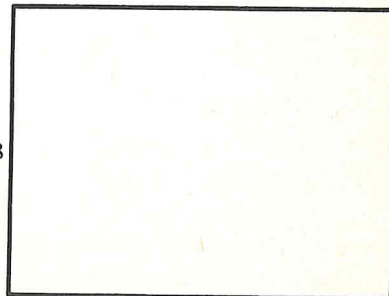
Mr. Wilson's backyard is shaped like a rectangle. It is 64' long and 48' wide.

26. Name the perimeter of Mr. Wilson's backyard.

27. Name the area of Mr. Wilson's backyard.

28. A box of grass seed costs 79¢ and will cover 512 square feet. How many boxes of grass seed will Mr. Wilson need to cover his backyard?

29. How much will Mr. Wilson pay for his grass seed?



64

Measuring space regions

For a rectangular prism let

l = measure of the length

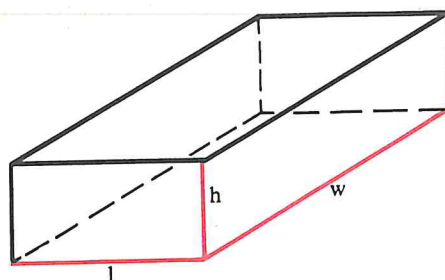
w = measure of the width

h = measure of the height

V = measure of the volume.

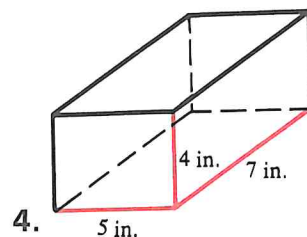
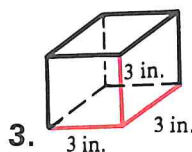
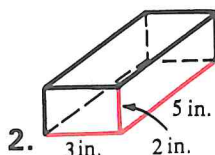
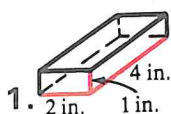
Then $V = l \times w \times h$.

The **volume** is $(l \times w \times h)$ units.

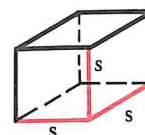


EXERCISES

Name the volume in each rectangular prism.



5. If s is the measure of the edge of a cube, then the equation for naming the volume in a cube is $V = \underline{\hspace{1cm}}?$.



Name the volume in

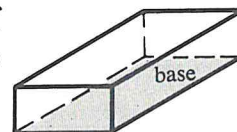
6. a cube with sides of length 12".

7. a cube with sides of length 36".

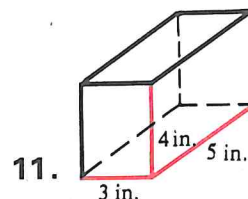
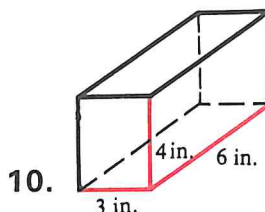
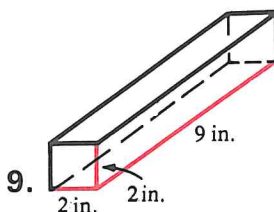
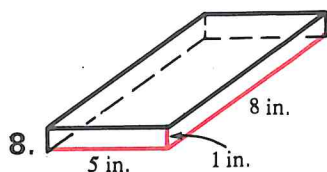
The base of a rectangular prism is a rectangular region with length of measure l and width of measure w . The measure of the area of the base is: $B = l \times w$. The measure of the volume is:

$$V = (\text{measure of the base}) \times (\text{measure of the height}) \text{ or}$$

$$V = B \times h.$$

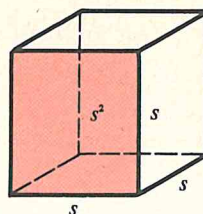


Name the area of the base of each prism, then name the volume.



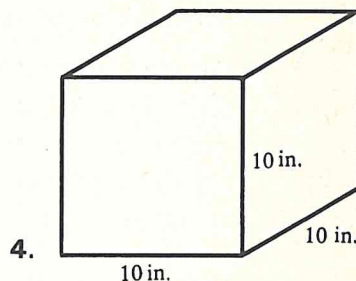
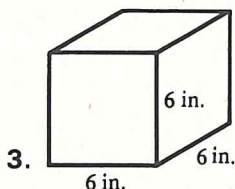
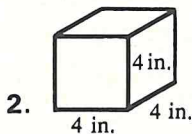
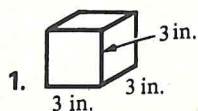
Area and closed surfaces

A **cube** is a simple closed surface formed by the union of 6 square regions. If s is the measure of the edge of a face, then the measure of the area of the face is s^2 . The **surface area** of a cube is $(6 \times s^2)$ square units.



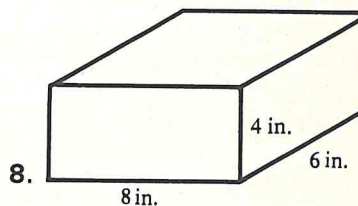
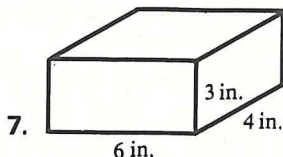
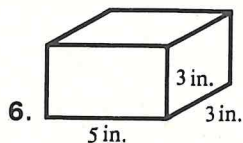
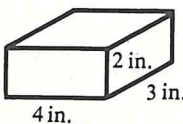
EXERCISES

Name the surface areas of the cubes.



If a rectangular prism has edges of measure l , w , and h , then there are 2 faces whose area is $(l \times w)$ units; 2 faces whose area is $(l \times h)$ units; and 2 faces whose area is $(w \times h)$ units. The measure of the surface area of a rectangular prism is named by $S = (2 \times l \times w) + (2 \times l \times h) + (2 \times w \times h)$.

Name the surface areas of the rectangular prisms.



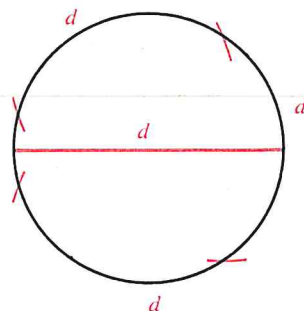
Name the volume in

9–12. the cubes in exercises 1–4.

13–16. the rectangular prisms in exercises 5–8.

More about circles

The distance around a circle is called its **circumference**. The circumference is a little longer than 3 diameters. The circumference of any circle divided by the diameter of the circle is *about* $\frac{22}{7}$ or $3\frac{1}{7}$. The name given to this number is the Greek letter pi (pronounced pie, and written π).



EXERCISES

To measure the circumference of a circle with radius 1"

Cut out a circular piece of paper with radius 1". Mark a starting point on the circle and on a line segment. Then roll the region along the line segment until the starting point touches the segment again. Measure the distance between the two points. This distance is *the circumference* of this circle.

Repeat the steps above with radii of 2" and 3", then copy and complete the table.

	radius	diameter(d)	circumference(C)	ratio of circumference to the diameter(C/d)
1.	1"	a	c	e
2.	2"	m	n	r
3.	3"	s	u	v

Use the equation $C = \pi \times d$ where d is the diameter and

$\pi \approx \frac{22}{7}$ ($\approx$ means "is approximately equal to") to

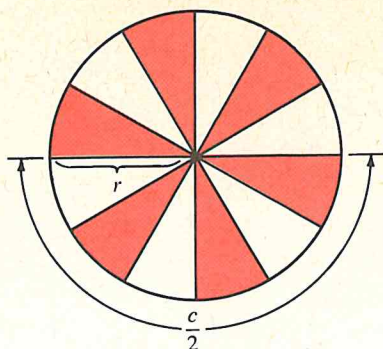
estimate the circumference of a circle with a diameter of

4. 7" 5. 14" 6. 21" 7. 28" 8. 4" 9. $3\frac{1}{2}$ " 10. 8"

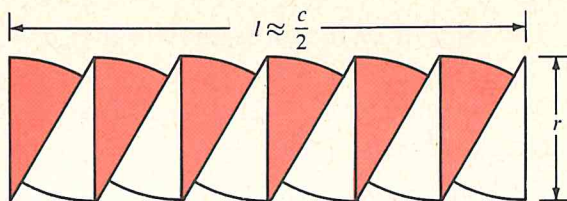
Since the diameter of a circle is twice the radius, the equation $C = \pi \times (2 \times r) = 2 \times \pi \times r$ will also name the circumference of a circle.

Estimate the circumference of a circle with radius of

11. 2" 12. 7" 13. 10" 14. 21" 15. 12" 16. 49" 17. 32"



Areas of circular regions



Separate a circular piece of paper into 12 congruent parts and arrange the pieces as in the figure above. The result is fairly close to a rectangular region with length half the circumference and a width equal to a radius. By naming the area of this region we can name the area of a circular region.

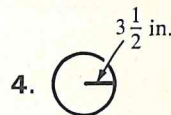
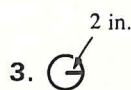
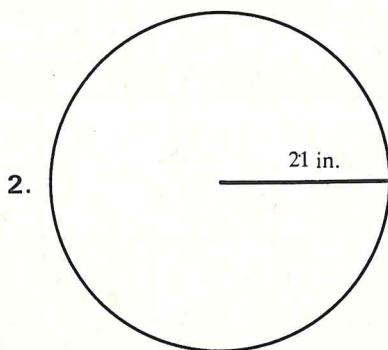
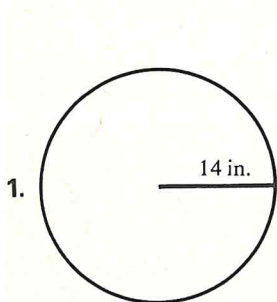
$$\begin{aligned}\text{The measure of the region} &= \left(\frac{1}{2} \times C\right) \times r \\ &= \frac{1}{2} \times (2 \times \pi \times r) \times r \\ &= \pi \times r \times r = \pi \times r^2.\end{aligned}$$

So the **area** of the circular region is $\pi \times r^2$ units.

EXERCISES

Use the equation $A = \pi \times r^2$ to estimate the areas of the circular regions. For example, if $r = 7''$,

$A = \pi \times 7^2 \approx \frac{22}{7} \times 49 \approx 154$. The area of the circular region is *about 154 square inches*.



Estimate the areas in circles with the following radii ($\pi \approx \frac{22}{7}$).

5. $r = 3''$

6. $r = 4''$

7. $r = 10''$

8. $r = 18''$

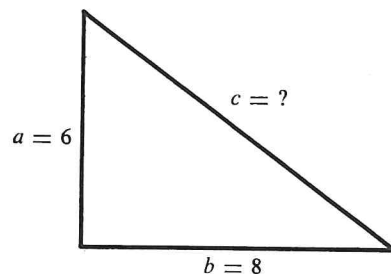
Theorem of Pythagoras

The **Theorem of Pythagoras** says that if a , b , and c are the measures of the sides of a *right triangle* and c is the measure of the side opposite the right angle, then $c^2 = a^2 + b^2$.

In the figure on the right,

$$c^2 = a^2 + b^2 = 6^2 + 8^2 = 100.$$

If $c^2 = 100$, then $c = \sqrt{100}$ or 10, so the length of $\overline{AB}$ is 10 units.



EXERCISES

Use the equation $c^2 = a^2 + b^2$ to see if the numbers in each of the sets can be the measures of the sides of a right triangle.

1. $\{5, 12, 13\}$
2. $\{8, 9, 12\}$
3. $\{11, 60, 61\}$
4. $\left\{\frac{1}{2}, \frac{2}{3}, \frac{5}{6}\right\}$

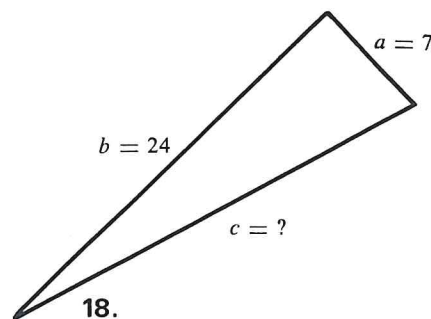
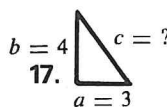
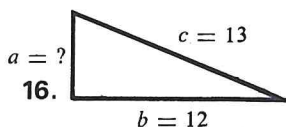
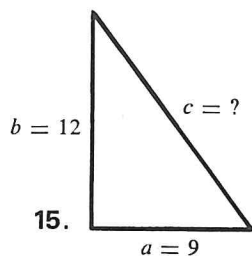
Complete the sentences.

5. $4^2 = 16$, so $\sqrt{16} = a$
6. $6^2 = 36$, so $\sqrt{36} = b$
7. $14^2 = 196$, so $\sqrt{196} = c$
8. $32^2 = 1024$, so $\sqrt{1024} = d$

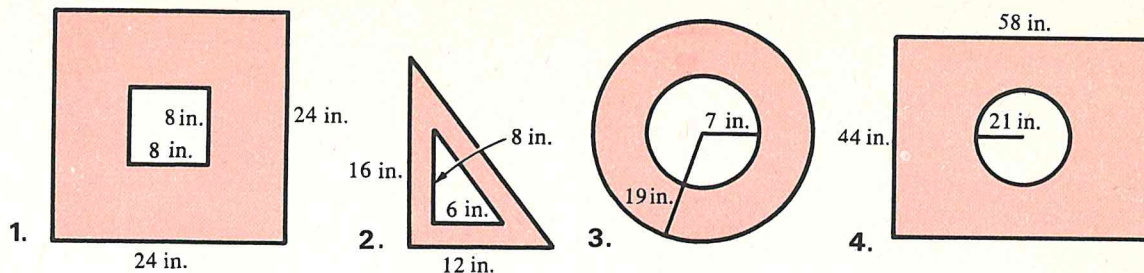
Name the square roots. You may need to review the averaging method shown on page 124.

9. $\sqrt{81}$
10. $\sqrt{225}$
11. $\sqrt{400}$
12. $\sqrt{169}$
13. $\sqrt{784}$
14. $\sqrt{1296}$

Use the Theorem of Pythagoras to name the missing measures of the right triangles.

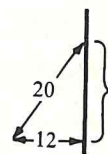
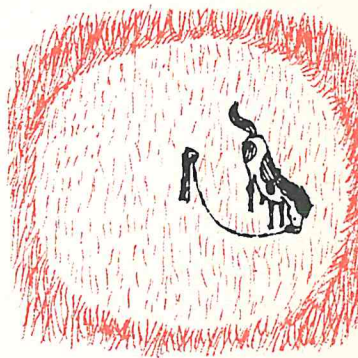


Name the area of each shaded region ($\pi \approx \frac{22}{7}$).

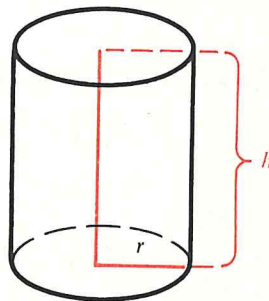


Solve the problems.

5. A refrigerator which holds 18 cubic feet sells for \$432. What is the cost per cubic foot?
6. A horse is tied to a stake by a rope 35' long.
 - a. Over what kind of region can the horse graze?
 - b. Name the area of the region over which he can graze.
 - c. If he walks all the way around the region with the rope taut, how far will he go?
7. Which has the greater surface area and volume, a cube 8'' on each side or a rectangular prism with length 12'', width 9'' and height 4''?
8. The top of a 20' ladder is placed against the bottom of a window on the side of a building. If the foot of the ladder is 12' from the building, how high is the window from the ground?



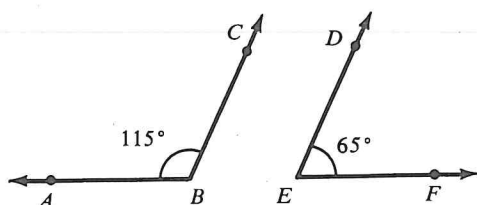
The volume in a circular cylinder may be named from the equation $V = (\pi \times r^2) \times h$, where $\pi \times r^2$ is the measure of the area of the base and h is the measure of the height. The volume is $(\pi \times r^2) \times h$ units.



Name the volume in each cylinder if

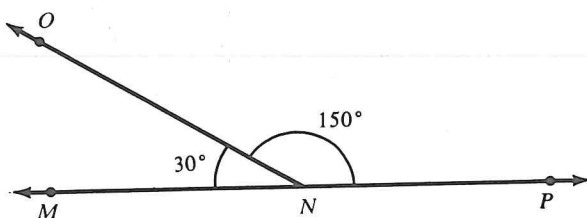
9. $r = 7''$, $h = 10''$ 10. $r = 5''$, $h = 14''$ 11. $r = 8''$, $h = 21''$

Linear pairs



$$m(\angle ABC) + m(\angle DEF) = 180$$

If the sum of the measures of 2 angles is 180, the angles form a **linear pair**.



$$m(\angle MNO) + m(\angle ONP) = 180$$

Angles ABC and DEF , and $\angle MNO$ and $\angle ONP$ form linear pairs.

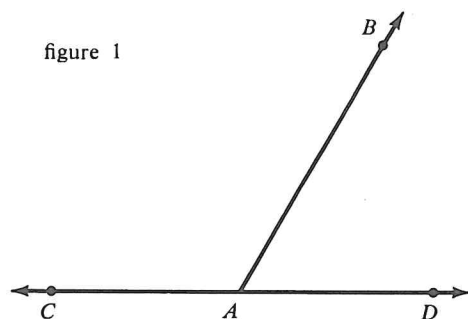
EXERCISES

Use figure 1 to solve these problems.

1. $m(\angle CAB) = a$ 2. $m(\angle BAD) = b$

3. Do angles CAB and BAD form a linear pair? If 2 angles have a common ray and the other rays of the angles form a line, will the angles always form a linear pair? Try some other examples.

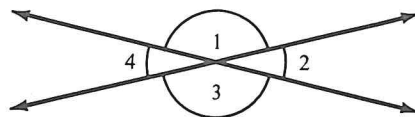
figure 1



In figure 2, name these measures in degrees.

4. $m(\text{angle } 1)$ 6. $m(\text{angle } 3)$
5. $m(\text{angle } 2)$ 7. $m(\text{angle } 4)$

figure 2



Use figure 2 to answer these questions.

8. Name the congruent angles.
9. Name 2 angles that form linear pairs with angle 1.
10. Name 2 angles that form linear pairs with angle 2.
11. Do any 2 intersecting lines form 2 pairs of congruent angles and 2 linear pairs? Try some other examples.
12. If the angles in a linear pair are congruent, how many degrees are there in each angle?

In figure 1, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

Use your protractor to name the measures in degrees.

1. $m(\text{angle } 1)$
2. $m(\text{angle } 4)$
3. $m(\text{angle } 5)$
4. $m(\text{angle } 8)$
5. $m(\text{angle } 2)$
6. $m(\text{angle } 3)$
7. $m(\text{angle } 6)$
8. $m(\text{angle } 7)$
9. Name the pairs of congruent angles.

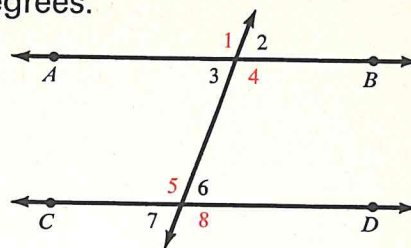


figure 1

10. Name the angles which form a linear pair with angle 1.
11. Name the angles which form a linear pair with angle 2.
12. If 2 parallel lines are intersected by a third line, do you think that there will always be 2 sets of 4 congruent angles and 2 sets of 4 linear pairs? Try some other examples.

In figure 2, $\overleftrightarrow{RD}$ is a line, and $\overrightarrow{OT}$ and $\overrightarrow{ON}$ are rays. Name these measures in degrees.

13. $m(\angle ROT)$
14. $m(\angle TON)$
15. $m(\angle NOD)$
16. $m(\angle TOD)$
17. $m(\angle RON)$
18. $m(\angle ROT) + m(\angle TOD)$
19. $m(\angle ROT) + m(\angle TON) + m(\angle NOD)$

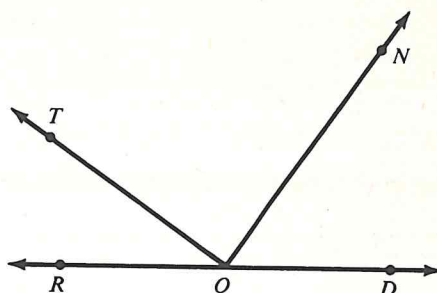


figure 2

In figure 3, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ is parallel to $\overleftrightarrow{GH}$.

20. Are angles 1, 4, 5, 8, 13, 16, 9, and 12 all congruent?
21. Are angles 2, 3, 6, 7, 10, 11, 14, and 15 all congruent?
22. Name 12 linear pairs. For example, write (1, 2), (1, 3) and so on.

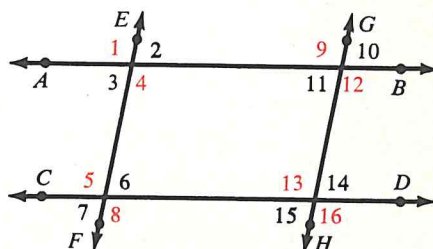
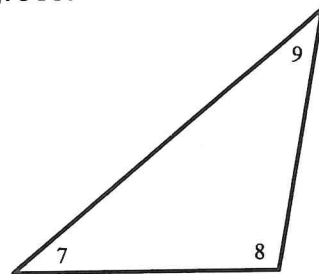
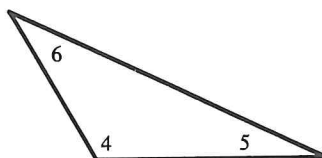
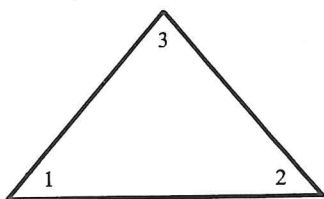


figure 3

● Angles of a triangle

Use your protractor to name the angle measures in degrees.



1. $m(\text{angle } 1) = a$

2. $m(\text{angle } 2) = b$

3. $m(\text{angle } 3) = c$

4. $\frac{\text{sum}}{\text{sum}} = d$

5. $m(\text{angle } 4) = e$

6. $m(\text{angle } 5) = f$

7. $m(\text{angle } 6) = g$

8. $\frac{\text{sum}}{\text{sum}} = h$

9. $m(\text{angle } 7) = i$

10. $m(\text{angle } 8) = j$

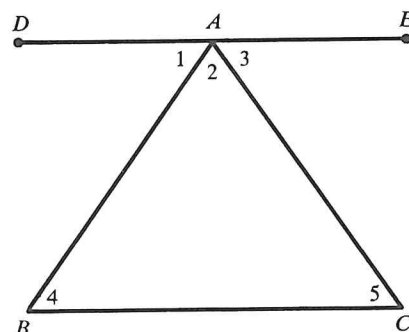
11. $m(\text{angle } 9) = k$

12. $\frac{\text{sum}}{\text{sum}} = l$

In the figure on the right, $\overline{DE}$ is parallel to $\overline{BC}$. Use a protractor to measure the angles, then answer the questions.

13. Is angle 1 congruent to angle 4?

14. Name an angle congruent to angle 5.



For the figure on the right, name the sum of these measures, in degrees.

15. $m(\text{angle } 1) + m(\text{angle } 2) + m(\text{angle } 3) = a$

16. $m(\text{angle } 4) + m(\text{angle } 2) + m(\text{angle } 5) = b$

Complete the sentence.

17. The sum of the measures of the 3 angles of a triangle, in degrees, is always ____?

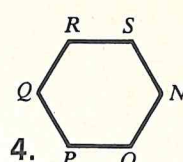
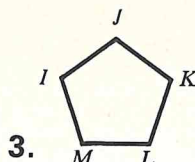
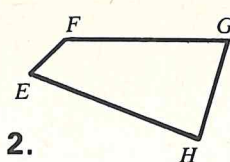
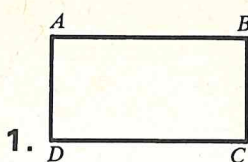
Answer the questions.

18. Can a triangle have 2 right angles? Why not?

19. A diagonal of a square separates the square into 2 triangles. What is the total number of degrees in the angles of the square?

Angles of polygons

List the sizes of the angles in each polygon.



Name the sum of the measures of the angles of

5. rectangle $ABCD$

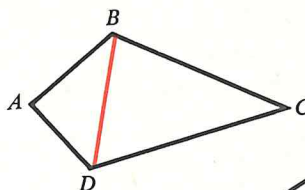
7. pentagon $IJKLM$

6. quadrilateral $EFGH$

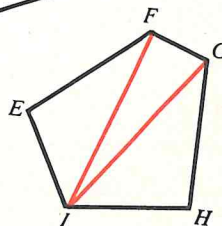
8. hexagon $NOPQRS$

Study the polygons on the right, then complete the sentences.

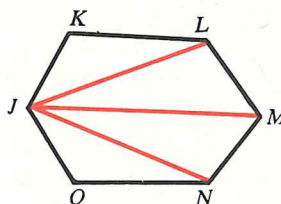
9. Diagonal $\overline{DB}$ separates quadrilateral $ABCD$ into ? triangles. The sum of the measures of the angles of $ABCD$ is ?.



10. Diagonals $\overline{IF}$ and $\overline{IG}$ separate pentagon $EFGHI$ into ? triangles. The sum of the measures of the angles of $EFGHI$ is ?.



11. Diagonals $\overline{JL}$, $\overline{JM}$, and $\overline{JN}$ separate hexagon $JKLMNO$ into ? triangles. The sum of the measures of the angles of $JKLMNO$ is ?.



Complete the table.

	polygon	no. of sides of polygon	no. of sides of polygon minus 2	total no. of degrees in angles of polygon
12.	triangle	3	1	180
13.	quadrilateral	a	e	c
14.	pentagon	n	r	s
15.	hexagon	u	v	z

Name the sum of the measures of the angles of

16. 7-sided polygons

18. 10-sided polygons

20. 15-sided polygons

17. 8-sided polygons

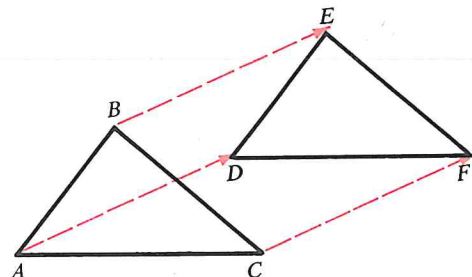
19. 12-sided polygons

21. 20-sided polygons



Congruent triangles

Remember that 2 triangles are **congruent** if they are the *same size and shape*. Triangle ABC is congruent to triangle DEF .



If 2 triangles are congruent, we can match congruent sides and angles.

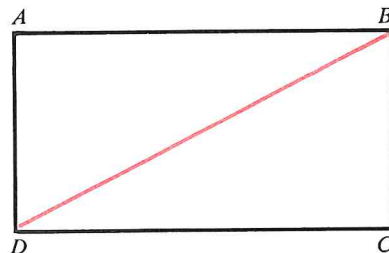
EXERCISES

For triangle DEF above, name the

1. side congruent to $\overline{AC}$
2. side congruent to $\overline{BC}$
3. side congruent to $\overline{BA}$
4. angle congruent to $\angle ACB$
5. angle congruent to $\angle BAC$
6. angle congruent to $\angle ABC$

For triangles ABD and CDB ,

7. name the measure of each side and pair the congruent sides.
8. name the measure of each angle and pair the congruent angles.
9. are triangles ABD and CDB congruent?

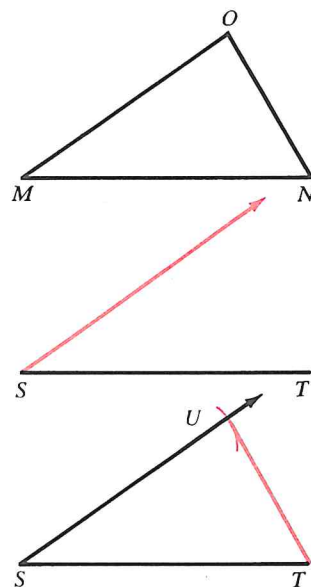


Follow the steps for drawing triangle STU congruent to triangle MON .

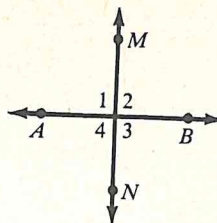
Step 1: Draw a line segment ST congruent to $\overline{MN}$. At S draw an angle congruent to $\angle OMN$.

Step 2: Use a compass to mark an arc of a circle with radius equal to the length of $\overline{MO}$, along the ray of your angle. This gives the point U . Draw $\overline{TU}$.

10. The above method matches 2 sides and the angle formed by these sides of MNO with 2 sides and the angle formed by these sides of STU . Check to see if the other matching sides and angles are congruent.

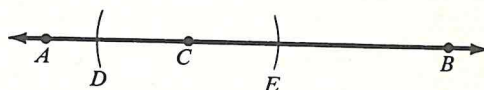
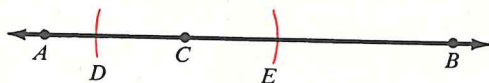


Two lines are **perpendicular** if they intersect to form right angles. In the figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{MN}$ are perpendicular. Angles 1, 2, 3 and 4 are right angles.



EXERCISES

Study the steps to draw a perpendicular to $\overleftrightarrow{AB}$ at point C .



Step 1:

With center C draw 2 arcs intersecting $\overleftrightarrow{AB}$ at D and E .

Step 2:

With D as center, draw an arc above $\overleftrightarrow{AB}$.



Step 3:

With center E using the same radius as in step 2, draw an arc intersecting the first arc in F .

Step 4:

Join F to C .

$\overleftrightarrow{FC}$ is perpendicular to $\overleftrightarrow{AB}$.

Use the steps above to help you.

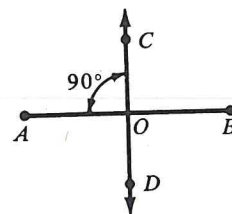
1. Draw a line segment 5'' long. Pick any point on the segment (not an endpoint) and draw a perpendicular to the line segment at this point.
2. Draw a right angle without using a protractor.
3. Draw a square with sides of length 3''.
4. Draw a right triangle with sides of length 3'', 4'', and 5''.

Study the figure at the top of the page.

5. Name the sum of angles 1, 2, 3, and 4.

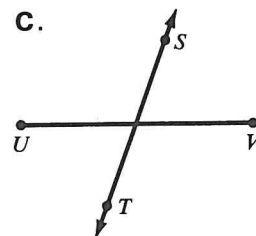
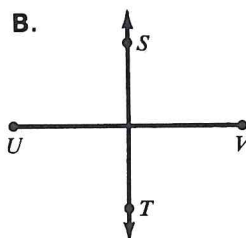
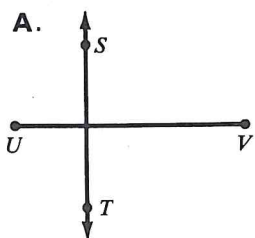
Separating into congruent parts

If a line separates a line segment into 2 congruent parts, it is called a **bisector**. $\overleftrightarrow{CD}$ is a bisector of $\overline{AB}$. $\overleftrightarrow{CD}$ is also perpendicular to $\overline{AB}$, so it is called the **perpendicular bisector** of $\overline{AB}$.



$\overline{AO}$ is congruent to $\overline{OB}$.

EXERCISES



In which of the diagrams above is $\overleftrightarrow{ST}$

1. a bisector of $\overline{UV}$?
2. perpendicular to $\overline{UV}$?
3. the perpendicular bisector of $\overline{UV}$?

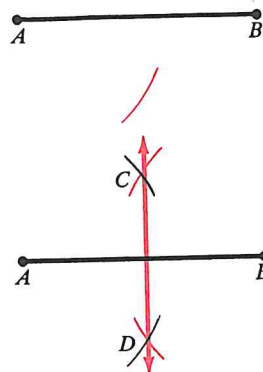
Follow these steps for drawing the perpendicular bisector of a line segment.

Step 1: Draw $\overline{AB}$.

Step 2: With A as center and radius greater than $\frac{1}{2} m(\overline{AB})$, mark arcs above and below $\overline{AB}$.

Step 3: With B as center and using the same radius as *Step 2*, mark arcs above and below $\overline{AB}$, intersecting the first arcs in points C and D .

Step 4: Connect points C and D .
 $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.



Draw a triangle and then draw the perpendicular bisectors of the three sides.

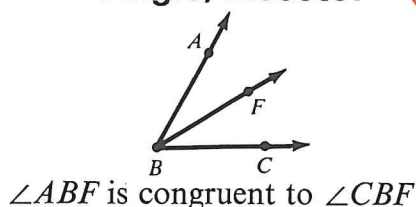
4. What do you notice about the perpendicular bisectors?

Angle, bisector



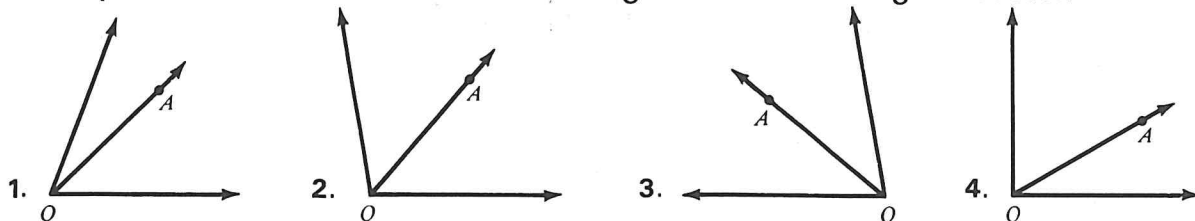
If a ray separates an angle into 2 congruent angles, it is called the **bisector** of the angle.

$\overrightarrow{BF}$ is the bisector of $\angle ABC$.



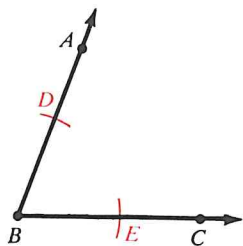
EXERCISES

Use a protractor to determine in which figures $\overrightarrow{OA}$ is an angle bisector.



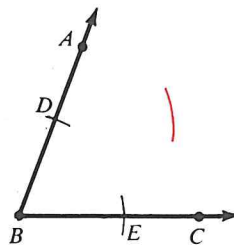
Follow these steps for drawing an angle bisector.

Step 1:



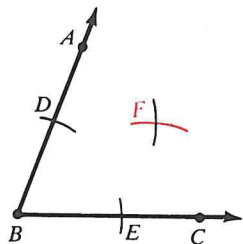
Given angle ABC , with B as center and any radius draw arcs intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in points D and E .

Step 2:



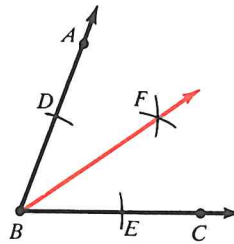
With D as center and radius greater than $\frac{1}{2} m(\overline{DE})$, draw an arc of a circle between rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

Step 3:



With E as center and the same radius used in step 2, draw an arc intersecting the first arc in F .

Step 4:

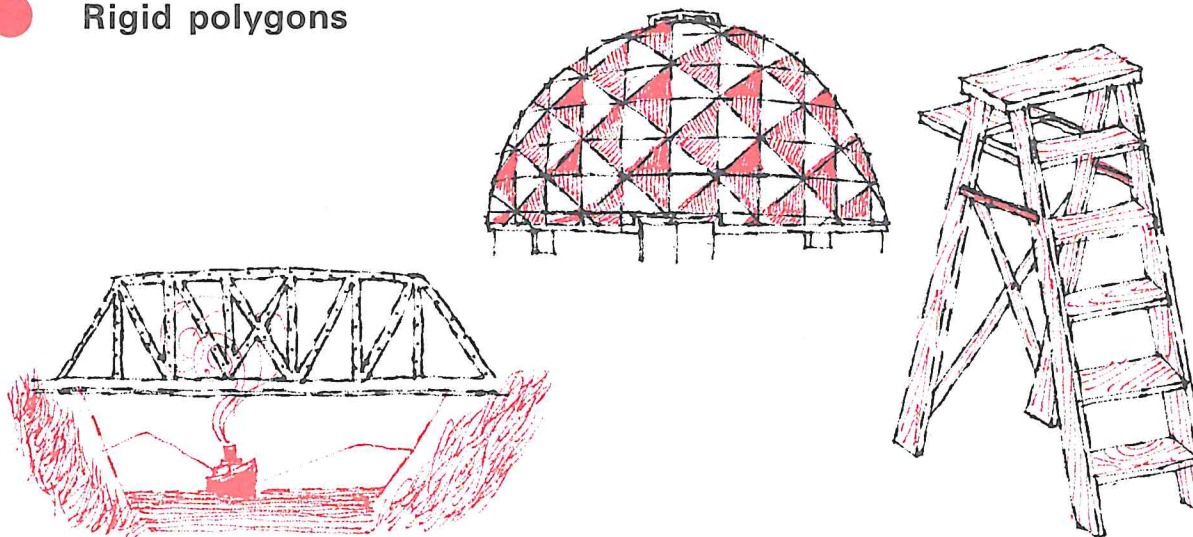


$\overrightarrow{BF}$ is the angle bisector.

Draw a triangle and then bisect the angles of the triangle.

5. What do you notice about the angle bisectors?

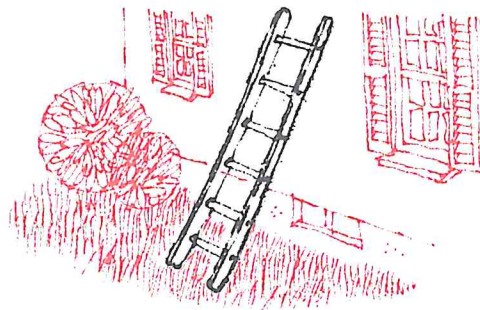
● Rigid polygons



Once a triangle is formed, its shape cannot be changed without changing the lengths of its sides. We say that triangles are **rigid**. Triangular shapes are often used to make buildings and bridges more rigid.

EXERCISES

1. Cut out 3 strips of cardboard and join them with paper fasteners to form a triangle. Try to change the shape of the triangle. Does it remain the same? Is it rigid?
2. Cut out 4 strips of cardboard and join them with paper fasteners to form a quadrilateral. Try to change the shape of the quadrilateral. Is it rigid? How can you make it rigid by adding another strip?
3. The second illustration above shows many triangles put together to form a geodesic dome. A roof made like this needs no center support. Do you know why? Can you think of other designs where triangular shapes are used?
4. Study the ladder in the figure on the right. Can you give a reason why it is not as sturdy as the step ladder in the illustration above?



Name the sums and differences.

1.
$$\begin{array}{r} 5326 \\ + 9849 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 7095 \\ - 4897 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 8592 \\ + 7648 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 5004 \\ - 3926 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 9875 \\ - 2978 \\ \hline \end{array}$$

Name the products.

6.
$$\begin{array}{r} 4387 \\ \times 9 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 5068 \\ \times 27 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 378 \\ \times 492 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 895 \\ \times 703 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 625 \\ \times 819 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 708 \\ \times 360 \\ \hline \end{array}$$

Name the quotients and remainders.

12. $59 \overline{)2242}$

16. $67 \overline{)8432}$

20. $42 \overline{)2394}$

24. $29 \overline{)7134}$

13. $385 \overline{)6160}$

17. $46 \overline{)23219}$

21. $94 \overline{)5828}$

25. $48 \overline{)22704}$

14. $496 \overline{)31744}$

18. $638 \overline{)84318}$

22. $73 \overline{)4957}$

26. $57 \overline{)24821}$

15. $94 \overline{)7862}$

19. $73 \overline{)43065}$

23. $88 \overline{)6548}$

27. $125 \overline{)47000}$

Study the function equation, then complete the ordered pairs.

28. $x + \frac{1}{2} = f(x): \left\{ \left(\frac{1}{2}, 1 \right), \left(\frac{1}{3}, a \right), \left(\frac{2}{5}, b \right), \left(\frac{6}{7}, c \right), \left(1\frac{3}{4}, d \right), \left(2\frac{3}{8}, e \right) \right\}$

29. $x - \frac{6}{5} = f(x): \left\{ \left(\frac{8}{5}, \frac{2}{5} \right), \left(\frac{11}{5}, f \right), \left(\frac{12}{7}, g \right), \left(1\frac{3}{4}, h \right), \left(3\frac{5}{9}, i \right), \left(4\frac{1}{2}, j \right) \right\}$

30. $\frac{3}{4} \times a = f(a): \left\{ \left(2, 1\frac{1}{2} \right), (4, a), (7, b), \left(\frac{4}{5}, c \right), \left(\frac{2}{3}, d \right), \left(\frac{6}{7}, e \right) \right\}$

31. $a \div \frac{2}{3} = f(a): \left\{ \left(\frac{2}{3}, 1 \right), \left(\frac{4}{9}, f \right), \left(\frac{11}{2}, g \right), \left(\frac{5}{6}, h \right), \left(\frac{17}{6}, i \right), \left(2\frac{1}{2}, j \right) \right\}$

Complete each sentence. Write $<$, $>$ or $=$ to replace each $\bigcirc$.

32. $3^2 + 5^2 \bigcirc 6^2$

39. $\sqrt{225} \div \sqrt{25} \bigcirc \sqrt{81} \div \sqrt{9}$

33. $1000 \div 25 \bigcirc 3000 \div 75$

40. $\sqrt{100} \times \sqrt{64} \bigcirc 9^2$

34. $13^2 + 84^2 \bigcirc 85^2$

41. $\sqrt{144} - \sqrt{100} \bigcirc 2 \times 0$

35. $1110 \div 3 \bigcirc 15 \times 24$

42. $13^2 \bigcirc 12 \times 14$

36. $72 \times 89 \bigcirc 50400 \div 8$

43. $13^2 - 12^2 \bigcirc 13 + 12$

37. $2^2 \times 3^2 \bigcirc 6^2$

44. $2^3 - 2^2 \bigcirc 2$

38. $\sqrt{4} \times \sqrt{9} \bigcirc \sqrt{36}$

45. $3^3 - 3^2 \bigcirc 3$

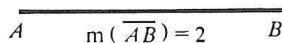


Ratios

The **ratio** of $m(\overline{AB})$ to $m(\overline{CD})$ is 2 to 4.

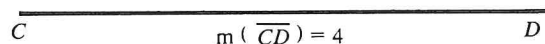
We may write this as a fraction:

$$\frac{m(\overline{AB})}{m(\overline{CD})} = \frac{2}{4}$$



2 to 4 and 1 to 2 are equal ratios.

$$\frac{2}{4} = \frac{1}{2}$$



EXERCISES

Name the missing number to make the ratios equal.

1. $\frac{1}{2} = \frac{a}{8}$

2. $\frac{b}{4} = \frac{6}{8}$

3. $\frac{3}{7} = \frac{9}{c}$

4. $\frac{4}{2} = \frac{10}{e}$

Measure each line segment, then name the ratios.



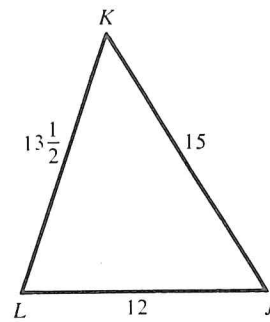
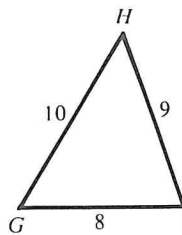
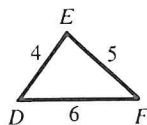
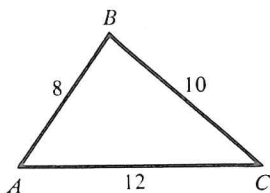
5. $\frac{m(\overline{AB})}{m(\overline{CD})} = a$

6. $\frac{m(\overline{AB})}{m(\overline{EF})} = b$

7. $\frac{m(\overline{CD})}{m(\overline{EF})} = c$

8. $\frac{m(\overline{EF})}{m(\overline{GH})} = d$

Using the figures, name the ratios.



9. $\frac{m(\overline{AB})}{m(\overline{DE})} = a$

11. $\frac{m(\overline{BC})}{m(\overline{EF})} = c$

13. $\frac{m(\overline{GH})}{m(\overline{LJ})} = e$

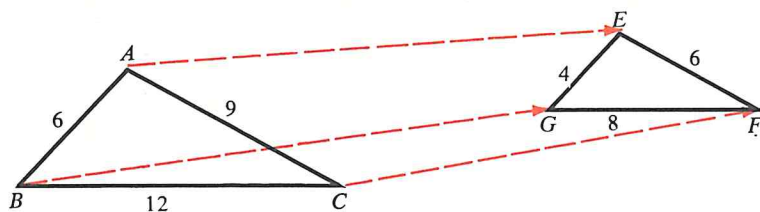
15. $\frac{m(\overline{GI})}{m(\overline{KJ})} = g$

10. $\frac{m(\overline{AC})}{m(\overline{DF})} = b$

12. $\frac{m(\overline{AB})}{m(\overline{DF})} = d$

14. $\frac{m(\overline{IH})}{m(\overline{LK})} = f$

16. $\frac{m(\overline{GI})}{m(\overline{LJ})} = h$



Triangles ABC and EGF have the same shape. They are called **similar** triangles. The pairs of sides, $\overline{AB}$ and $\overline{EG}$, $\overline{AC}$ and $\overline{EF}$, $\overline{BC}$ and $\overline{GF}$ are called **matching** sides.

EXERCISES

Use the units given on the triangles above to name the ratios in simplest form.

- The ratio of the measures of $\overline{AB}$ and $\overline{EG}$ is, $\frac{m(\overline{AB})}{m(\overline{EG})} = \underline{\quad? \quad}$.
- The ratio of the measures of $\overline{AC}$ and $\overline{EF}$ is, $\frac{m(\overline{AC})}{m(\overline{EF})} = \underline{\quad? \quad}$.
- The ratio of the measures of $\overline{BC}$ and $\overline{GF}$ is, $\frac{m(\overline{BC})}{m(\overline{GF})} = \underline{\quad? \quad}$.

Complete the sentence.

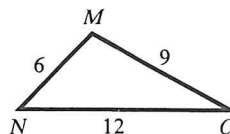
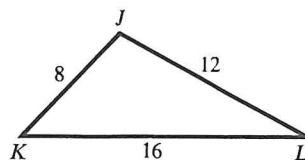
- The ratios of the measures of matching sides of similar triangles are $\underline{\quad? \quad}$.

Name the ratios for triangles JKL and MNO .

- $\frac{m(\overline{MN})}{m(\overline{JK})} = a$
- $\frac{m(\overline{NO})}{m(\overline{KL})} = b$
- $\frac{m(\overline{OM})}{m(\overline{LJ})} = c$

Triangles JKL and MNO are similar.

- Name the measure of the angles of each triangle.
- Pair the congruent angles in part 8.
- Do you think it is true that if 2 triangles are similar, then the matching angles will be congruent?
- Name the ratio of the perimeter of MNO to the perimeter of JKL .



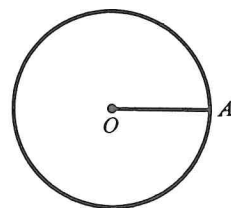
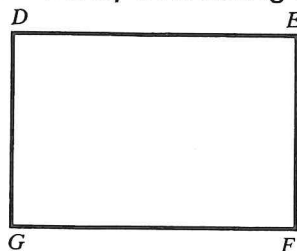
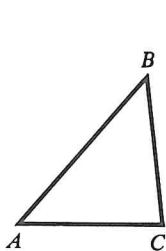
Using similar figures

The matching objects in these pictures are **similar**. The matching angles will be congruent and the matching line segments will form equal ratios. Photographers, map readers, builders, engineers and dress designers all use the idea of **similarity**.



EXERCISES

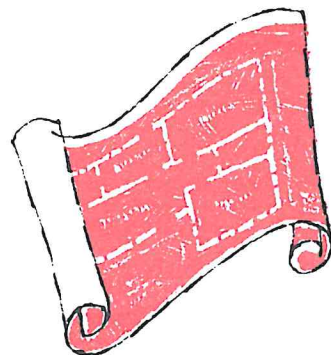
We often use the idea of similarity in making scale drawings.



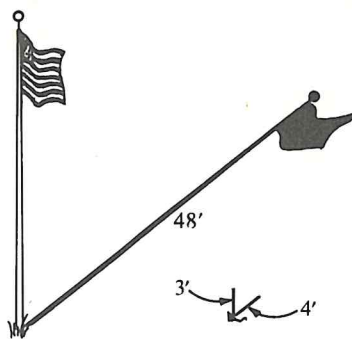
1. Draw a triangle similar to ABC with sides 4 times as long.
2. Draw a rectangle similar to $DEFG$ with sides 3 times as long.
3. Draw a circle with a radius 4 times the measure of $\overline{OA}$.

Answer the questions.

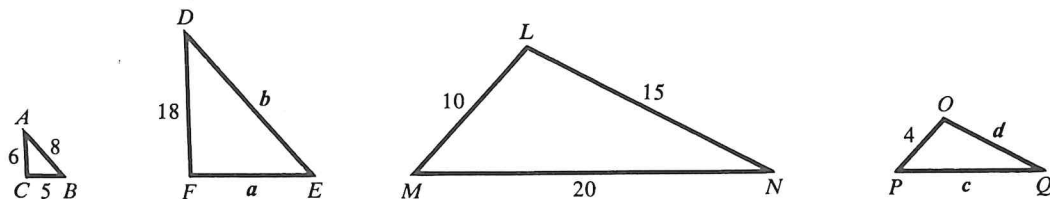
4. Are all circles similar?
5. The ratio of the radius of a circle with radius 7 to the radius of another circle with radius 14 is $\frac{1}{2}$. Name the ratio of the circumferences of these circles.
6. A builder uses a blueprint in building a house. The ratios of the length shown on the blueprint to the actual lengths in the house will all be the same. If $\frac{1}{8}$ " represents 1', how long will be the side represented by **a.** $\frac{30}{8}$ " **b.** $2\frac{1}{8}$ "?
7. On the blueprint, what will be the length of the line segment that represents a side: **a.** 48' **b.** 20'?



1. Mrs. Wilson's 6th grade class uses ratios to name the height of the flagpole in front of the school. They know that a 3' stick makes a 4' shadow and that the flagpole casts a shadow of 48'. If n is the number of feet in the length of the flagpole, then they know that the ratios $\frac{3}{4}$ and $\frac{n}{48}$ must be equal. Name the height of the flagpole.

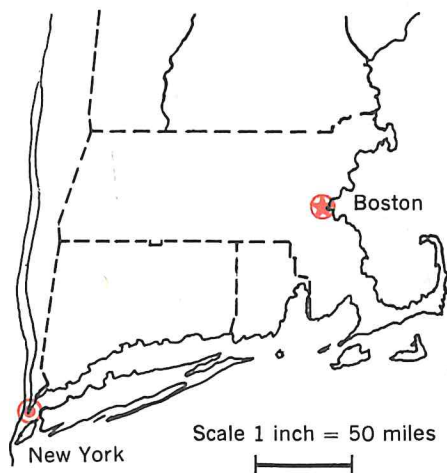


2. Use ratios to name the missing measures of the sides of these similar triangles.



3. Sarah was studying a large map of the United States. At the bottom of the map a scale was given: 1 inch to 50 miles. Sarah knows that the distance from New York to Boston is 4'' on the map, so she knows that the actual distance is $4 \times 50 = 200$ miles. Name the number of miles between cities if the distance on the map is:

- a. 3'' c. $7\frac{1}{2}$ '' e. $12\frac{1}{4}$ ''
b. $2\frac{1}{2}$ '' d. $6\frac{1}{5}$ '' f. $9\frac{3}{4}$ ''

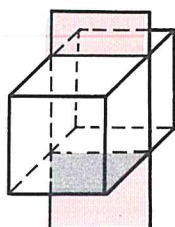


4. Using the scale in exercise 3, name the number of inches on a map if the distance in miles is:

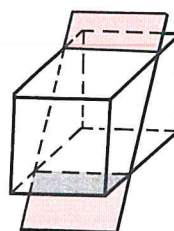
- a. 350 b. 725 c. 975 d. 260 e. 820 f. 585

5. Betty and her friends decided to make a model of the museum they visited. They plan to make the model 24 inches wide. If the museum is 216 feet wide, 342 feet long and 45 feet high, what will be the length and height of the model?

● Intersecting planes and closed surfaces



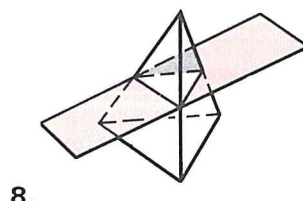
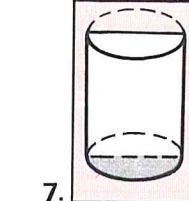
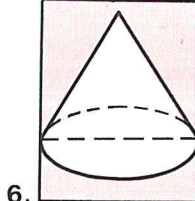
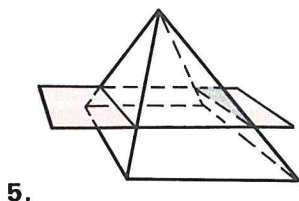
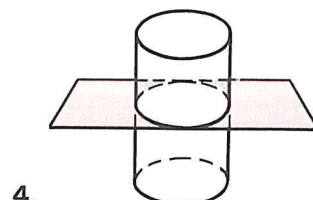
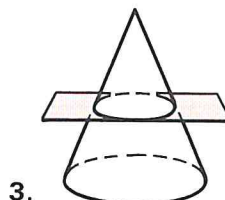
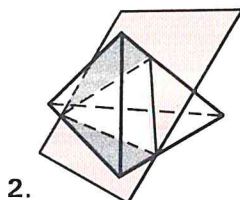
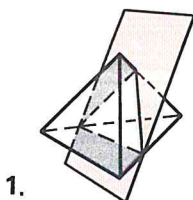
The intersection of a cube and a plane parallel to a face of the cube is a square.



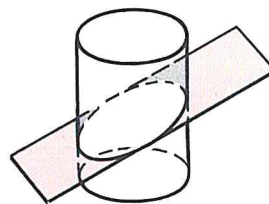
The intersection of a cube and a plane *not* parallel to a face may be a rectangle.

EXERCISES

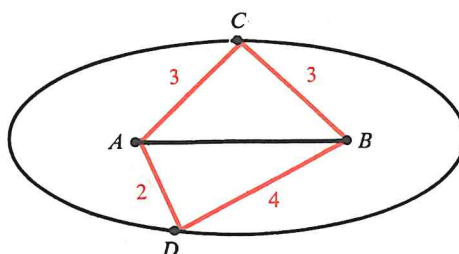
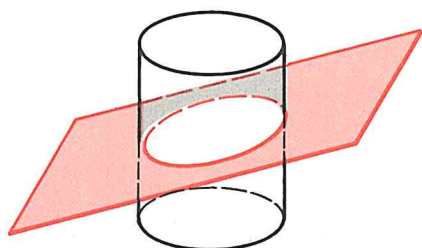
Name the intersection of the planes and space figures.



The intersection of a cylinder and a plane not parallel or perpendicular to the base of the cylinder is called an **ellipse**.
An ellipse looks like this:



9. Can you think of any examples of elliptical shapes?
10. The intersection of a cone and a plane not perpendicular to the base of the cone, may be a circle or an ellipse. When will it be a circle?
- *11. When will the intersection of a plane and a cube be a triangle?



The intersection of a cylinder and a plane not parallel or perpendicular to the base of the cylinder forms a simple closed curve called an **ellipse**. C and D are 2 points on the ellipse. A and B are called the **foci** of the ellipse.

EXERCISES

Use the figure above to name the measures.

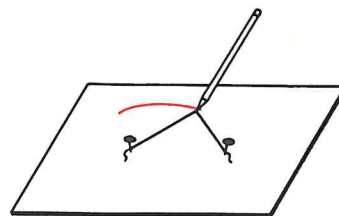
1. $m(\overline{AC})$
2. $m(\overline{BC})$
3. $m(\overline{AC}) + m(\overline{BC})$
4. $m(\overline{AD})$
5. $m(\overline{BD})$
6. $m(\overline{AD}) + m(\overline{BD})$

Complete the sentence.

7. In an ellipse where A and B are foci, and C and D are 2 points in the ellipse, $m(\overline{AC}) + m(\overline{BC})$? $m(\overline{AD}) + m(\overline{BD})$

Find the answers.

8. You can draw an ellipse by fixing the ends of a piece of string with thumbtacks and moving a pencil along the string, keeping the string taut as in the diagram. The ends of the string will be at the foci. Use this method to draw several ellipses.
9. You can change the size and shape of an ellipse by changing the length of the string or by moving the foci closer together or farther apart. What do you notice about the shape when the foci are brought closer and closer together?
10. Ellipses are very common curves. Did you know that
 - a. the path of the earth around the sun is an ellipse?
 - b. a space craft in orbit around the earth may follow an elliptical path?



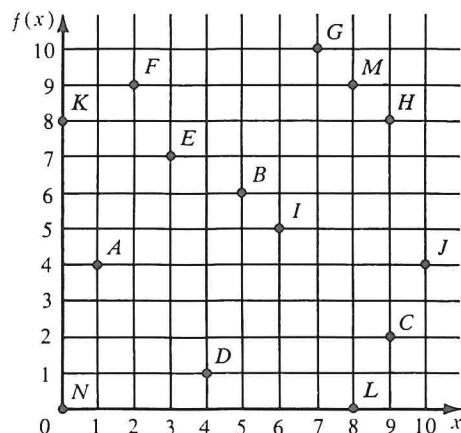
See if you can discover more information about these examples of an ellipse.



Working in the number plane

How much do you remember about the number plane?

1. A number plane is formed by ____?____ sets of number lines perpendicular to each other.
2. Points in a number plane are located by using ____?____ numbers.
3. We write ordered ____?____ to locate these numbers.
4. The 2 number lines intersect in a point called the origin. The ordered pair for the origin is ____?____.
5. Name the ordered pairs for the points labeled *A* to *M* on this number plane.
6. Name the ordered pair labeled *N* on the number plane.



Draw a 10 by 10 number plane. Locate and join the points of each set.

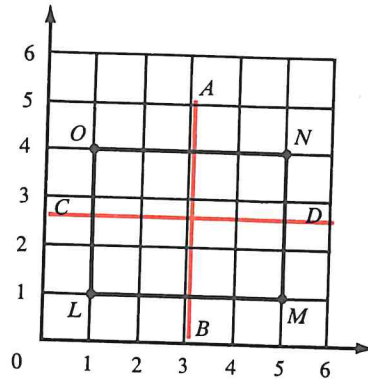
7. $A = \{(2, 4), (3, 5), (4, 6), (5, 7), (6, 8)\}$
8. $B = \{(6, 8), (7, 7), (8, 6), (9, 5), (10, 4)\}$
9. $C = \{(2, 4), (3, 3), (4, 2), (5, 1), (6, 0)\}$
10. $D = \{(6, 0), (7, 1), (8, 2), (9, 3), (10, 4)\}$
11. Name the union of the line segments formed in exercises 7–10.

Answer the questions.

12. The set of number pairs in exercise 7 *satisfies* the equation $x + 2 = f(x)$, where x names the first number in the ordered pair and $f(x)$ names the second number. What is the equation for the ordered pairs in set *B*, in set *C*, in set *D*.
13. Does the unit on each axis of a number plane have to be the same size?

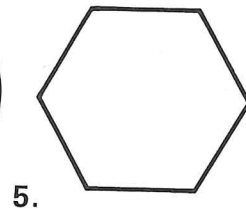
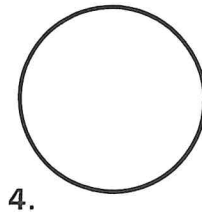
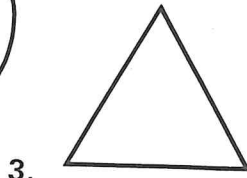
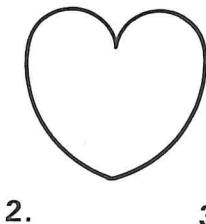
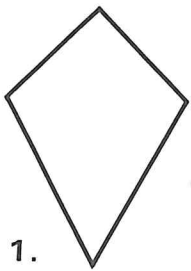
If we find a line which separates a curve into 2 congruent parts so that, when the curve is folded about the line the congruent parts match, the curve is **symmetrical**. The line is a **fold line** or a **line of symmetry**.

$\overline{AB}$ and $\overline{CD}$ are lines of symmetry for $LMNO$.



EXERCISES

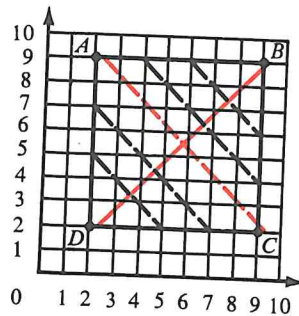
Copy each of the figures and see how many lines of symmetry you can show for each one.



6. Draw other figures and tell whether each one is symmetrical.

Find the answers.

7. Square $ABCD$ is symmetrical about the diagonal $\overline{DB}$. We may match the points in the sides of $ABCD$ according to their distance from $\overline{DB}$. For example, point A matches point C ; (2, 9) matches (9, 2). Name the ordered pair for the point which matches:



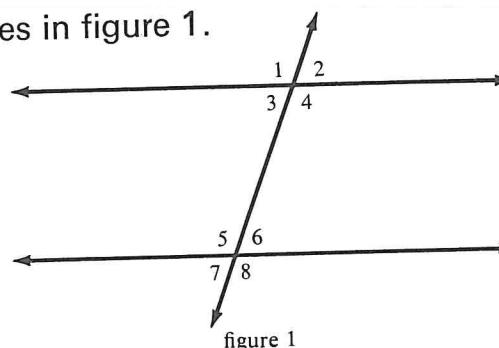
- | | | | |
|-----------|-----------|-----------|-----------|
| a. (2, 8) | d. (2, 3) | g. (5, 9) | j. (8, 2) |
| b. (2, 7) | e. (9, 3) | h. (6, 9) | k. (5, 2) |
| c. (2, 5) | f. (3, 9) | i. (8, 9) | l. (2, 2) |

8. In exercise 7, what point would match the general point $(x, f(x))$?

CHECKING UP

Name the measures, in degrees, for the angles in figure 1.

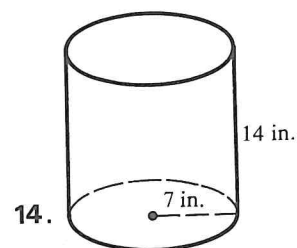
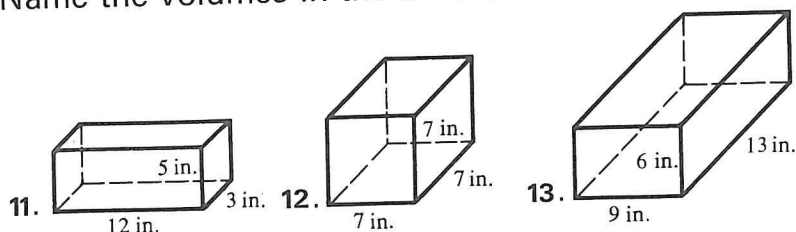
1. $m(\text{angle } 2)$
2. $m(\text{angle } 1)$
3. $m(\text{angle } 3)$
4. $m(\text{angle } 4)$
5. $m(\text{angle } 6)$
6. $m(\text{angle } 5)$
7. $m(\text{angle } 7)$
8. $m(\text{angle } 8)$



In figure 1 name

9. 3 angles congruent to angle 1.
10. 4 angles which form linear pairs with angle 1.

Name the volumes in the closed surfaces.



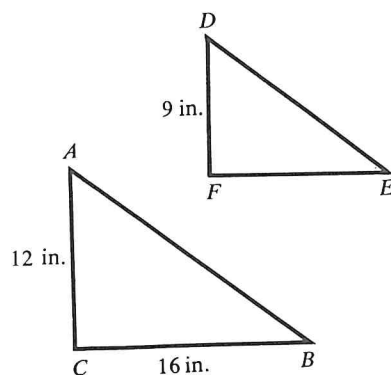
- 15-17. Name the surface areas of the closed surfaces in exercises 11-13.

O is a circle with radius $\frac{7}{2}$ in.

18. Name the diameter.
19. Name the circumference.
20. Name the area.

In figure 2, ABC and DEF are similar.

21. Use the Theorem of Pythagoras to name the length of side $\overline{AB}$.
22. Use ratios to name the length of sides $\overline{DE}$ and $\overline{FE}$.



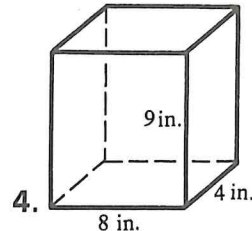
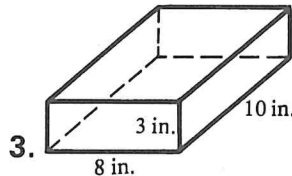
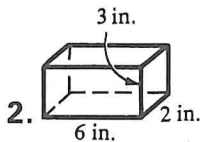
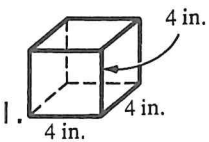
Complete the sentences.

23. The sum of the measures of the angles of a triangle is ____?
24. The sum of the measures of the angles of a quadrilateral is ____?
25. A ray which separates an angle into 2 congruent parts is called an angle ____?



In **measurement**, we compare a unit with the object being measured.

Name the volume and surface area in the closed surfaces.



The equation for naming the **circumference** of a circle is: $C = \pi \times d$. The equation for naming the **area of a circular region** is: $A = \pi \times r^2$ ($\pi \approx \frac{22}{7}$ or $3\frac{1}{7}$).

Estimate the circumference and area of circles with radii

5. 1"

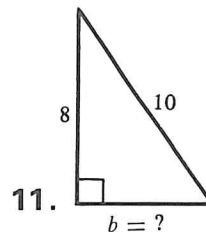
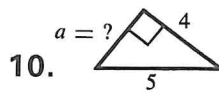
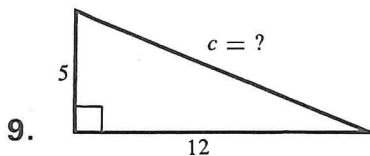
6. 7"

7. 14"

8. 11"

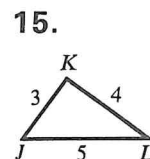
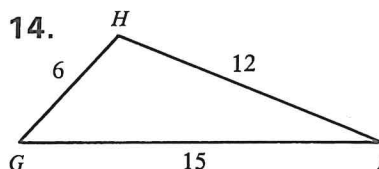
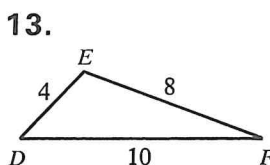
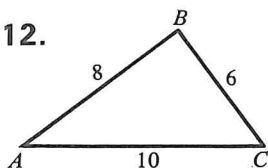
The **Theorem of Pythagoras** is: $c^2 = a^2 + b^2$ where a , b , and c are the *measures* of the sides of a right triangle.

Use the Theorem of Pythagoras to name the missing measures of the right triangles.



Similar triangles have the **ratio** of their matching sides equal.

Name the similar triangles and the ratio of their sides.





LOOKING WAY BACK

Name these amounts.

$$\begin{array}{r} 1. \quad \$2.56 \\ \quad 4.34 \\ + 2.69 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad \$34.86 \\ \quad 29.56 \\ + 40.08 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad \$17.83 \\ \quad 24.79 \\ + 33.04 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad \$66.14 \\ \quad 32.69 \\ + 8.46 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad \$14.38 \\ \quad 27.62 \\ + 8.79 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad \$2.49 \\ - 1.56 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad \$18.69 \\ - 9.47 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad \$46.32 \\ - 5.09 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad \$50.06 \\ - 37.38 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad \$48.26 \\ - 16.34 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad \$7.29 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad \$7.86 \\ \times 17 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad \$39.41 \\ \times 27 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad \$56.54 \\ \times 38 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad \$19.97 \\ \times 64 \\ \hline \end{array}$$

$$16. \quad 7 \overline{) \$20.79}$$

$$17. \quad 9 \overline{) \$34.47}$$

$$18. \quad 19 \overline{) \$52.63}$$

$$19. \quad 12 \overline{) \$85.44}$$

Name the sums or differences.

$$\begin{array}{r} 20. \quad 655879 \\ + 239352 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 486294 \\ + 847819 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 64306 \\ - 52499 \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 773085 \\ - 649788 \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 3102408 \\ - 66795 \\ \hline \end{array}$$

Name the sums or differences in simplest form.

$$\begin{array}{r} 25. \quad 2\frac{3}{5} \\ + 4\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 7\frac{4}{7} \\ - 2\frac{3}{7} \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 6\frac{1}{6} \\ + 3\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 8\frac{7}{9} \\ + 13\frac{6}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 4\frac{3}{4} \\ + 9\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 81\frac{7}{16} \\ - 39\frac{11}{16} \\ \hline \end{array}$$

Name the products or quotients in simplest form.

$$31. \quad \frac{2}{3} \times \frac{7}{8}$$

$$33. \quad \frac{8}{9} \div \frac{2}{3}$$

$$35. \quad \frac{7}{12} \times \frac{4}{5}$$

$$37. \quad \frac{3}{8} \times \frac{16}{15}$$

$$32. \quad \frac{9}{5} \times \frac{10}{3}$$

$$34. \quad \frac{5}{2} \div \frac{5}{6}$$

$$36. \quad \frac{11}{7} \div \frac{22}{14}$$

$$38. \quad \frac{12}{5} \times \frac{10}{24}$$

Complete each sentence. Write $<$, $>$ or $=$ to replace each $\bigcirc$.

$$39. \quad \frac{1}{2} + \frac{1}{6} \bigcirc \frac{1}{3} + \frac{1}{4}$$

$$41. \quad \frac{5}{8} \times \frac{4}{3} \times \frac{18}{15} \bigcirc 1$$

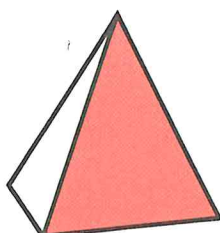
$$43. \quad \frac{2}{3} - \frac{1}{5} \bigcirc \frac{2}{5} - \frac{1}{3}$$

$$40. \quad 9^2 + 9^2 \bigcirc 8^2 + 10^2$$

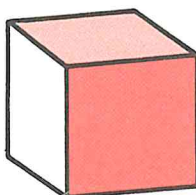
$$42. \quad \left(\frac{3}{4}\right)^2 \bigcirc \left(\frac{2}{3}\right)^2$$

$$44. \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \bigcirc 1$$

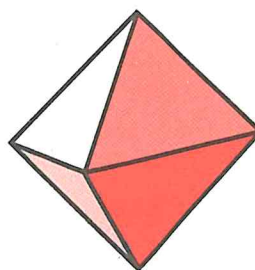
A simple closed surface whose faces are polygons is called a **polyhedron**. If all the faces are equilateral and congruent, then the polyhedron is said to be **regular**. Surprisingly enough, there are only 5 regular polyhedra. Here are the 5 with their models.



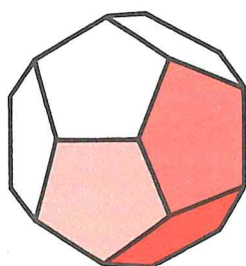
Tetrahedron



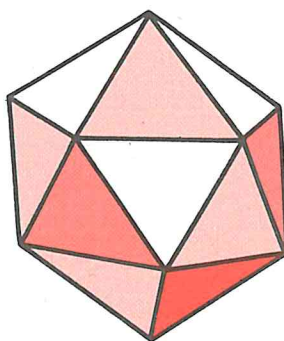
Hexahedron



Octahedron



Dodecahedron



Icosahedron

Copy and complete the table.

figure	shape of face	no. of vertices(V)	no. of faces(F)	no. of edges(E)	$V + F - E$
tetrahedron					
hexahedron					
octahedron					
dodecahedron					
icosahedron					

The equation $V + F - E = 2$ was discovered by a Swiss mathematician named Euler. See if this equation is true for other space figures (prisms, pyramids, etc.).

CHAPTER 10 • Decimals, Per Cents

Decimal numeration

one thousand	one hundred	ten	one
10^3	10^2	10^1	1

Ten is the *base* of our system of numeration. The Latin word for ten is “decem,” so our system of numeration is a **decimal system**.

EXERCISES

Name the place value of the 4 in the numerals.

1. 4638

2. 6438

3. 6348

4. 6384

Name the places to complete the chart.

5.

<i>a</i>	hundred thousands	ten thousands	<i>c</i>	hundreds	<i>e</i>	ones
----------	-------------------	---------------	----------	----------	----------	------

Name the exponents to complete the chart.

6.

10^a	10^c	10^e	10^n	1
ten thousands	thousands	hundreds	tens	ones

Complete the sentence.

7. In our system of numeration, the value of each place is one _____ of the value of the place immediately to its left.

Write compact numerals for each of the numbers.

8. 3 hundred thousand, 4 hundred fifty-six
9. $(2 \times 10^6) + (5 \times 10^5) + (4 \times 10^2) + (3 \times 10^1) + 9$
10. $(4 \times 10^5) + (6 \times 10^4) + (2 \times 10^3) + (3 \times 10^1) + 2$

Name the total value of the 6 in each of the numerals.

11. 267

12. 608

13. 6972

14. 4601

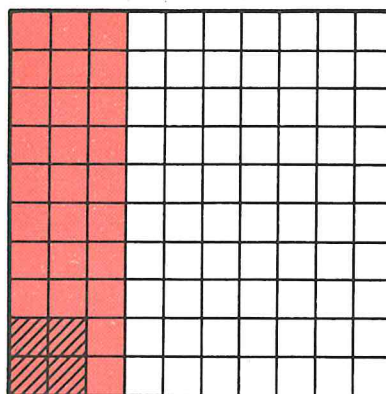
15. 62,157



$\frac{3}{10}$ of the region is red. $\frac{3}{10}$ can be written as .3 and read as “3 tenths” or “point 3”.

$\frac{4}{100}$ of the region is shaded. $\frac{4}{100}$ can be written as .04 and read as “4 hundredths” or “point zero 4.”

.3 and .04 are called **decimal fractions**. The point is called the **decimal point**. The first place to the right of the decimal point is the **tenths’ place**; the second place is the **hundredths’ place**.



Fractional numbers greater than 1 may be named by decimal fractions. 2.34 means $2\frac{34}{100}$.

EXERCISES

Write the fractions in decimal form.

1. $\frac{1}{10}$
2. $\frac{3}{10}$
3. $\frac{5}{10}$
4. $\frac{6}{100}$
5. $\frac{9}{100}$
6. $\frac{49}{100}$

Write the decimal fractions as fractions.

7. .7
8. .9
9. .27
10. .06
11. .95
12. .17

Write the mixed numerals in decimal form.

13. $2\frac{6}{10}$
14. $5\frac{7}{100}$
15. $8\frac{3}{10}$
16. $14\frac{2}{100}$
17. $9\frac{7}{10}$
18. $3\frac{9}{10}$

Write the decimal fractions as mixed numerals.

19. 2.13
20. 6.7
21. 13.04
22. 8.8
23. 9.07
24. 21.5

Write the numeral in each of the sentences, inserting a decimal point to make the sentence sensible.

25. Sandra weighs 953 lb, and Bill is 726 in. tall.
26. The wind averaged 3475 miles per hour.
27. The airplane averaged 5786 miles per hour.
28. The cyclist averaged 245 miles per hour.

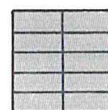
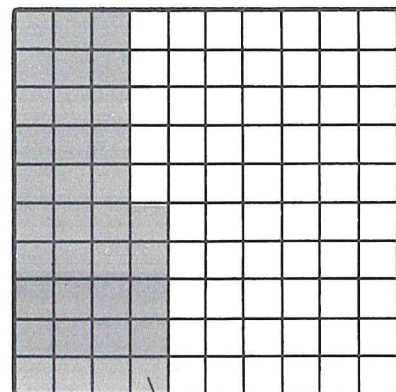


Thousandths

Each column is $\frac{1}{10}$ of the region and each square is $\frac{1}{100}$ of the region. If each of the squares can contain 10 small rectangles, the region can contain $10 \times 100 = 1000$ small rectangles. Each small rectangle is $\frac{1}{1000}$ (one thousandth) of the region.

$\frac{1}{1000}$ may be written as .001.

348 small rectangles are shaded, that is, $\frac{348}{1000}$ of the region is shaded. $\frac{348}{1000}$ can be written as .348 and read “348 thousandths.”



EXERCISES

Rename the numbers in decimal form.

- | | | | | | |
|---------------------|---------------------|-----------------------|------------------------|------------------------|-----------------------|
| 1. $\frac{17}{100}$ | 4. $\frac{67}{100}$ | 7. $\frac{9}{100}$ | 10. $\frac{761}{1000}$ | 13. $\frac{973}{1000}$ | 16. $\frac{13}{1000}$ |
| 2. $\frac{23}{100}$ | 5. $\frac{32}{100}$ | 8. $\frac{127}{1000}$ | 11. $\frac{625}{1000}$ | 14. $\frac{807}{1000}$ | 17. $\frac{23}{1000}$ |
| 3. $\frac{52}{100}$ | 6. $\frac{93}{100}$ | 9. $\frac{429}{1000}$ | 12. $\frac{290}{1000}$ | 15. $\frac{515}{1000}$ | 18. $\frac{7}{1000}$ |

Rename the numbers as fractions with denominators of 100 or 1000.

- | | | | |
|---------|----------|----------|----------|
| 19. .51 | 22. .123 | 25. .487 | 28. .049 |
| 20. .67 | 23. .997 | 26. .320 | 29. .30 |
| 21. .24 | 24. .906 | 27. .573 | 30. .007 |

Solve the problems.

31. Rita said, “ $\frac{60}{100}$ of the region is shaded.” Betty said, “ $\frac{6}{10}$ of the region is shaded.” Could both Betty and Rita be right?
32. Bob said, “ $\frac{370}{1000}$ of a region is shaded.” Brad said, “ $\frac{37}{100}$ of the same region is shaded.” Could both Bob and Brad be right?

The position of the decimal point



thousands	hundreds	tens	Ones
1000	100	10	1

tenths	hundredths	thousandths
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

The first place to the right of the decimal point is for tenths, the second place is for hundredths, the third place is for thousandths.

.473 means 4 tenths, 7 hundredths and 3 thousandths and can

be written as an expanded numeral: $.473 = \frac{4}{10} + \frac{7}{100} + \frac{3}{1000}$

EXERCISES

Complete the sentences.

- The first place to the left of ones' place is ____?____ place.
- The first place to the right of ones' place is ____?____ place.
- The second place to the left of ones' place is ____?____ place.
- The second place to the right of ones' place is ____?____ place.
- The third place to the left of ones' place is ____?____ place.
- The third place to the right of ones' place is ____?____ place.

Complete the sentences.

- .56 means ____?____ tenths and ____?____ hundredths.
- .79 means 7 ____?____ and 9 ____?____.
- .257 means ____?____ tenths, ____?____ hundredths, and ____?____ thousandths.
- .813 means 8 ____?____, 1 ____?____, and 3 ____?____.

Rename the decimal fractions as expanded numerals.

11. $.35 = \frac{a}{10} + \frac{c}{100}$

14. $.576 = j$

17. $.056 = m$

12. $.182 = \frac{a}{10} + \frac{c}{100} + \frac{e}{1000}$

15. $.209 = k$

18. $2.3 = r$

13. $.674 = \frac{a}{10} + \frac{c}{100} + \frac{e}{1000}$

16. $.49 = l$

19. $4.67 = s$

Decimal fractions in addition and subtraction

Study these examples.

$$\begin{array}{r} \frac{3}{10} \\ + \frac{4}{10} \\ \hline \frac{7}{10} \end{array} \quad \begin{array}{r} .3 \\ + .4 \\ \hline .7 \end{array}$$

$$\begin{array}{r} \frac{7}{10} \\ - \frac{4}{10} \\ \hline \frac{3}{10} \end{array} \quad \begin{array}{r} .7 \\ - .4 \\ \hline .3 \end{array}$$

$$\begin{array}{r} \frac{14}{100} \\ + \frac{27}{100} \\ \hline \frac{41}{100} \end{array} \quad \begin{array}{r} .14 \\ + .27 \\ \hline .41 \end{array}$$

$$\begin{array}{r} \frac{41}{100} \\ - \frac{27}{100} \\ \hline \frac{14}{100} \end{array} \quad \begin{array}{r} .41 \\ - .27 \\ \hline .14 \end{array}$$

EXERCISES

Name the sums.

$$\begin{array}{r} 1. \quad .2 \\ + .3 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad .8 \\ + .4 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad .43 \\ + .23 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad .18 \\ + .19 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 6.3 \\ + 2.5 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 8.23 \\ + 6.45 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad .5 \\ + .4 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad .8 \\ + .2 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad .25 \\ + .61 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad .29 \\ + .07 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 4.5 \\ + 9.3 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 7.17 \\ + 3.38 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad .6 \\ + .3 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad .3 \\ + .9 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad .47 \\ + .29 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad .15 \\ + .08 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 6.2 \\ + 7.4 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 7.34 \\ + 6.37 \\ \hline \end{array}$$

Name the differences.

$$\begin{array}{r} 19. \quad .9 \\ - .4 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad .7 \\ - .3 \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 1.3 \\ - .8 \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad .67 \\ - .38 \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad .81 \\ - .37 \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad .70 \\ - .48 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad .8 \\ - .6 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad .8 \\ - .3 \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 1.2 \\ - .5 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad .54 \\ - .41 \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad .73 \\ - .65 \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad .69 \\ - .45 \\ \hline \end{array}$$

Name the sums or differences.

$$\begin{array}{r} 31. \quad \$.68 \\ + .27 \\ \hline \end{array}$$

$$\begin{array}{r} 34. \quad \$.63 \\ - .47 \\ \hline \end{array}$$

$$\begin{array}{r} 37. \quad \$ 3.2 \\ + 4.6 \\ \hline \end{array}$$

$$\begin{array}{r} 40. \quad \$ 5.29 \\ + 2.13 \\ \hline \end{array}$$

$$\begin{array}{r} 32. \quad \$.45 \\ + .36 \\ \hline \end{array}$$

$$\begin{array}{r} 35. \quad \$.39 \\ - .28 \\ \hline \end{array}$$

$$\begin{array}{r} 38. \quad \$ 8.1 \\ - 6.1 \\ \hline \end{array}$$

$$\begin{array}{r} 41. \quad \$ 6.21 \\ - .18 \\ \hline \end{array}$$

$$\begin{array}{r} 33. \quad \$.28 \\ + .46 \\ \hline \end{array}$$

$$\begin{array}{r} 36. \quad \$.56 \\ - .38 \\ \hline \end{array}$$

$$\begin{array}{r} 39. \quad \$ 7.43 \\ + .37 \\ \hline \end{array}$$

$$\begin{array}{r} 42. \quad \$ 8.34 \\ - .18 \\ \hline \end{array}$$

Decimal fractions in addition and subtraction

Study these methods.

Expanded numerals

$$\begin{array}{r} 5.3 = 5 + .3 \\ + 3.8 = + 3 + .8 \\ \hline 8 + 1.1 = 9.1 \end{array}$$

$$\begin{array}{r} 5.3 = \cancel{5}^4 + \cancel{.3}^{1.3} \\ - 3.8 = - \cancel{3} - \cancel{.8} \\ \hline 1 + .5 = 1.5 \end{array}$$

Three-step

$$\begin{array}{r} 5.3 \\ + 3.8 \\ \hline 1.1 \\ \hline 8.0 \\ \hline 9.1 \end{array}$$

$$\begin{array}{r} \cancel{5}^4 \cancel{.3}^{1.3} \\ - \cancel{3.8} \\ \hline .5 \\ \hline 1.0 \\ \hline 1.5 \end{array}$$

Short form

$$\begin{array}{r} 1 \\ 5.3 \\ + 3.8 \\ \hline 9.1 \end{array}$$

$$\begin{array}{r} \cancel{5}^4 \cancel{.3}^{1.3} \\ - \cancel{3.8} \\ \hline 1.5 \end{array}$$

EXERCISES

Use expanded numerals to name the sums.

1. $\begin{array}{r} 2.7 \\ + 3.4 \\ \hline \end{array}$

3. $\begin{array}{r} 6.8 \\ + 2.7 \\ \hline \end{array}$

5. $\begin{array}{r} 3.7 \\ + 5.5 \\ \hline \end{array}$

7. $\begin{array}{r} 4.6 \\ + 5.7 \\ \hline \end{array}$

9. $\begin{array}{r} 6.5 \\ + 3.9 \\ \hline \end{array}$

11. $\begin{array}{r} 1.9 \\ + 7.9 \\ \hline \end{array}$

2. $\begin{array}{r} 5.3 \\ + 4.9 \\ \hline \end{array}$

4. $\begin{array}{r} 4.6 \\ + 4.2 \\ \hline \end{array}$

6. $\begin{array}{r} 8.6 \\ + 1.9 \\ \hline \end{array}$

8. $\begin{array}{r} 3.8 \\ + 3.8 \\ \hline \end{array}$

10. $\begin{array}{r} 2.8 \\ + 4.3 \\ \hline \end{array}$

12. $\begin{array}{r} 5.6 \\ + 2.8 \\ \hline \end{array}$

13–24. Use the 3-step method to name the sums in exercises 1–12.

25–36. Use the short method to name the sums in exercises 1–12.

Use expanded numerals to name the differences.

37. $\begin{array}{r} 8.3 \\ - 3.9 \\ \hline \end{array}$

39. $\begin{array}{r} 7.5 \\ - 2.6 \\ \hline \end{array}$

41. $\begin{array}{r} 5.2 \\ - 4.3 \\ \hline \end{array}$

43. $\begin{array}{r} 3.2 \\ - 1.4 \\ \hline \end{array}$

45. $\begin{array}{r} 9.4 \\ - 5.6 \\ \hline \end{array}$

47. $\begin{array}{r} 7.3 \\ - 3.4 \\ \hline \end{array}$

38. $\begin{array}{r} 9.2 \\ - 4.4 \\ \hline \end{array}$

40. $\begin{array}{r} 4.1 \\ - 2.7 \\ \hline \end{array}$

42. $\begin{array}{r} 6.4 \\ - 1.5 \\ \hline \end{array}$

44. $\begin{array}{r} 7.3 \\ - 2.7 \\ \hline \end{array}$

46. $\begin{array}{r} 8.1 \\ - 6.2 \\ \hline \end{array}$

48. $\begin{array}{r} 9.1 \\ - 7.1 \\ \hline \end{array}$

49–60. Use the 3-step method to name the differences in exercises 37–48.

61–72. Use the short method to name the differences in exercises 37–48.

Decimal fractions in addition and subtraction

Study these methods.

Expanded numerals

$$\begin{array}{r} 5.63 = 5 + .63 \\ + 3.78 = + 3 + .78 \\ \hline 8 \quad 1.41 = 9.41 \end{array}$$

Four-step

$$\begin{array}{r} 5.63 \\ + 3.78 \\ \hline .11 \\ 1.30 \\ 8.00 \\ \hline 9.41 \end{array}$$

Short form

$$\begin{array}{r} 5.63 \\ + 3.78 \\ \hline 9.41 \end{array}$$

$$\begin{array}{r} 5.63 = \overset{4}{\cancel{5}} + \overset{1.63}{\cancel{.63}} \\ - 3.78 = - 3 - \cancel{.78} \\ \hline 1 + .85 = 1.85 \end{array}$$

$$\begin{array}{r} \overset{4 \ 15 \ 13}{\cancel{5.63}} \\ - 3.78 \\ \hline .05 \\ .80 \\ 1.00 \\ \hline 1.85 \end{array}$$

$$\begin{array}{r} 5.63 \\ - 3.78 \\ \hline 1.85 \end{array}$$

EXERCISES

Use expanded numerals to name the sums.

1. $\begin{array}{r} 4.32 \\ + 2.94 \\ \hline \end{array}$

3. $\begin{array}{r} 5.53 \\ + 1.63 \\ \hline \end{array}$

5. $\begin{array}{r} 1.87 \\ + 6.06 \\ \hline \end{array}$

7. $\begin{array}{r} 7.73 \\ + 2.82 \\ \hline \end{array}$

9. $\begin{array}{r} 3.57 \\ + 4.85 \\ \hline \end{array}$

2. $\begin{array}{r} 3.81 \\ + 6.64 \\ \hline \end{array}$

4. $\begin{array}{r} 2.37 \\ + 3.91 \\ \hline \end{array}$

6. $\begin{array}{r} 5.29 \\ + 3.65 \\ \hline \end{array}$

8. $\begin{array}{r} 1.94 \\ + 8.61 \\ \hline \end{array}$

10. $\begin{array}{r} 6.46 \\ + 3.98 \\ \hline \end{array}$

11–20. Use the 4-step method to name the sums in exercises 1–10.

21–30. Use the short method to name the sums in exercises 1–10.

Use expanded numerals to name the differences.

31. $\begin{array}{r} 6.26 \\ - 3.54 \\ \hline \end{array}$

33. $\begin{array}{r} 7.26 \\ - 2.44 \\ \hline \end{array}$

35. $\begin{array}{r} 5.49 \\ - 4.63 \\ \hline \end{array}$

37. $\begin{array}{r} 5.67 \\ - 3.76 \\ \hline \end{array}$

39. $\begin{array}{r} 7.29 \\ - 4.37 \\ \hline \end{array}$

32. $\begin{array}{r} 9.38 \\ - 5.75 \\ \hline \end{array}$

34. $\begin{array}{r} 8.17 \\ - 6.35 \\ \hline \end{array}$

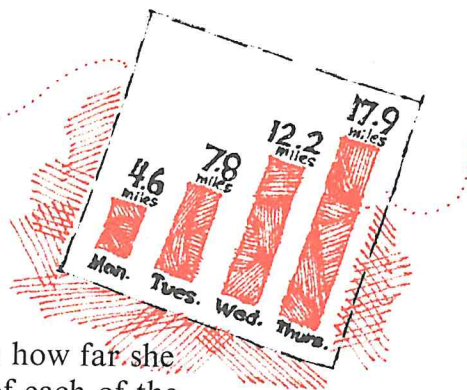
36. $\begin{array}{r} 9.05 \\ - 8.22 \\ \hline \end{array}$

38. $\begin{array}{r} 8.76 \\ - 6.93 \\ \hline \end{array}$

40. $\begin{array}{r} 6.08 \\ - 5.16 \\ \hline \end{array}$

41–50. Use the 4-step method to name the differences in exercises 31–40.

51–60. Use the short method to name the differences in exercises 31–40.



1. Judy's new bicycle has a cyclometer which tells how far she travels. She recorded the mileages at the end of each of the first four days she had the bicycle. They were 4.6 mi., 7.8 mi., 12.2 mi. and 17.9 mi. How far did Judy travel on **a.** the second day, **b.** the third day, **c.** the fourth day?
2. By the end of the first week Judy had traveled 23.8 mi.; by the end of the second week she had traveled 39.4 mi. How far did she travel the second week?
3. On a Saturday ride Judy noted these mileages: from home to Mandy's, 2.6 mi.; from Mandy's to Philip's, 3.1 mi.; from Philip's to Jill's, 1.9 mi. and from Jill's to Judy's home, 1.4 mi. How far was the round trip?
4. How far would Judy travel if she rode to Mandy's and back?
5. How far is it **a.** from Mandy's to Jill's? **b.** from Mandy's past Philip's and Jill's to Judy's?
6. Judy's school is 7.3 miles from her house. If she rode 2.7 miles on her bicycle, how much farther would she have to go to get to school?
7. At the start of one trip, the cyclometer read 50.7 miles. At the finish it read 68.4 miles. If the trip was from Judy's home to Mary's (7.4 miles) and Mary's home to Betty's (5.6 miles) and then home, how far is it from Betty's home to Judy's home?
- *8. Judy's father drove Judy and her bicycle 4.3 miles. Judy then rode her bicycle 2.4 more miles, then turned around and rode home. If the cyclometer read 72.8 miles at the start of the trip, what did it read when she arrived at home?

Decimal fractions in multiplication

Study these examples.

$$3 \times .4 = 3 \times \frac{4}{10} = \frac{12}{10} = 1\frac{2}{10} = 1.2$$

$$3 \times .04 = 3 \times \frac{4}{100} = \frac{12}{100} = .12$$

$$3 \times .004 = 3 \times \frac{4}{1000} = \frac{12}{1000} = .012$$

EXERCISES

Name the products as decimal fractions.

1. $4 \times \frac{2}{10}$

4. $45 \times \frac{5}{10}$

7. $8 \times \frac{11}{100}$

10. $5 \times \frac{31}{1000}$

2. $5 \times \frac{3}{10}$

5. $2 \times \frac{8}{100}$

8. $4 \times \frac{16}{100}$

11. $13 \times \frac{27}{1000}$

3. $6 \times \frac{7}{10}$

6. $9 \times \frac{7}{100}$

9. $4 \times \frac{2}{1000}$

12. $27 \times \frac{34}{1000}$

Name the products as decimal fractions.

13. $7 \times .3$ 15. $5 \times .11$ 17. $.03 \times 6$ 19. $4 \times .123$ 21. $.74 \times 6$
 14. $5 \times .9$ 16. $8 \times .07$ 18. $.14 \times 7$ 20. $6 \times .209$ 22. $4 \times .568$

Complete the chart.

	$\times$	4	10	15	23	100	125	1000
23.	6	24	<i>a</i>	<i>c</i>	<i>e</i>	<i>n</i>	<i>r</i>	<i>s</i>
24.	.6	<i>a</i>	6	<i>e</i>	<i>n</i>	<i>r</i>	<i>s</i>	600
25.	.06	<i>a</i>	<i>c</i>	<i>e</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>
26.	.006	<i>a</i>	<i>c</i>	<i>e</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>

Complete the sentences.

27. If one factor is a counting number and the other factor has 1 decimal place, the product will have ____?____ decimal places.
28. If one factor is a counting number and the other factor has 2 decimal places, the product will have ____?____ decimal places.
29. If one factor is a counting number and the other factor has 3 decimal places, the product will have ____?____ decimal places.

Decimal fractions in multiplication



Study these methods.

Four-step

$$\begin{array}{r} 4.26 \\ \times 3 \\ \hline .18 \\ .60 \\ \hline 12.00 \\ 12.78 \end{array}$$

Short form

$$\begin{array}{r} 4.26 \\ \times 3 \\ \hline 12.78 \end{array}$$

Three-step

$$\begin{array}{r} 4.26 \\ \times 23 \\ \hline 12.78 \\ 85.20 \\ \hline 97.98 \end{array}$$

EXERCISES

Name the products by the 4-step method.

1. $\begin{array}{r} 2.43 \\ \times 2 \\ \hline \end{array}$

3. $\begin{array}{r} 3.69 \\ \times 5 \\ \hline \end{array}$

5. $\begin{array}{r} 3.12 \\ \times 5 \\ \hline \end{array}$

7. $\begin{array}{r} 4.28 \\ \times 8 \\ \hline \end{array}$

9. $\begin{array}{r} 5.62 \\ \times 3 \\ \hline \end{array}$

2. $\begin{array}{r} 3.15 \\ \times 3 \\ \hline \end{array}$

4. $\begin{array}{r} 7.76 \\ \times 9 \\ \hline \end{array}$

6. $\begin{array}{r} 4.41 \\ \times 6 \\ \hline \end{array}$

8. $\begin{array}{r} 6.75 \\ \times 9 \\ \hline \end{array}$

10. $\begin{array}{r} 3.17 \\ \times 5 \\ \hline \end{array}$

Name the products by the short method.

11. $\begin{array}{r} 2.45 \\ \times 7 \\ \hline \end{array}$

13. $\begin{array}{r} 7.05 \\ \times 8 \\ \hline \end{array}$

15. $\begin{array}{r} 8.91 \\ \times 5 \\ \hline \end{array}$

17. $\begin{array}{r} 29.04 \\ \times 7 \\ \hline \end{array}$

19. $\begin{array}{r} 4.63 \\ \times 7 \\ \hline \end{array}$

12. $\begin{array}{r} 7.64 \\ \times 8 \\ \hline \end{array}$

14. $\begin{array}{r} 9.16 \\ \times 7 \\ \hline \end{array}$

16. $\begin{array}{r} 10.96 \\ \times 6 \\ \hline \end{array}$

18. $\begin{array}{r} 12.29 \\ \times 8 \\ \hline \end{array}$

20. $\begin{array}{r} 29.83 \\ \times 5 \\ \hline \end{array}$

Name the products.

21. $\begin{array}{r} 3.28 \\ \times 14 \\ \hline \end{array}$

24. $\begin{array}{r} 6.25 \\ \times 48 \\ \hline \end{array}$

27. $\begin{array}{r} 6.02 \\ \times 27 \\ \hline \end{array}$

30. $\begin{array}{r} 7.18 \\ \times 62 \\ \hline \end{array}$

33. $\begin{array}{r} 31.02 \\ \times 15 \\ \hline \end{array}$

22. $\begin{array}{r} 5.37 \\ \times 35 \\ \hline \end{array}$

25. $\begin{array}{r} 5.32 \\ \times 59 \\ \hline \end{array}$

28. $\begin{array}{r} 7.45 \\ \times 24 \\ \hline \end{array}$

31. $\begin{array}{r} 3.96 \\ \times 28 \\ \hline \end{array}$

34. $\begin{array}{r} 61.10 \\ \times 39 \\ \hline \end{array}$

23. $\begin{array}{r} 4.86 \\ \times 46 \\ \hline \end{array}$

26. $\begin{array}{r} 1.89 \\ \times 86 \\ \hline \end{array}$

29. $\begin{array}{r} 9.25 \\ \times 26 \\ \hline \end{array}$

32. $\begin{array}{r} 16.87 \\ \times 42 \\ \hline \end{array}$

35. $\begin{array}{r} 72.16 \\ \times 15 \\ \hline \end{array}$

What is the price of

36. 6 chairs at \$7.95 each?

38. 17 lawn rakes at \$3.59 each?

37. 26 chairs at \$7.95 each?

39. 42 snow shovels at \$4.86 each?

Decimal fractions in multiplication

Study these examples.

$$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \quad \frac{5}{10} \times \frac{7}{10} = \frac{35}{100}$$

$$.1 \times .1 = .01 \quad .5 \times .7 = .35$$

tenths $\times$ tenths = hundredths

5 tenths $\times$ 7 tenths =
35 hundredths

$$\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000} \quad \frac{5}{10} \times \frac{7}{100} = \frac{35}{1000}$$

$$.1 \times .01 = .001 \quad .5 \times .07 = .035$$

tenths $\times$ hundredths = thousandths

5 tenths $\times$ 7 hundredths =
35 thousandths

EXERCISES

Complete the sentences.

1. 7 tenths $\times$ 4 tenths = 28 ____?
2. 3 tenths $\times$ 8 tenths = 24 ____?
3. 5 tenths $\times$ 3 hundredths = 15 ____?
4. 7 hundredths $\times$ 6 tenths = 42 ____?
5. $.4 \times .8 = 32$ ____?
6. $.9 \times .2 = 18$ ____?
7. $.06 \times .8 = 48$ ____?
8. $.9 \times .03 = 27$ ____?

Complete the sentences.

9. If there is 1 decimal place in each of the 2 factors, there will be ____? decimal places in the product.
10. If there is 1 decimal place in one factor and 2 decimal places in the other factor, there will be ____? decimal places in the product.
11. The number of decimal places in the product is found by ____? the decimal places in each factor.

Match each product on the left with the correct product on the right.

- | | | | | | |
|------------------------|---------|---------|---------|--------|-------|
| 12. $.8 \times .6$: | 48, | 4.8, | .48, | .048, | .0048 |
| 13. $.7 \times .31$: | 2.17, | .217, | 21.7, | 217, | .0217 |
| 14. $.3 \times .24$: | 7.2, | .72, | .072, | 72, | .0072 |
| 15. 24×3.1 : | 744, | .744, | .0744, | 74.4, | 7.44 |
| 16. 1.76×22 : | 3.872, | 38.72, | .3872, | 387.2, | 2872 |
| 17. 63×2.13 : | 134.19, | 13.419, | 1.3419, | 134.9, | 13419 |

Decimal fractions in multiplication



Study these examples.

$$.6 \times .4 = \frac{6}{10} \times \frac{4}{10} \\ = \frac{24}{100} = .24$$

$$\begin{array}{r} .6 \\ \times .4 \\ \hline .24 \end{array}$$

$$.6 \times .28 = \frac{6}{10} \times \frac{28}{100} \\ = \frac{168}{1000} = .168$$

$$\begin{array}{r} .28 \\ \times .6 \\ \hline .168 \end{array}$$

EXERCISES

Name the products as decimal fractions.

1. $5 \times \frac{3}{10}$

5. $\frac{5}{100} \times \frac{3}{10}$

9. $\frac{7}{10} \times \frac{6}{100}$

13. $\frac{9}{10} \times \frac{8}{10}$

2. $3 \times \frac{5}{10}$

6. $6 \times \frac{7}{10}$

10. $\frac{7}{100} \times \frac{6}{10}$

14. $\frac{9}{10} \times \frac{8}{100}$

3. $\frac{3}{10} \times \frac{5}{10}$

7. $7 \times \frac{6}{10}$

11. $8 \times \frac{9}{10}$

15. $\frac{9}{100} \times \frac{8}{10}$

4. $\frac{5}{10} \times \frac{3}{100}$

8. $\frac{7}{10} \times \frac{6}{10}$

12. $9 \times \frac{8}{10}$

16. $9 \times \frac{8}{100}$

Name the products.

17. $5 \times .3$

20. $.5 \times .03$

23. $7 \times .6$

26. $.07 \times .6$

29. $.9 \times .8$

18. $3 \times .5$

21. $.05 \times .3$

24. $.7 \times .6$

27. $8 \times .9$

30. $.9 \times .08$

19. $.3 \times .5$

22. $6 \times .7$

25. $.7 \times .06$

28. $9 \times .8$

31. $.09 \times .8$

Name the products.

32. $\begin{array}{r} .4 \\ \times .3 \\ \hline \end{array}$

34. $\begin{array}{r} .6 \\ \times .4 \\ \hline \end{array}$

36. $\begin{array}{r} .13 \\ \times .2 \\ \hline \end{array}$

38. $\begin{array}{r} .34 \\ \times .6 \\ \hline \end{array}$

40. $\begin{array}{r} .91 \\ \times .8 \\ \hline \end{array}$

42. $\begin{array}{r} .84 \\ \times .2 \\ \hline \end{array}$

33. $\begin{array}{r} .5 \\ \times .8 \\ \hline \end{array}$

35. $\begin{array}{r} .4 \\ \times .9 \\ \hline \end{array}$

37. $\begin{array}{r} .25 \\ \times .7 \\ \hline \end{array}$

39. $\begin{array}{r} .82 \\ \times .5 \\ \hline \end{array}$

41. $\begin{array}{r} .63 \\ \times .6 \\ \hline \end{array}$

43. $\begin{array}{r} .76 \\ \times .3 \\ \hline \end{array}$

Solve the problems.

44. If .6 of the region is red and .8 of the red region is shaded, how much of the region is red and shaded?

45. If .4 of the region is red and .25 of the red region is shaded, how much of the region is red and shaded?

Decimal fractions in multiplication

Follow these steps for naming the product 3.24×4.7

Step 1: Estimate the product. Since 3.24 is about 3 and 4.7 is about 5, the product 3.24×4.7 is about $3 \times 5 = 15$.

$$\begin{array}{r} 3.24 \\ \times 4.7 \\ \hline \end{array}$$

Step 2: Multiply. $324 \times 47 = 15228$

Step 3: Place the decimal point. $.24 \times .7$ would give thousandths, so place the decimal point with 3 digits on its right.

$$\begin{array}{r} 324 \\ \times 47 \\ \hline 2268 \\ 12960 \\ \hline 15228 \end{array}$$

Step 4: Check with the estimate. 15.228 is about 15.

EXERCISES

Complete the sentences.

1. If there are 2 decimal places in the product and one of the factors is tenths, the other factor must be ____.
2. The product of tenths and ____ is thousandths.
3. If there is 1 decimal place in one factor and 3 in the product, there must be ____ places in the other factor.

Estimate, then name the products.

4. $\begin{array}{r} 3.56 \\ \times 2.3 \\ \hline \end{array}$

7. $\begin{array}{r} 5.44 \\ \times 7.5 \\ \hline \end{array}$

10. $\begin{array}{r} 4.63 \\ \times 7.2 \\ \hline \end{array}$

13. $\begin{array}{r} 2.05 \\ \times 9.2 \\ \hline \end{array}$

16. $\begin{array}{r} 2.74 \\ \times 8.3 \\ \hline \end{array}$

5. $\begin{array}{r} 4.17 \\ \times 5.6 \\ \hline \end{array}$

8. $\begin{array}{r} 7.93 \\ \times 7.2 \\ \hline \end{array}$

11. $\begin{array}{r} 5.16 \\ \times 4.2 \\ \hline \end{array}$

14. $\begin{array}{r} 1.17 \\ \times 5.7 \\ \hline \end{array}$

17. $\begin{array}{r} 8.31 \\ \times 4.4 \\ \hline \end{array}$

6. $\begin{array}{r} 6.02 \\ \times 8.1 \\ \hline \end{array}$

9. $\begin{array}{r} 2.37 \\ \times 3.4 \\ \hline \end{array}$

12. $\begin{array}{r} 7.34 \\ \times 3.8 \\ \hline \end{array}$

15. $\begin{array}{r} 3.42 \\ \times 6.1 \\ \hline \end{array}$

18. $\begin{array}{r} 9.32 \\ \times 5.7 \\ \hline \end{array}$

Name the products.

19. 7.81×2.3

21. $85.4 \times .27$

23. 1.11×9.9

25. $20.1 \times .47$

20. 6.94×3.7

22. $82.3 \times .59$

24. $57.2 \times .38$

26. $36.2 \times .07$

Decimal fractions in division

Study these examples.

$$\begin{array}{r} 3.37 \\ 6 \overline{)20.22} \\ \underline{18.00} \\ 2.22 \\ \underline{1.80} \\ .42 \\ \underline{.42} \end{array}$$

Check

$$\begin{array}{l} 3.00 \times 6 = 18.00 \\ .30 \times 6 = 1.80 \\ .07 \times 6 = .42 \\ 3.37 \times 6 = 20.22 \end{array}$$

$$\begin{array}{r} 1.21 \\ 32 \overline{)38.72} \\ \underline{32.00} \\ 6.72 \\ \underline{6.40} \\ .32 \\ \underline{.32} \end{array}$$

Check

$$\begin{array}{l} 1.00 \times 32 = 32.00 \\ .20 \times 32 = 6.40 \\ .01 \times 32 = .32 \\ 1.21 \times 32 = 38.72 \end{array}$$

EXERCISES

Name the quotients.

1. $2 \overline{)2.84}$

4. $6 \overline{)19.32}$

7. $8 \overline{)81.12}$

10. $9 \overline{)62.82}$

2. $7 \overline{)27.23}$

5. $6 \overline{)32.22}$

8. $4 \overline{)72.36}$

11. $8 \overline{)58.48}$

3. $5 \overline{)18.35}$

6. $3 \overline{)10.26}$

9. $8 \overline{)19.28}$

12. $7 \overline{)75.25}$

Name the quotients.

13. $21 \overline{)44.94}$

16. $23 \overline{)25.76}$

19. $54 \overline{)165.78}$

22. $29 \overline{)95.99}$

14. $23 \overline{)75.9}$

17. $84 \overline{)109.2}$

20. $37 \overline{)45.14}$

23. $72 \overline{)85.68}$

15. $21 \overline{)46.62}$

18. $32 \overline{)198.4}$

21. $14 \overline{)114.24}$

24. $83 \overline{)172.64}$

Name each of the quotients with the decimal point in the correct place.

25. $\begin{array}{r} 216 \\ 5 \overline{)10.80} \end{array}$

26. $\begin{array}{r} 726 \\ 3 \overline{)21.78} \end{array}$

27. $\begin{array}{r} 117 \\ 15 \overline{)17.55} \end{array}$

28. $\begin{array}{r} 243 \\ 21 \overline{)51.03} \end{array}$

29. $\begin{array}{r} 357 \\ 24 \overline{)85.68} \end{array}$

Match the quotient on the left with the correct quotient on the right.

30. $17.05 \div 5:$.341, 3.41, 34.1, 34, .00341

31. $69.75 \div 31:$ 2.25, .225, .0225, 22.5, 225.0

32. $18.81 \div 3:$.627, 627, 62.7, 6.27, .0627

33. $87.72 \div 51:$ 172, 1.72, 17.2, .0172, .00172

34. $88.56 \div 24:$ 3.69, 36.9, 369, .0369, .00369

Decimal fractions in division

Study the examples.

$$\begin{array}{r} .069 \\ 7 \overline{) .483} \\ \underline{.420} \\ .063 \\ \underline{.063} \\ 0 \end{array}$$

Check

$$.06 \times 7 = .420$$

$$.009 \times 7 = .063$$

$$.069 \times 7 = .483$$

$$\begin{array}{r} .579 \\ 3 \overline{) 1.737} \\ \underline{1.500} \\ .237 \\ \underline{.210} \\ .027 \\ \underline{.027} \\ 0 \end{array}$$

Check

$$.5 \times 3 = 1.500$$

$$.07 \times 3 = .210$$

$$.009 \times 3 = .027$$

$$.579 \times 3 = 1.737$$

EXERCISES

Name each of the quotients with the decimal point in the correct place.

$$1. \quad 8 \overline{) 1.384} \quad \begin{array}{r} 173 \\ 8 \overline{) 1384} \end{array}$$

$$3. \quad 5 \overline{) 1.860} \quad \begin{array}{r} 372 \\ 5 \overline{) 1860} \end{array}$$

$$5. \quad 3 \overline{) .246} \quad \begin{array}{r} 82 \\ 3 \overline{) 246} \end{array}$$

$$7. \quad 4 \overline{) .268} \quad \begin{array}{r} 67 \\ 4 \overline{) 268} \end{array}$$

$$2. \quad 4 \overline{) 1.076} \quad \begin{array}{r} 269 \\ 4 \overline{) 1076} \end{array}$$

$$4. \quad 6 \overline{) .324} \quad \begin{array}{r} 54 \\ 6 \overline{) 324} \end{array}$$

$$6. \quad 2 \overline{) 1.084} \quad \begin{array}{r} 542 \\ 2 \overline{) 1084} \end{array}$$

$$8. \quad 9 \overline{) .675} \quad \begin{array}{r} 75 \\ 9 \overline{) 675} \end{array}$$

Name the quotients. Check your work.

$$9. \quad 2 \overline{) .234}$$

$$16. \quad 7 \overline{) .889}$$

$$23. \quad 4 \overline{) 3.444}$$

$$30. \quad 6 \overline{) 4.044}$$

$$10. \quad 3 \overline{) .639}$$

$$17. \quad 4 \overline{) .852}$$

$$24. \quad 3 \overline{) 2.376}$$

$$31. \quad 4 \overline{) 1.528}$$

$$11. \quad 2 \overline{) .684}$$

$$18. \quad 9 \overline{) .675}$$

$$25. \quad 5 \overline{) 2.160}$$

$$32. \quad 2 \overline{) 1.234}$$

$$12. \quad 5 \overline{) .410}$$

$$19. \quad 8 \overline{) .344}$$

$$26. \quad 2 \overline{) 1.134}$$

$$33. \quad 3 \overline{) 1.629}$$

$$13. \quad 7 \overline{) .469}$$

$$20. \quad 3 \overline{) .936}$$

$$27. \quad 3 \overline{) 1.182}$$

$$34. \quad 5 \overline{) 2.080}$$

$$14. \quad 7 \overline{) .266}$$

$$21. \quad 7 \overline{) 6.552}$$

$$28. \quad 8 \overline{) 2.096}$$

$$35. \quad 3 \overline{) 2.466}$$

$$15. \quad 2 \overline{) .846}$$

$$22. \quad 8 \overline{) 1.176}$$

$$29. \quad 2 \overline{) 1.562}$$

$$36. \quad 6 \overline{) 4.938}$$

Solve the problems.

37. If .896 inches of rain fell during 7 hours, what was the average amount of rain per hour?

38. If 9.488 inches of snow fell during four weeks, what was the average amount of snow per week?



Study the examples.

$$\begin{array}{r}
 3.37 \\
 6 \overline{)20.26} \\
 \underline{18.00} \\
 2.26 \\
 \underline{1.80} \\
 .46 \quad Q = 3.37 \\
 \underline{.42} \\
 .04 \quad R = .04
 \end{array}$$

$$\begin{array}{r}
 1.21 \\
 32 \overline{)38.87} \\
 \underline{32.00} \\
 6.87 \\
 \underline{6.40} \\
 .47 \quad Q = 1.21 \\
 \underline{.32} \\
 .15 \quad R = .15
 \end{array}$$

EXERCISES

Solve the equations.

1. $23.4 = (6 \times 3.7) + a$

2. $45.6 = (9 \times 4.8) + b$

3. $36.6 = (5 \times 7.3) + c$

4. $46.8 - (8 \times 5.4) = d$

5. $39.5 - (6 \times 6.5) = e$

6. $51.6 - (8 \times 6.4) = f$

7. $38.2 - (9 \times 4.2) = g$

8. $58.1 - (7 \times 7.8) = h$

9. $52.3 = (8 \times 6.3) + i$

10. $62.5 = (7 \times 8.9) + j$

Name the quotients and remainders.

11. $4 \overline{)12.87}$

15. $9 \overline{)14.83}$

19. $24 \overline{)71.48}$

23. $59 \overline{)73.86}$

12. $6 \overline{)43.85}$

16. $8 \overline{)57.14}$

20. $19 \overline{)63.54}$

24. $47 \overline{)98.65}$

13. $7 \overline{)27.09}$

17. $6 \overline{)38.63}$

21. $23 \overline{)42.85}$

25. $38 \overline{)92.41}$

14. $8 \overline{)14.35}$

18. $7 \overline{)83.29}$

22. $63 \overline{)81.74}$

26. $36 \overline{)48.64}$

Name the cost to the nearest dime for one item if

27. 4 items cost \$17.35.

30. 5 items cost \$33.46.

28. 6 items cost \$21.99.

31. 9 items cost \$164.50.

29. 8 items cost \$33.46.

32. 3 items cost \$286.40.

Name the quotients to the nearest tenth.

33. $7 \overline{)59.46}$

34. $8 \overline{)37.25}$

35. $6 \overline{)32.85}$

36. $9 \overline{)43.61}$

Decimal fractions in division

Study the examples.

$$\begin{aligned} 4 \div \frac{2}{10} &= (10 \times 4) \div \left(10 \times \frac{2}{10}\right) & 7 \div \frac{4}{100} &= (100 \times 7) \div \left(100 \times \frac{4}{100}\right) \\ &= 40 \div 2 & &= 700 \div 4 \\ &= 20 & &= 175 \\ 4 \div .2 &= (10 \times 4) \div (10 \times .2) & 7 \div .04 &= (100 \times 7) \div (100 \times .04) \\ &= 40 \div 2 & &= 700 \div 4 \\ &= 20 & &= 175 \end{aligned}$$

EXERCISES

Solve the equations.

1. $3 \div \frac{5}{10} = (10 \times 3) \div \left(10 \times \frac{5}{10}\right) = 30 \div a = b$
2. $6 \div \frac{4}{100} = (100 \times 6) \div \left(100 \times \frac{4}{100}\right) = 600 \div c = d$
3. $5\frac{6}{10} \div \frac{8}{10} = \left(10 \times \frac{56}{10}\right) \div \left(10 \times \frac{8}{10}\right) = 56 \div e = f$
4. $2\frac{1}{100} \div \frac{3}{100} = \left(100 \times \frac{201}{100}\right) \div \left(100 \times \frac{3}{100}\right) = 201 \div g = h$
5. $3 \div .2 = (10 \times 3) \div (10 \times .2) = 30 \div i = j$
6. $5 \div .05 = (100 \times 5) \div (100 \times .05) = 500 \div k = l$
7. $4.8 \div .6 = (10 \times 4.8) \div (10 \times .6) = 48 \div m = n$
8. $2.7 \div .09 = (100 \times 2.7) \div (100 \times .09) = 270 \div p = r$

Name the quotients.

- | | | | |
|------------------|--------------------|--------------------|--------------------|
| 9. $4 \div .2$ | 15. $3.6 \div .06$ | 21. $2.5 \div .05$ | 27. $5.4 \div .09$ |
| 10. $7 \div .2$ | 16. $2.4 \div .4$ | 22. $1.8 \div .3$ | 28. $4.2 \div .6$ |
| 11. $6 \div .04$ | 17. $3.6 \div .09$ | 23. $2.8 \div .07$ | 29. $3.2 \div .4$ |
| 12. $9 \div .6$ | 18. $4.8 \div .8$ | 24. $2.1 \div .07$ | 30. $7.2 \div .08$ |
| 13. $5 \div .02$ | 19. $7.2 \div .09$ | 25. $4.5 \div .05$ | 31. $8.1 \div .09$ |
| 14. $4 \div .5$ | 20. $4.2 \div .7$ | 26. $6.4 \div .8$ | 32. $6.3 \div .7$ |



Study the examples.

$$.7 \overline{)4.2} = 7 \overline{)42} \qquad .41 \overline{)49.2} = 41 \overline{)4920}$$

$$\begin{array}{r} 120 \\ 41 \overline{)4920} \\ \underline{4100} \\ 820 \\ \underline{820} \end{array}$$

To name the quotient $4.2 \div .7$

Step 1: Multiply both dividend and divisor by 10.

Step 2: Complete the division.

To name the quotient $49.2 \div .41$

Step 1: Multiply both dividend and divisor by 100.

Step 2: Complete the division.

EXERCISES

Name the quotients.

$$1. .9 \overline{)3.6} \qquad 5. .8 \overline{)7.2} \qquad 9. .2 \overline{)1.4} \qquad 13. .4 \overline{)3.2} \qquad 17. .6 \overline{)4.2}$$

$$2. .6 \overline{)4.8} \qquad 6. .6 \overline{)3.6} \qquad 10. .4 \overline{)2.8} \qquad 14. .9 \overline{)5.4} \qquad 18. .9 \overline{)8.1}$$

$$3. .9 \overline{)2.7} \qquad 7. .7 \overline{)6.3} \qquad 11. .9 \overline{)7.2} \qquad 15. .3 \overline{)2.7} \qquad 19. .8 \overline{)6.4}$$

$$4. .5 \overline{)4.5} \qquad 8. .9 \overline{)4.5} \qquad 12. .8 \overline{)4.8} \qquad 16. .7 \overline{)3.5} \qquad 20. .4 \overline{)3.6}$$

$$21. .27 \overline{)56.7} \qquad 25. .31 \overline{)37.2} \qquad 29. .11 \overline{)69.3} \qquad 33. .29 \overline{)93.8}$$

$$22. .38 \overline{)121.6} \qquad 26. .23 \overline{)73.6} \qquad 30. .26 \overline{)54.6} \qquad 34. .18 \overline{)75.6}$$

$$23. .15 \overline{)94.5} \qquad 27. .44 \overline{)92.4} \qquad 31. .37 \overline{)77.7} \qquad 35. .42 \overline{)88.2}$$

$$24. .47 \overline{)103.4} \qquad 28. .13 \overline{)40.3} \qquad 32. .52 \overline{)57.2} \qquad 36. .33 \overline{)75.9}$$

Answer the questions.

37. How many .31 inches are there in 96.1 inches?
38. How many .12 inches are there in 64.8 inches?
39. How many .26 inches are there in 59.8 inches?
40. How many .47 inches are there in 61.1 inches?



Working with decimal fractions

Solve the equations.

1. $9 \times a = 8.1$

7. $.90 + .81 = g$

13. $14.6 + 7.8 = m$

2. $9 \times b = .81$

8. $.90 - .81 = h$

14. $16.3 + n = 24$

3. $9 \times 8.1 = c$

9. $4 \times 3.54 = i$

15. $p - 3.6 = 17.2$

4. $9 \times .81 = d$

10. $3.6 \times 4.6 = j$

16. $18.6 - q = 12.9$

5. $8.1 + e = 9$

11. $7.2 \div 8 = k$

17. $4.3 \times 2.3 = r$

6. $9 + 8.1 = f$

12. $7.20 \div 8 = l$

18. $4.3 \times 23 = s$

If you are buying dress material by the yard, name the cost of

19. 3.6 yards at \$5 a yard

25. 8 yards at \$7.30 a yard

20. 2.8 yards at \$6 a yard

26. 10 yards at \$15.40 a yard

21. 7.2 yards at \$4 a yard

27. 3.6 yards at \$5.25 a yard

22. 4.6 yards at \$3 a yard

28. 2.8 yards at \$6.45 a yard

23. 4 yards at \$4.60 a yard

29. 7.2 yards at \$4.45 a yard

24. 6 yards at \$5.60 a yard

30. 6.7 yards at \$8.69 a yard

Name the sums or differences.

31.
$$\begin{array}{r} 4.63 \\ + 7.82 \\ \hline \end{array}$$

33.
$$\begin{array}{r} 16.96 \\ + 24.03 \\ \hline \end{array}$$

35.
$$\begin{array}{r} 18.98 \\ + 2.46 \\ \hline \end{array}$$

37.
$$\begin{array}{r} 13.74 \\ - 6.81 \\ \hline \end{array}$$

39.
$$\begin{array}{r} 41.60 \\ - 23.49 \\ \hline \end{array}$$

41.
$$\begin{array}{r} 11.63 \\ - 1.84 \\ \hline \end{array}$$

32.
$$\begin{array}{r} 8.57 \\ + 9.74 \\ \hline \end{array}$$

34.
$$\begin{array}{r} 43.55 \\ + 16.73 \\ \hline \end{array}$$

36.
$$\begin{array}{r} 13.87 \\ + 8.13 \\ \hline \end{array}$$

38.
$$\begin{array}{r} 39.73 \\ - 9.59 \\ \hline \end{array}$$

40.
$$\begin{array}{r} 27.21 \\ - 18.04 \\ \hline \end{array}$$

42.
$$\begin{array}{r} 72.25 \\ - 71.75 \\ \hline \end{array}$$

Name the products or quotients.

43.
$$\begin{array}{r} 4.86 \\ \times 7 \\ \hline \end{array}$$

45.
$$\begin{array}{r} 3.94 \\ \times 23 \\ \hline \end{array}$$

47.
$$\begin{array}{r} 6.83 \\ \times 45 \\ \hline \end{array}$$

49.
$$\begin{array}{r} 37.4 \\ \times 16 \\ \hline \end{array}$$

51.
$$\begin{array}{r} 28.46 \\ \times 93 \\ \hline \end{array}$$

53.
$$\begin{array}{r} 2.8 \\ \times 4.6 \\ \hline \end{array}$$

44.
$$\begin{array}{r} 3.42 \\ \times 9 \\ \hline \end{array}$$

46.
$$\begin{array}{r} 2.39 \\ \times 13 \\ \hline \end{array}$$

48.
$$\begin{array}{r} 7.75 \\ \times 62 \\ \hline \end{array}$$

50.
$$\begin{array}{r} 29.3 \\ \times 18 \\ \hline \end{array}$$

52.
$$\begin{array}{r} 27.25 \\ \times 45 \\ \hline \end{array}$$

54.
$$\begin{array}{r} 4.8 \\ \times 3.7 \\ \hline \end{array}$$

55. $7 \overline{)9.52}$

58. $.9 \overline{)4.68}$

61. $.14 \overline{)9.94}$

64. $62 \overline{)83.85}$

67. $37 \overline{)71.96}$

56. $.8 \overline{)5.28}$

59. $12 \overline{)73.2}$

62. $11 \overline{)25.31}$

65. $46 \overline{)51.07}$

68. $26 \overline{)83.71}$

57. $.4 \overline{)8.36}$

60. $23 \overline{)39.1}$

63. $.36 \overline{).792}$

66. $73 \overline{)270.1}$

69. $62 \overline{)303.8}$



Name the sums.

$$\begin{array}{r} 1. \quad 22 \\ 47 \\ 68 \\ 30 \\ 91 \\ 75 \\ + 42 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 317 \\ 205 \\ 611 \\ 940 \\ 582 \\ 761 \\ + 842 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 9546 \\ 7283 \\ 4756 \\ 3009 \\ 6471 \\ + 5018 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 10651 \\ 9374 \\ 35069 \\ 2001 \\ 18567 \\ + 45983 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 147020 \\ 357692 \\ 101398 \\ 475611 \\ 98601 \\ + 627418 \\ \hline \end{array}$$

Name the differences.

$$\begin{array}{r} 6. \quad 307 \\ - 199 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 685 \\ - 237 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 3976 \\ - 2834 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 59123 \\ - 27654 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 80000 \\ - 56073 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 839200 \\ - 647832 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 647302 \\ - 480613 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 701235 \\ - 594867 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 3452764 \\ - 1619382 \\ \hline \end{array}$$

Name the products.

$$\begin{array}{r} 15. \quad 37 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 59426 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 23 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 37954 \\ \times 217 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 576 \\ \times 8 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 760961 \\ \times 25 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 395 \\ \times 72 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 2130762 \\ \times 35 \\ \hline \end{array}$$

Name the quotients.

$$23. \quad 3 \overline{)5761}$$

$$24. \quad 53 \overline{)8456}$$

$$25. \quad 47 \overline{)10239}$$

$$26. \quad 237 \overline{)396214}$$

Name the sums or differences.

$$\begin{array}{r} 27. \quad \frac{1}{3} \\ + \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 3\frac{1}{2} \\ - 2\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 5\frac{1}{8} \\ + 3\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 4\frac{3}{4} \\ + 5\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 31. \quad 2\frac{3}{10} \\ + 4\frac{2}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 32. \quad 4\frac{1}{3} \\ - 1\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 33. \quad 23\frac{13}{15} \\ - 7\frac{2}{3} \\ \hline \end{array}$$

Name the products or quotients.

$$34. \quad \frac{2}{3} \times 24$$

$$36. \quad \frac{3}{4} \div \frac{1}{2}$$

$$38. \quad \frac{1}{2} \div \frac{3}{2}$$

$$40. \quad 6\frac{1}{7} \times 2\frac{1}{3}$$

$$42. \quad 2\frac{2}{5} \div \frac{3}{10}$$

$$35. \quad \frac{5}{6} \times \frac{2}{3}$$

$$37. \quad \frac{6}{7} \times \frac{14}{3}$$

$$39. \quad 2\frac{5}{6} \times 3\frac{1}{4}$$

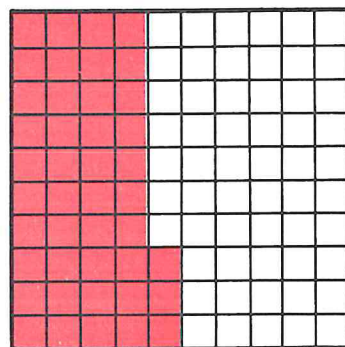
$$41. \quad 1\frac{5}{6} \div \frac{5}{8}$$

$$43. \quad 3\frac{2}{7} \times 2\frac{5}{9}$$



Per cent

$\frac{43}{100} = .43$ of the region is red. 43% of the region is red. $\frac{43}{100}$, .43, and 43% are different names for the same number. 43% is read "43 per cent." (Per cent means per hundred.) The per cent sign, %, is a numeral for the fraction $\frac{1}{100}$.



$$43\% = 43 \times \frac{1}{100} = \frac{43}{100} = .43$$

EXERCISES

Name the fractions as per cents.

1. $\frac{47}{100}$
3. $\frac{72}{100}$
5. $\frac{24}{100}$
7. $\frac{9}{100}$
9. $\frac{2}{100}$
11. $\frac{75}{100}$
2. $\frac{56}{100}$
4. $\frac{18}{100}$
6. $\frac{61}{100}$
8. $\frac{27}{100}$
10. $\frac{33}{100}$
12. $\frac{1}{100}$

Name the per cents as decimal fractions.

13. 15%
15. 62%
17. 71%
19. 9%
21. 58%
23. 5%
14. 36%
16. 83%
18. 46%
20. 45%
22. 6%
24. 100%

Name the decimals as per cents.

25. .23
27. .56
29. .82
31. .05
33. .60
35. .08
26. .48
28. .39
30. .75
32. .50
34. .80
36. .92

Complete the chart.

37.

fraction	$\frac{26}{100}$	$\frac{53}{100}$	$\frac{78}{100}$	u	v	x	$\frac{48}{100}$	i	n	$\frac{67}{100}$
decimal fraction	.26	.53	t	.39	h	.91	k	.09	w	d
per cent	26%	a	c	e	84%	s	m	z	4%	r

Complete the sentences.

38. In a per cent, the fraction $\frac{1}{100}$ is shown by the sign ____?
39. A per cent is a fraction with a denominator of ____?

Per cents in addition and subtraction

Study the
examples.

$$\begin{array}{r} .35 \\ + .49 \\ \hline .84 \end{array}$$

$$\begin{array}{r} 35\% \\ + 49\% \\ \hline 84\% \end{array}$$

$$\begin{array}{r} .74 \\ - .56 \\ \hline .18 \end{array}$$

$$\begin{array}{r} 74\% \\ - 56\% \\ \hline 18\% \end{array}$$

EXERCISES

Name the sums or differences.

1. $\begin{array}{r} 9\% \\ + 7\% \\ \hline \end{array}$

4. $\begin{array}{r} 21\% \\ - 16\% \\ \hline \end{array}$

7. $\begin{array}{r} 32\% \\ + 9\% \\ \hline \end{array}$

10. $\begin{array}{r} 42\% \\ - 37\% \\ \hline \end{array}$

13. $\begin{array}{r} 48\% \\ + 26\% \\ \hline \end{array}$

16. $\begin{array}{r} 57\% \\ + 46\% \\ \hline \end{array}$

2. $\begin{array}{r} 14\% \\ + 8\% \\ \hline \end{array}$

5. $\begin{array}{r} 45\% \\ + 10\% \\ \hline \end{array}$

8. $\begin{array}{r} 31\% \\ - 12\% \\ \hline \end{array}$

11. $\begin{array}{r} 62\% \\ - 15\% \\ \hline \end{array}$

14. $\begin{array}{r} 39\% \\ - 21\% \\ \hline \end{array}$

17. $\begin{array}{r} 61\% \\ - 29\% \\ \hline \end{array}$

3. $\begin{array}{r} 8\% \\ - 3\% \\ \hline \end{array}$

6. $\begin{array}{r} 73\% \\ - 67\% \\ \hline \end{array}$

9. $\begin{array}{r} 36\% \\ + 36\% \\ \hline \end{array}$

12. $\begin{array}{r} 97\% \\ - 48\% \\ \hline \end{array}$

15. $\begin{array}{r} 78\% \\ + 9\% \\ \hline \end{array}$

18. $\begin{array}{r} 42\% \\ + 57\% \\ \hline \end{array}$

Study this equation : $60\% + 40\% = \frac{60}{100} + \frac{40}{100} = \frac{100}{100} = 100\% = 1$.
Solve the equations.

19. $80\% + 20\% = a\% = b$

22. $50\% + 50\% = f$

20. $90\% + 10\% = c\% = d$

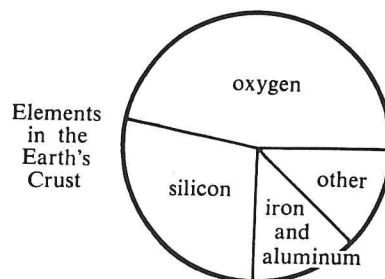
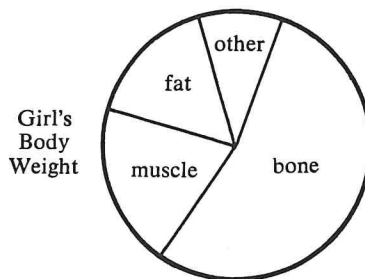
23. $35\% + 65\% = g$

21. $30\% + 70\% = e$

24. $45\% + 55\% = h$

Solve the problems.

25. If fat and muscle make up 36% of a girl's body by weight, and you know that 20% is muscle, name the per cent of a girl's body that is fat.
26. If bone and fat together make up 70% of a girl's body by weight and you know that 16% is fat, name the per cent of a girl's body that is bone.
27. If oxygen and silicon make up 75% of the earth's crust, and you know that 28% is silicon, name the per cent of the earth's crust that is oxygen.
28. If 41% of the earth's crust is iron, aluminum, and silicon, and you know that 28% is silicon, name the per cent of the earth's crust that is iron and aluminum.





Common fractions and per cents

Study the equations.

$$\begin{aligned}\frac{3}{4} &= \frac{3}{4} \times 1 \\ &= \frac{3}{4} \times 100\% \\ &= \left(\frac{3}{4} \times 100\right)\% \\ &= 75\%\end{aligned}$$

$$\begin{aligned}75\% &= \frac{75}{100} \\ &= \frac{3 \times 25}{4 \times 25} \\ &= \frac{3}{4}\end{aligned}$$

EXERCISES

Solve the equations.

$$\begin{aligned}1. \quad \frac{1}{5} &= \frac{1}{5} \times 1 \\ &= \frac{1}{5} \times a\% \\ &= \left(\frac{1}{5} \times b\right)\% \\ &= c\%\end{aligned}$$

$$\begin{aligned}3. \quad \frac{1}{2} &= \frac{1}{2} \times 1 \\ &= \frac{1}{2} \times g\% \\ &= \left(\frac{1}{2} \times h\right)\% \\ &= i\%\end{aligned}$$

$$\begin{aligned}5. \quad \frac{9}{20} &= \frac{9}{20} \times 1 \\ &= \frac{9}{20} \times m\% \\ &= \left(\frac{9}{20} \times n\right)\% \\ &= s\%\end{aligned}$$

$$\begin{aligned}2. \quad \frac{1}{10} &= \frac{1}{10} \times 1 \\ &= \frac{1}{10} \times d\% \\ &= \left(\frac{1}{10} \times e\right)\% \\ &= f\%\end{aligned}$$

$$\begin{aligned}4. \quad \frac{2}{5} &= \frac{2}{5} \times 1 \\ &= \frac{2}{5} \times j\% \\ &= \left(\frac{2}{5} \times k\right)\% \\ &= l\%\end{aligned}$$

$$\begin{aligned}6. \quad \frac{11}{25} &= \frac{11}{25} \times 1 \\ &= \frac{11}{25} \times p\% \\ &= \left(\frac{11}{25} \times q\right)\% \\ &= r\%\end{aligned}$$

Name the fractional numbers as per cents.

7. $\frac{1}{20}$

10. $\frac{4}{5}$

13. $\frac{8}{10}$

16. $\frac{7}{20}$

19. $\frac{3}{25}$

22. $\frac{7}{50}$

25. $\frac{23}{50}$

8. $\frac{1}{25}$

11. $\frac{3}{10}$

14. $\frac{9}{10}$

17. $\frac{9}{20}$

20. $\frac{7}{25}$

23. $\frac{9}{50}$

26. $\frac{25}{50}$

9. $\frac{3}{5}$

12. $\frac{7}{10}$

15. $\frac{3}{20}$

18. $\frac{19}{20}$

21. $\frac{1}{50}$

24. $\frac{17}{50}$

27. $\frac{1}{4}$

Name the per cents as fractional numbers.

28. 10%

31. 40%

34. 70%

37. 35%

40. 65%

29. 20%

32. 50%

35. 15%

38. 45%

41. 85%

30. 30%

33. 60%

36. 25%

39. 55%

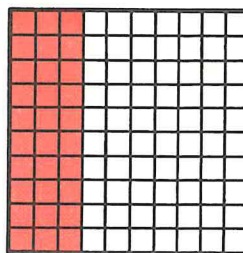
42. 95%

$\frac{3}{10}$ of the region is shaded. $\frac{30}{100}$ of the region is

shaded. $.30 = .3$ of the region is shaded.

30% of the region is shaded.

$\frac{3}{10}$, $\frac{30}{100}$, $.3$, and 30% are **equivalent fractions**.



EXERCISES

Study and complete the chart.

	fraction in simplest form	fractions as hundredths	decimal fraction	per cent
1.	$\frac{1}{2}$	$\frac{1}{2} \times \frac{50}{50} = \frac{50}{100}$	$\frac{50}{100} = .50$ $= .5$	$\frac{50}{100} = 50\%$
2.	$\frac{1}{4}$	$\frac{1}{4} \times \frac{25}{25} = \frac{25}{100}$	$\frac{25}{100} = a$	$\frac{25}{100} = c$
3.	$\frac{1}{5}$	$\frac{1}{5} \times \frac{c}{c} = \frac{20}{100}$	$\frac{20}{100} = d$	$\frac{20}{100} = e$
4.	$\frac{3}{4}$	$\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$	$\frac{75}{100} = f$	$\frac{75}{100} = m$
5.	$\frac{3}{5}$	$\frac{3}{5} \times \frac{20}{20} = \frac{a}{100}$	$\frac{60}{100} = b$	$\frac{60}{100} = n$
6.	$\frac{1}{10}$	$\frac{1}{10} \times \frac{10}{10} = \frac{10}{100}$	$\frac{10}{100} = j$	$\frac{10}{100} = r$
7.	$\frac{7}{10}$	$\frac{7}{10} \times \frac{10}{10} = \frac{n}{100}$	$\frac{70}{100} = m$	$\frac{70}{100} = s$
8.	$\frac{1}{20}$	$\frac{1}{20} \times \frac{n}{n} = \frac{e}{100}$	$\frac{5}{100} = q$	$\frac{5}{100} = u$
9.	$\frac{3}{20}$	$\frac{3}{20} \times \frac{5}{5} = \frac{s}{100}$	$\frac{15}{100} = t$	$\frac{15}{100} = v$
10.	$\frac{13}{20}$	$\frac{13}{20} \times \frac{5}{5} = \frac{v}{100}$	$\frac{65}{100} = .65$	$\frac{65}{100} = w$
11.	$\frac{1}{25}$	$\frac{1}{25} \times \frac{v}{v} = \frac{s}{100}$	$\frac{4}{100} = r$	$\frac{4}{100} = z$



Naming a per cent of a number

Study these equations.

$$\begin{aligned}\frac{3}{4} \times 400 &= \left(3 \times \frac{1}{4}\right) \times 400 \\ &= 3 \times \left(\frac{1}{4} \times 400\right) \\ &= 3 \times 100 = 300\end{aligned}$$

$$\begin{aligned}75\% \text{ of } 400 &= \left(75 \times \frac{1}{100}\right) \times 400 \\ &= 75 \times \left(\frac{1}{100} \times 400\right) \\ &= 75 \times 4 = 300\end{aligned}$$

EXERCISES

Solve the equations.

1. $\frac{3}{4} \times 12 = a$

6. $\frac{4}{5} \times 20 = f$

11. $28\% \text{ of } 100 = k$

2. $\frac{3}{4} \times 16 = b$

7. $12\% \text{ of } 300 = g$

12. $48\% \text{ of } 200 = l$

3. $15\% \text{ of } 200 = c$

8. $14\% \text{ of } 400 = h$

13. $36\% \text{ of } 300 = m$

4. $35\% \text{ of } 200 = d$

9. $18\% \text{ of } 200 = i$

14. $27\% \text{ of } 400 = n$

5. $\frac{2}{5} \times 10 = e$

10. $17\% \text{ of } 100 = j$

15. $38\% \text{ of } 400 = p$

Study the equations.

$$\begin{aligned}25\% \text{ of } 48 &= 25\% \times 48 \\ &= \frac{1}{4} \times 48 \\ &= 12\end{aligned}$$

$$\begin{aligned}75\% \text{ of } 48 &= \frac{3}{4} \times 48 \\ &= \left(3 \times \frac{1}{4}\right) \times 48 \\ &= 3 \times \left(\frac{1}{4} \times 48\right) \\ &= 3 \times 12 = 36\end{aligned}$$

Solve the equations.

16. $25\% \text{ of } 16 = a$

23. $50\% \text{ of } 80 = h$

30. $20\% \text{ of } 30 = p$

17. $25\% \text{ of } 32 = b$

24. $75\% \text{ of } 16 = i$

31. $20\% \text{ of } 60 = q$

18. $25\% \text{ of } 64 = c$

25. $75\% \text{ of } 32 = j$

32. $40\% \text{ of } 15 = r$

19. $25\% \text{ of } 80 = d$

26. $75\% \text{ of } 64 = k$

33. $40\% \text{ of } 25 = s$

20. $50\% \text{ of } 16 = e$

27. $75\% \text{ of } 80 = l$

34. $40\% \text{ of } 35 = t$

21. $50\% \text{ of } 32 = f$

28. $20\% \text{ of } 15 = m$

35. $40\% \text{ of } 60 = u$

22. $50\% \text{ of } 64 = g$

29. $20\% \text{ of } 25 = n$

36. $65\% \text{ of } 60 = v$



Study these methods.

A

$$25\% \text{ of } 28 = \frac{1}{4} \times 28 = 7$$

$$\begin{aligned} 75\% \text{ of } 28 &= \frac{3}{4} \times 28 \\ &= 3 \times \left(\frac{1}{4} \times 28\right) \\ &= 21 \end{aligned}$$

B

$$24\% \text{ of } 400 = .24 \times 400 = 96$$

$$\begin{aligned} 24\% \text{ of } 40 &= .24 \times 40 \\ &= 9.6 \end{aligned}$$

EXERCISES

Use method A to solve the equations.

- | | | |
|--------------------------|-------------------------|---------------------------|
| 1. 25% of 36 = <i>a</i> | 5. 10% of 90 = <i>e</i> | 9. 75% of 36 = <i>i</i> |
| 2. 25% of 96 = <i>b</i> | 6. 20% of 90 = <i>f</i> | 10. 75% of 96 = <i>j</i> |
| 3. 25% of 352 = <i>c</i> | 7. 60% of 90 = <i>g</i> | 11. 75% of 352 = <i>k</i> |
| 4. 25% of 684 = <i>d</i> | 8. 90% of 90 = <i>h</i> | 12. 75% of 684 = <i>l</i> |

Use method B to solve the equations.

- | | | |
|---------------------------|---------------------------|---------------------------|
| 13. 12% of 400 = <i>a</i> | 18. 17% of 300 = <i>f</i> | 23. 45% of 80 = <i>k</i> |
| 14. 16% of 400 = <i>b</i> | 19. 42% of 300 = <i>g</i> | 24. 65% of 80 = <i>l</i> |
| 15. 28% of 400 = <i>c</i> | 20. 57% of 300 = <i>h</i> | 25. 14% of 120 = <i>m</i> |
| 16. 63% of 400 = <i>d</i> | 21. 15% of 80 = <i>i</i> | 26. 14% of 150 = <i>n</i> |
| 17. 15% of 300 = <i>e</i> | 22. 35% of 80 = <i>j</i> | 27. 36% of 140 = <i>p</i> |

Solve the word problems.

28. If 24% of the students in Phil's school come by bus, 14% come by subway, and 32% by train, what per cent come by other means?
29. If there are 700 children in Phil's school, use the per cents in exercise 28 to name the number of children who come by
- a. bus, b. subway, c. train, d. other means.

Notice that $\frac{1}{3} \times 100 = 33\frac{1}{3}$, so $33\frac{1}{3}\% = \frac{1}{3}$.

Name _____

- | | | |
|-----------------------------|------------------------------|-----------------------------|
| 30. $33\frac{1}{3}\%$ of 24 | 31. $33\frac{1}{3}\%$ of 300 | 32. $33\frac{1}{3}\%$ of 60 |
|-----------------------------|------------------------------|-----------------------------|

Per cents

We see per cents in newspapers, on television and in many other instances.

A reduction on the marked price (sometimes called a discount) may be shown as a per cent of the marked price.



To name 10% of \$8, we think $\frac{1}{10}$ of 800 = 80 or $.10 \times 800 = 80$. The amount of the reduction is 80¢.

EXERCISES

Name the amounts to complete the charts.

1.

marked price	\$10	\$12	\$16	\$18	\$21	\$24	\$30	\$40
rate of reduction as a % of the marked price	15%	10%	10%	10%	10%	25%	10%	15%
amount of reduction	\$1.50	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>

2.

marked price	\$12	\$15	\$18	\$27	\$40	\$30	\$64	\$50	<i>h</i>
rate of reduction as a % of the marked price	75%	<i>a</i>	50%	$33\frac{1}{3}\%$	<i>d</i>	<i>e</i>	25%	<i>g</i>	10%
amount of reduction	\$9	\$12	<i>b</i>	<i>c</i>	\$30	\$20	<i>f</i>	\$40	\$9

Solve the problems.

3. A girl's blouse was marked at \$8. If there was a 20% discount, what was the selling price of the blouse?

4. A baseball bat that usually sells for \$6.70 is reduced 10%. What is the amount of reduction? How much does the bat sell for after the reduction?

Profit and loss

If a shirt cost the storekeeper \$5 and was sold for \$6, the **profit** was \$1.

If a shirt cost the storekeeper \$10 and was sold for \$9, the **loss** was \$1.

Profit and loss may be shown as a per cent of the cost.

$$\begin{aligned}\frac{\text{profit on shirt}}{\text{cost of shirt}} &= \frac{1}{5} \\ &= 20\%\end{aligned}$$

$$\begin{aligned}\frac{\text{loss on shirt}}{\text{cost of shirt}} &= \frac{1}{10} \\ &= 10\%\end{aligned}$$

EXERCISES

Name the amounts to complete the chart.

1.

cost price	\$10	\$10	\$20	\$30	\$40	\$12	\$16	\$18	\$25
profit	20%	10%	15%	20%	<i>n</i>	<i>e</i>	25%	33 $\frac{1}{3}$ %	20%
selling price	\$12	<i>a</i>	<i>m</i>	<i>c</i>	\$60	\$15	<i>r</i>	<i>s</i>	<i>u</i>

Name the amounts to complete the chart.

2.

cost price	\$10	\$10	\$20	\$25	\$30	\$8	\$15	\$18	\$75
loss	20%	10%	25%	20%	<i>n</i>	<i>e</i>	33 $\frac{1}{3}$ %	33 $\frac{1}{3}$ %	20%
selling price	\$8	<i>a</i>	<i>m</i>	<i>c</i>	\$15	\$6	<i>r</i>	<i>s</i>	<i>u</i>

Name the *profit* or *loss* as a per cent of the *cost price*.

3.

cost price	\$10	\$15	\$10	\$25	\$25	\$30	\$75	\$60	\$50
sale price	\$8	\$18	\$12	\$30	\$20	\$40	\$100	\$40	\$60
amount of profit or loss	\$2	<i>a</i>	<i>c</i>	<i>e</i>	<i>m</i>	<i>n</i>	<i>r</i>	<i>s</i>	<i>u</i>
profit or loss as a % of the cost price	$\frac{2}{10} = 20\%$	<i>v</i>	<i>x</i>	<i>z</i>	<i>i</i>	<i>t</i>	<i>h</i>	<i>k</i>	<i>w</i>

● Using sets and fractional numbers

1. If the rainfall is 3.9 inches in March, 2.7 inches in April, and 3.3 inches in May, what is the total rainfall for these 3 months?

2. At 9 a.m. the temperature was 49.7 degrees. At 1 p.m. it was 56.4 degrees. How many degrees warmer was it at 1 p.m. than at 9 a.m.?

3. If a man rides a bicycle at 8.75 miles an hour, how far does he travel in 3 hours?

4. The longer side of a rectangle is 4.6 inches long, the shorter side is 3.7 inches long. Name the perimeter.

5. The lengths of 6 line segments are 3.6, 5.8, 9.4, 4.8, 6.3, and 4.3 inches. What is the average of these lengths?

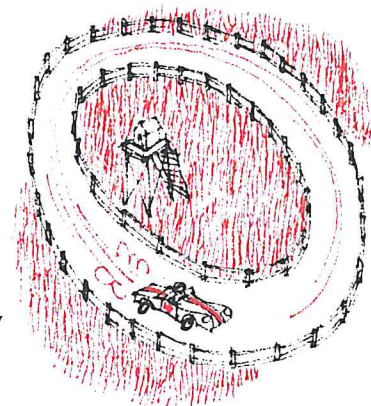
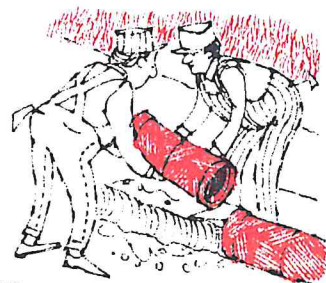
6. If 3.5 inches of pipe are cut from a 11.2 inch length of pipe, what length of pipe is left?

7. If the perimeter of a square is 34.4 feet, name the length of the sides of the square.

8. Name the area of a square region with sides of length 3.7 inches.

9. Name the volume of a cube with sides of length 4.9 inches.

10. If a racing car travels 12.5 miles in 5 minutes, how many miles per minute is that?





Name the fractions in decimal form.

1. $\frac{3}{10}$ 2. $1\frac{7}{10}$ 3. $\frac{35}{100}$ 4. $\frac{807}{1000}$ 5. $\frac{23}{1000}$ 6. $\frac{826}{1000}$

Rename the decimal fractions as fractions.

7. .08 8. .92 9. .902 10. .14 11. .069 12. .726

Rename the decimal fractions as expanded fractions.

13. .06 14. .26 15. .192 16. .105 17. .938 18. .007

Name the sums or differences.

- | | | | | |
|---|---|---|---|---|
| 19. $\begin{array}{r} .3 \\ + .5 \\ \hline \end{array}$ | 21. $\begin{array}{r} .39 \\ + .48 \\ \hline \end{array}$ | 23. $\begin{array}{r} 6.47 \\ + 2.03 \\ \hline \end{array}$ | 25. $\begin{array}{r} 5.28 \\ - 3.69 \\ \hline \end{array}$ | 27. $\begin{array}{r} 11.51 \\ + 13.79 \\ \hline \end{array}$ |
| 20. $\begin{array}{r} .67 \\ - .12 \\ \hline \end{array}$ | 22. $\begin{array}{r} .80 \\ - .37 \\ \hline \end{array}$ | 24. $\begin{array}{r} 8.76 \\ - 7.49 \\ \hline \end{array}$ | 26. $\begin{array}{r} 7.49 \\ + 3.82 \\ \hline \end{array}$ | 28. $\begin{array}{r} 23.41 \\ - 6.95 \\ \hline \end{array}$ |

Name the products or quotients.

- | | | | | |
|--|---|--|---|--|
| 29. $\begin{array}{r} .5 \\ \times .7 \\ \hline \end{array}$ | 30. $\begin{array}{r} .29 \\ \times .6 \\ \hline \end{array}$ | 31. $\begin{array}{r} .19 \\ \times 1.7 \\ \hline \end{array}$ | 32. $\begin{array}{r} 3.56 \\ \times .22 \\ \hline \end{array}$ | 33. $\begin{array}{r} 7.95 \\ \times 2.82 \\ \hline \end{array}$ |
| 34. $3 \overline{)16.41}$ | 35. $14 \overline{)21.82}$ | 36. $37 \overline{)92.89}$ | 37. $83 \overline{)127.65}$ | |

Name the fractions as decimals in per cent form.

38. $\frac{4}{5}$ 39. $\frac{23}{100}$ 40. $\frac{3}{8}$ 41. .07 42. .92 43. .60

Name decimal fractions for the per cents.

44. 6% 45. 18% 46. 20% 47. 57% 48. 100% 49. 500%

Solve the equations.

- | | | |
|--------------------------------|----------------------|-----------------------|
| 50. $\frac{2}{3} \times 9 = a$ | 53. 50% of 60 = d | 56. 65% of 70 = g |
| 51. 15% of 200 = b | 54. 42% of 200 = e | 57. 10% of \$22 = h |
| 52. 75% of 32 = c | 55. 14% of 130 = f | 58. 15% of \$80 = i |



LOOKING BACK

.578 means 5 tenths, 7 hundredths, and 8 thousandths and can be written as an expanded fraction: $.578 = \frac{5}{10} + \frac{7}{100} + \frac{8}{1000}$.

Rename the decimal fractions as expanded fractions.

1. .942

2. .29

3. .370

4. .481

5. .706

Expanded numerals may be used to name the sum or differences of decimal fractions.

$$\begin{array}{r} 5.4 = 5 + .4 \\ + 3.9 = 3 + .9 \\ \hline 8 + 1.3 = 9.3 \end{array}$$

$$\begin{array}{r} 5.4 = 5 + .4 \\ - 3.9 = 3 - .9 \\ \hline 1 + .5 = 1.5 \end{array}$$

Name the sums or differences.

6. $\begin{array}{r} 3.27 \\ + 5.61 \\ \hline \end{array}$

7. $\begin{array}{r} 6.09 \\ + 3.82 \\ \hline \end{array}$

8. $\begin{array}{r} 7.15 \\ - 5.37 \\ \hline \end{array}$

9. $\begin{array}{r} 16.89 \\ - 4.99 \\ \hline \end{array}$

10. $\begin{array}{r} 8.76 \\ + 2.08 \\ \hline \end{array}$

11. $\begin{array}{r} 20.04 \\ - 16.68 \\ \hline \end{array}$

To solve the equation $3.7 \times 5.16 = n$, name the product 37×516 , then place the decimal in the correct place.

Remember:

tenths $\times$ tenths = hundredths

tenths $\times$ hundredths = thousandths

$$\begin{array}{r} 516 \\ \times 37 \\ \hline 3612 \\ 15480 \\ \hline 19092 \end{array}$$

$$3.7 \times 5.16 = 19.092$$

Name the products.

12. $\begin{array}{r} .6 \\ \times .9 \\ \hline \end{array}$

13. $\begin{array}{r} 3.7 \\ \times 4 \\ \hline \end{array}$

14. $\begin{array}{r} 6.8 \\ \times .7 \\ \hline \end{array}$

15. $\begin{array}{r} 5.29 \\ \times .6 \\ \hline \end{array}$

16. $\begin{array}{r} 7.64 \\ \times 1.3 \\ \hline \end{array}$

17. $\begin{array}{r} 9.23 \\ \times 5.4 \\ \hline \end{array}$

The per cent sign means "per hundred."

$$8\% = \frac{8}{100} = .08$$

$$37\% = \frac{37}{100} = .37$$

Solve the equations.

18. $\frac{1}{2} = a\%$

19. $\frac{7}{100} = .b$

20. $\frac{32}{100} = c\%$

21. $.54 = d\%$

22. $\frac{1}{3} = e\%$



Quite frequently, scientists and engineers write numbers in *scientific notation*. These numbers are expressed as a pair of factors—one factor a decimal fraction between 1 and 10 and the other factor a power of 10; for example

$$\begin{aligned} 53 &= 5.3 \times 10 \\ 63,800 &= 6.38 \times 10^4 \\ 93,870,000 &= 9.387 \times 10^7 \end{aligned}$$

Name the numbers using scientific notation.

- | | | | |
|--------|---------|-----------|----------------|
| 1. 35 | 3. 5730 | 5. 30,500 | 7. 291,600 |
| 2. 854 | 4. 6984 | 6. 93,450 | 8. 735,000,000 |

Just For Fun

Try this on your friends. Ask them to think of a number from 10 to 17 and point out the columns in which the number can be found.

1	2	3	4	5	6	7	8
10	11	10	11	11	13	16	14
12	14	15	14	13	17	15	16
16	13	17	12	15	10	12	17

If they say second, fourth and eighth column you say 14. If they say first, fourth and seventh, you say 12. Can you see why?

Here is another game of the same kind.

1	2	3	4
8	4	2	1
9	5	3	3
10	6	6	5
11	7	7	7
12	12	10	9
13	13	11	11
14	14	14	13
15	15	15	15

This chart is made up by using base 2 numerals. Can you see how?

 The possible and the impossible

We know we cannot name a boy under 14 who has run a mile in less than 4 minutes. On the other hand, we can name many men who have run a mile in less than 4 minutes. When we know the set from which we can make our choice (*our replacement set*), we know whether we can find a solution or not.

EXERCISES

Write *yes* or *no* to show whether you can answer the question from the given replacement set.

1. Name an animal that can leap over 10 feet. {elephants, crocodiles, hippopotami}
2. Name a liquid that can be drunk. {milk, water, bread, potatoes, soda}
3. Name a sport played with a round ball. {football, hockey, baseball, basketball}
4. Name a food that can be eaten. {oranges, candy, apples, celery}
5. Name an animal that has hair. {snakes, lizards, turtles}
6. Name a number greater than 7. {1, 2, 3, 4, 5}
7. Name an even number. {0, 2, 4, 6, 8}
8. Name a red fruit that can be eaten. {orange, peach, pear}
9. Name a four-legged animal. {chicken, bird, giraffe}
10. Name a prime number greater than 11. {2, 3, 5, 7, . . . }
11. Name an even number less than 10. {16, 18, 24, 32, 36}
12. Name a number less than 10 that can be divided by 4. {1, 3, 5, 7, 9}
- 13–19. Some of the questions in exercises 1–12 cannot be answered from the given replacement sets. Name objects to change the sets so the questions can be answered from the new sets.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

$$3 + 4 = 7 \leftarrow \text{whole numbers}$$

$$2 \times 5 = 10 \leftarrow \text{whole numbers}$$

The answer for the addition or multiplication of two whole numbers is *always* a whole number. We say that the set of whole numbers is **closed for addition and multiplication**.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

$$6 \div 2 = 3 \leftarrow \text{whole number}$$

$$3 \div 4 = \frac{3}{4} \leftarrow \text{not a whole number}$$

When one whole number is divided by another whole number the quotient is *not always* a whole number. We say that the set of whole numbers is **not closed for division**.

$$B = \{1, 3, 5, 7, 9, \dots\}$$

$$3 + 5 = 8 \leftarrow \text{even number} \quad 7 \times 9 = 63 \leftarrow \text{odd number}$$

The sum of 2 *odd* numbers is an *even*, not an odd number, so the set of odd numbers is **not closed for addition**. The product of 2 odd numbers is an odd number, so the set of odd numbers is **closed for multiplication**.

EXERCISES

Write examples to show that

- the sum of 2 even numbers is always an even number.
- the product of 2 even numbers is always an even number.
- the product of 2 primes is not a prime.
- the quotient of 2 whole numbers is not always a whole number.

For each set there is an operation shown. Write *C* or *NC* to show whether the set is *Closed* or *Not Closed* for the operation.

	set	operation
5.	{prime numbers} = {2, 3, 5, 7, 11, 13, ...}	addition
6.	{even numbers} = {0, 2, 4, 6, 8, 10, ...}	multiplication
7.	{whole numbers} = {0, 1, 2, 3, 4, 5, ...}	subtraction
8.	{odd numbers} = {1, 3, 5, 7, 9, ...}	subtraction
9.	{whole numbers} = {0, 1, 2, 3, 4, 5, ...}	addition
10.	{multiples of 5} = {0, 5, 10, 15, 20, ...}	addition
11.	{multiples of 3} = {0, 3, 6, 9, 12, ...}	multiplication

A new set of numbers

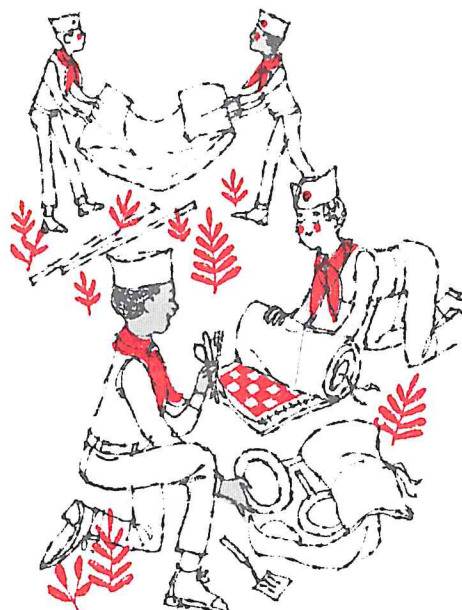
We know we cannot give away \$10 if we have only \$8, nor can we take 6 apples from a set of 4 apples, since no set can contain fewer than zero members. If our answer has to be a whole number, the equations $n = 8 - 10$ or $n = 4 - 6$ have no solutions. The set of whole numbers is **not closed for subtraction**. If we want to be sure we can get an answer to **any** subtraction with whole numbers, we have to use a new set of numbers. This new set of numbers is used in many everyday situations as these exercises show.

EXERCISES

Solve the problems.

The Boy Scouts plan to leave on a camping trip in the afternoon. They call the hour when they will leave, “H hour”. They call 8 hours before H hour, “H hour minus 8”, 7 hours before H hour, “H hour minus 7”, and so on.

- What would the Boy Scouts call
 - 4 hours before H hour?
 - 9 hours before H hour?
 - 1 hour before H hour?
 - 2 hours after H hour?
 - 5 hours after H hour?
 - 6 hours after H hour minus 8?
 - 5 hours after H hour minus 7?
 - 5 hours after H hour minus 5?
- How many hours are there between H hour minus 5 and H hour minus 3?



The scientists are “counting down” to blast off. They “hold” if there is anything wrong.

- If there is one hold at 96 seconds before blast-off and another at 88 seconds before blast-off, what is the difference in countdown time between these holds?
- If there is one hold at 75 seconds and another 16 seconds later in countdown time, how many seconds before blast-off is the second hold?



A new set of numbers

Bobby keeps a bank account. He makes an arrangement with the bank to borrow money if he needs it. When he has money in the bank, this shows in his account in black. When he owes the bank money, this shows in red. Bobby is "in the black" when the balance is on his side, "in the red" when the balance is against him.

paid in	paid out	balance
		\$14
	\$10	\$ 4
	\$ 3	\$ 1
	\$ 1	\$ 0
	\$ 5	\$ 5
	\$ 2	\$ 7

EXERCISES

Solve the problems.

1. If Bobby has \$8 in the bank and spends \$8, what is his balance?
2. If Bobby has \$8 in the bank and spends \$10, is his balance in the black or in the red? What is his balance?

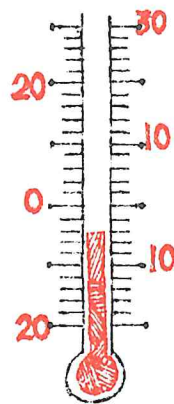
Complete the chart. Write the numeral in black if Bobby has money in the bank, in red if he owes the bank.

3.

balance before paying	16	10	8	4	5	6	8	9
paid out	7	15	12	9	8	6	7	12
balance after paying	9	5	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>

Our thermometer shows 4° below zero.

4. What is the temperature if the temperature rises 10° from 14° below zero? falls 6° from 9° below zero?
5. How many degrees does the temperature rise if it rises from 12° below zero to 7° below zero?
6. How many degrees does the temperature fall if it falls from 4° below zero to 12° below zero?



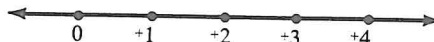
Complete the chart.

7.

first temperature in degrees below zero	4	4	3	15	14	9	8	17
rise	2			5			<i>e</i>	<i>f</i>
fall		8	7		<i>c</i>	<i>d</i>		
second temperature in degrees below zero	2	12	<i>a</i>	<i>b</i>	15	20	4	13

The integers

On the number line the greater numbers are on the right of zero. We have seen on the last two pages that we may want to use numbers that come before (are less than) zero.



EXERCISES

We shall use the symbols $\square \rightarrow$ and $\leftarrow \square$ for moving to the right and left on the number line. $\square 4 \rightarrow$ means "move 4 places to the right." $\leftarrow \square 3$ means "move 3 places to the left." Complete the chart. Use a number line to help you if necessary.

1.

start at	6	7	5	9	8	16	6	4
move	$\square 2 \rightarrow$	$\leftarrow \square 6$	$\square c \rightarrow$	$\square d \rightarrow$	$\square 6 \rightarrow$	$\square 9 \rightarrow$	$\leftarrow \square g$	$\leftarrow \square 4$
stop at	<i>a</i>	<i>b</i>	23	15	<i>e</i>	<i>f</i>	2	<i>h</i>



On this number line we show the **integers** or **signed whole numbers**. The numbers to the right of zero are **positive integers**, those to the left of zero are **negative integers**. We use the + sign for the positive, the - sign for negative integers: +4 is read "positive 4", -3 is read "negative 3".

Complete the charts. Use a number line to help you if necessary.

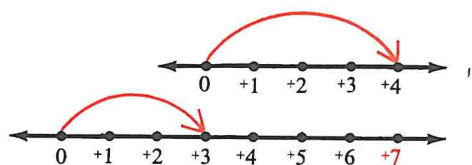
2.

start at	+7	+6	+4	+3	+2	-8	-5	-14
move	$\leftarrow \square 8$	$\leftarrow \square 16$	<i>c</i>	<i>d</i>	$\square 4 \rightarrow$	$\square 4 \rightarrow$	$\square 5 \rightarrow$	$\square 20 \rightarrow$
stop at	<i>a</i>	<i>b</i>	-7	-9	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>

3.

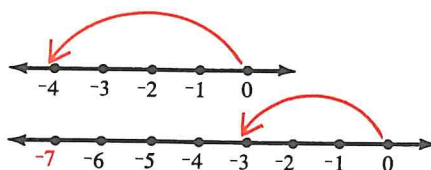
start at	+4	0	-6	+3	0	-9	-4	-4
move	$\leftarrow \square 7$	$\leftarrow \square 4$	$\square 4 \rightarrow$	$\leftarrow \square 6$	$\leftarrow \square h$	$\leftarrow \square 12$	$\square r \rightarrow$	$\square 4 \rightarrow$
stop at	<i>a</i>	<i>c</i>	<i>e</i>	<i>i</i>	-19	<i>p</i>	-1	<i>s</i>

Adding integers



The arrows show that moving 4 to the right on the number line is the same as adding +4.

$$+3 + +4 =$$

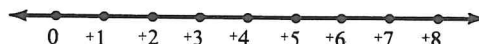
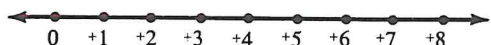
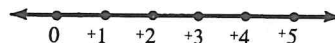
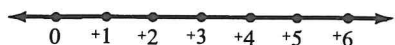


The arrows show that moving 4 to the left on the number line is the same as adding -4.

$$-3 + -4 = -7$$

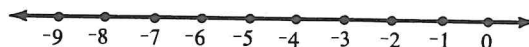
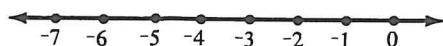
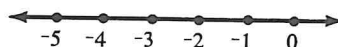
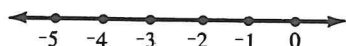
EXERCISES

Use the number lines to help you name the sums.



- | | |
|--------------|--------------|
| 1. $+2 + +1$ | 4. $+2 + +4$ |
| 2. $+2 + +2$ | 5. $+2 + +5$ |
| 3. $+2 + +3$ | 6. $+2 + +6$ |

- | | |
|--------------|---------------|
| 7. $+3 + 0$ | 10. $+3 + +3$ |
| 8. $+3 + +1$ | 11. $+3 + +4$ |
| 9. $+3 + +2$ | 12. $+3 + +5$ |



- | | |
|---------------|---------------|
| 13. $-2 + 0$ | 16. $-2 + -3$ |
| 14. $-2 + -1$ | 17. $-2 + -4$ |
| 15. $-2 + -2$ | 18. $-2 + -5$ |
| 19. $-4 + 0$ | 22. $-4 + -3$ |
| 20. $-4 + -1$ | 23. $-4 + -4$ |
| 21. $-4 + -2$ | 24. $-4 + -5$ |

Complete the sentences.

25. Moving to the right on the number line is the same as adding a ? integer.
26. Moving to the left on the number line is the same as adding a ? integer.
27. To name the sum of two integers with the same sign (both + or both -) we add the numbers and use the same ? .
- *28. Why is 0 neither negative nor positive?

Adding integers

Study the number patterns.

$$+2 + +3 = +5$$

$$+2 + +2 = +4$$

$$+2 + +1 = +3$$

$$+2 + 0 = +2$$

$$+2 + -1 = +1$$

$$+2 + -2 = 0$$

$$+2 + -3 = -1$$

$$+2 + -4 = -2$$

$$-2 + -3 = -5$$

$$-2 + -2 = -4$$

$$-2 + -1 = -3$$

$$-2 + 0 = -2$$

$$-2 + +1 = -1$$

$$-2 + +2 = 0$$

$$-2 + +3 = +1$$

$$-2 + +4 = +2$$

EXERCISES

Complete the number patterns.

1.

first addend	+4	+4	+4	<i>d</i>	<i>e</i>	<i>f</i>	+4	+4	+4
second addend	+2	+1	0	-1	-2	-3	<i>g</i>	<i>h</i>	<i>i</i>
sum	<i>a</i>	<i>b</i>	<i>c</i>	+3	+2	+1	0	-1	-2

2.

first addend	-4	-4	-4	-4	<i>e</i>	<i>f</i>	-4	-4	-4
second addend	-2	-1	0	+1	+2	+3	<i>g</i>	<i>h</i>	<i>i</i>
sum	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	-2	-1	0	+1	+2

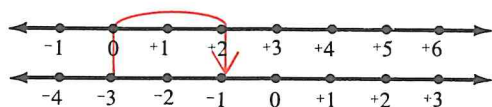
Complete the sentences.

- In an addition pattern if one addend remains the same, the greater the other addend, the ____?____ the sum.
- In an addition pattern if one addend remains the same, the less the other addend, the ____?____ the sum.

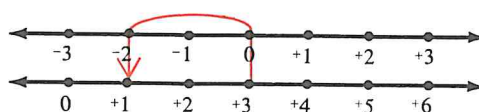
Use the rule on the left to complete the number pairs.

- Add +4: $\{(+4, a), (+3, b), (+2, c), (+1, d), (0, e), (-1, f)\}$
- Add +3: $\{(+2, g), (+1, h), (0, i), (-1, j), (-2, k), (-3, l)\}$
- Add -2: $\{(+4, m), (+3, n), (+2, p), (+1, q), (0, r), (-1, s)\}$
- Add -3: $\{(0, t), (-1, u), (-2, v), (-3, w), (-4, x), (-5, y)\}$

Study the number lines.



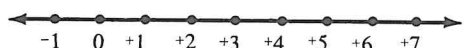
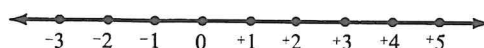
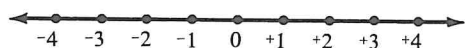
$$-3 + +2 = -1$$



$$+3 + -2 = +1$$

EXERCISES

Use the number lines to help you name the sums.

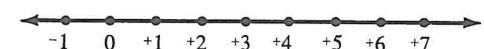
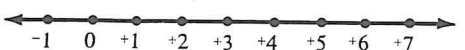
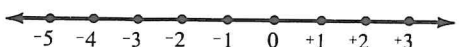


1. $+3 + +1$ 3. $+3 + +3$

13. $-2 + +1$ 15. $-2 + +3$

2. $+3 + +2$ 4. $+3 + +4$

14. $-2 + +2$ 16. $-2 + +5$

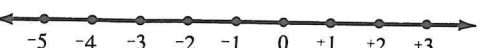
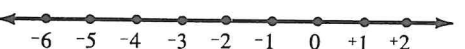
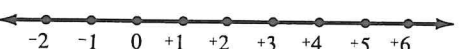


5. $+4 + -2$ 7. $+4 + -3$

17. $+5 + +1$ 19. $+5 + +2$

6. $+4 + -1$ 8. $+4 + -5$

18. $+5 + -3$ 20. $+5 + -4$



9. $-4 + +1$ 11. $-4 + +4$

21. $-1 + -2$ 23. $-1 + -1$

10. $-4 + +2$ 12. $-4 + +6$

22. $-1 + +1$ 24. $-1 + -4$

Name the sums. Use number lines if necessary.

25. $+5 + +3$

31. $+6 + -6$

37. $-8 + 0$

43. $0 + -8$

26. $+5 + +1$

32. $+6 + -2$

38. $-8 + +4$

44. $0 + +6$

27. $+5 + -2$

33. $+6 + +2$

39. $-8 + -4$

45. $0 + -1$

28. $-5 + +6$

34. $-6 + +6$

40. $+8 + 0$

46. $-4 + -1$

29. $-5 + +3$

35. $-6 + +2$

41. $+8 + -4$

47. $-8 + +9$

30. $-5 + 0$

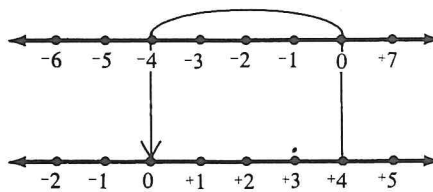
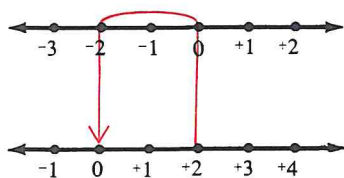
36. $-6 + -2$

42. $+8 + +4$

48. $-6 + +8$



Opposites



The red arrows show that $+2 + -2 = 0$.

The black arrows show that $+4 + -4 = 0$.

Number lines may also be used to show that $+1 + -1 = 0$; $+3 + -3 = 0$; $+5 + -5 = 0$; $+6 + -6 = 0$ and so on.

Two numbers whose sum is zero are **opposites**. $+2$ and -2 are **opposites**.

EXERCISES

Name the sums.

1. $+2 + -2$ 2. $+7 + -7$ 3. $-12 + +12$ 4. $-91 + +91$

Complete the sentences.

5. $+4$ is 4 units to the ____?____ of zero.
 6. -4 is 4 units to the ____?____ of zero.
 7. If 2 numbers are opposites, they are the same number of units from ____?____, one to the right and the other to the left.

Name the opposites of the numbers.

8. $+5$ 9. -3 10. $+17$ 11. -6 12. -28 13. $+10$ 14. -80

Pair the opposites. For example, $(+6, -6)$.

- | A | B |
|---------------|------|
| 15. $+6$ | $+7$ |
| 16. -5 | -2 |
| 17. $+1 + +1$ | -6 |
| 18. $+1 + -5$ | $+4$ |
| 19. $-5 + +3$ | $+5$ |
| 20. $+(-7)$ | $+2$ |
| 21. $+(-9)$ | $+9$ |

Study the number patterns.

$$+3 - +1 = +2$$

$$+3 - +2 = +1$$

$$+3 - +3 = 0$$

$$+3 - +4 = -1$$

$$+3 - +5 = -2$$

$$+3 - +6 = -3$$

$$+2 - +2 = 0$$

$$+2 - +1 = +1$$

$$+2 - 0 = +2$$

$$+2 - -1 = +3$$

$$+2 - -2 = +4$$

$$+2 - -3 = +5$$

$$-2 - +2 = -4$$

$$-2 - +1 = -3$$

$$-2 - 0 = -2$$

$$-2 - -1 = -1$$

$$-2 - -2 = 0$$

$$-2 - -3 = +1$$

EXERCISES

Complete the number patterns.

1. $+4 - +1 = a$

$$+4 - +2 = b$$

$$+4 - +3 = c$$

$$+4 - +4 = d$$

$$+4 - +5 = e$$

$$+4 - +6 = f$$

$$+4 - +7 = g$$

2. $+1 - +2 = a$

$$+1 - +1 = b$$

$$+1 - 0 = c$$

$$+1 - -1 = d$$

$$+1 - -2 = e$$

$$+1 - -3 = f$$

$$+1 - -4 = g$$

3. $-5 - +3 = a$

$$-5 - +2 = b$$

$$-5 - +1 = c$$

$$-5 - 0 = d$$

$$-5 - -1 = e$$

$$-5 - -2 = f$$

$$-5 - -3 = g$$

Complete the sentences.

4. In a subtraction pattern if the sum remains the same, the greater the given addend the ____? the missing addend (difference).
5. In a subtraction pattern if the sum remains the same, the less the given addend, the ____? the difference.

We know that $+4 + -4 = 0$, so $+4 = 0 - -4$, that is $+4 = - -4$. We use this fact to solve the equation $+6 - -4 = n$. Instead of subtracting -4 , we shall add its *opposite* $+4$. $+6 + +4 = +10$.

Name the differences using the idea of *opposites*.

6. $+6 - +4$

10. $+2 - -3$

14. $+3 - -5$

18. $-2 - -7$

7. $+5 - +3$

11. $-2 - +3$

15. $+6 - -6$

19. $-9 - -7$

8. $+7 - -5$

12. $-4 - -2$

16. $-4 - -5$

20. $-1 - -3$

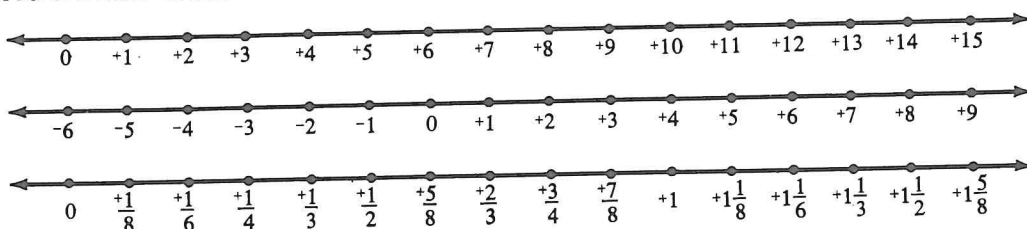
9. $+5 - -4$

13. $-5 - -6$

17. $-8 - -3$

21. $-3 - -1$

Rational numbers



The first number line shows only positive integers (whole numbers).
The second number line shows negative as well as positive integers.
The third number line shows fractional numbers.

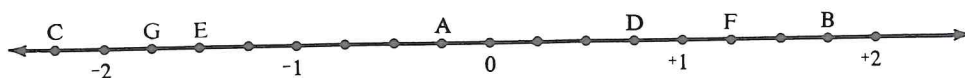
From these 3 number lines we may ask if it is possible to have negative as well as positive fractional numbers. We may show some of these numbers on a number line like the one below:



These are signed fractional numbers. They are called **rational numbers**, or for short, **rationals**. The word rational comes from the word **ratio** which we have learned is the name for a fractional number used to compare 2 sets as on page 286.

EXERCISES

Name the rational numbers on the number line labeled



1. A 2. B 3. C 4. D 5. E 6. F 7. G

Name the products of the rational numbers.

8. $\frac{+1}{2} \times \frac{+1}{3}$ 10. $\frac{+3}{7} \times \frac{+7}{9}$ 12. $\frac{+5}{6} \times \frac{+2}{9}$ 14. $\frac{+2}{9} \times \frac{+3}{11}$
9. $\frac{+2}{5} \times \frac{+3}{4}$ 11. $\frac{+1}{2} \times \frac{+2}{3}$ 13. $\frac{+4}{7} \times \frac{+3}{8}$ 15. $\frac{+7}{10} \times \frac{+3}{10}$

Name the sums of the rational numbers.

16. $\frac{+2}{3} + \frac{+1}{3}$ 18. $\frac{+6}{7} + \frac{+6}{7}$ 20. $\frac{+4}{5} + \frac{+4}{5}$ 22. $\frac{+2}{3} + \frac{+5}{6}$
17. $\frac{+3}{4} + \frac{+3}{8}$ 19. $\frac{+2}{5} + \frac{+3}{10}$ 21. $\frac{+1}{2} + \frac{+3}{4}$ 23. $\frac{+7}{8} + \frac{+5}{8}$

On page 339 we found that the integers are closed for addition and subtraction. On page 340 we introduced negative fractional numbers (the rationals) and this set is also closed for division. We may now describe a rational number as the quotient when any integer a is divided by any integer b (b cannot be 0).

$$a \div b = \frac{a}{b}, (b \neq 0) \text{ is called a rational number.}$$

EXERCISES

Remember that the quotient $a \div b = c$ is the number which, when multiplied by b , gives a as a product.

Using this idea, answer these questions.

1. Name the product when 0 is one of the factors.
2. Is it possible to find a number which when multiplied by 0 gives 7?
3. Is it possible to name any quotient with 0 as the divisor?

Name the quotients.

- | | | | | |
|---------------|----------------|----------------|-----------------|------------------|
| 4. $4 \div 8$ | 6. $6 \div 10$ | 8. $1 \div 11$ | 10. $4 \div 1$ | 12. $16 \div 2$ |
| 5. $3 \div 7$ | 7. $14 \div 5$ | 9. $0 \div 2$ | 11. $8 \div 13$ | 13. $11 \div 17$ |

Use mixed numerals to name the rational numbers.

- | | | | | |
|-----------------|-------------------|-------------------|--------------------|--------------------|
| 14. $15 \div 4$ | 17. $8 \div 3$ | 20. $11 \div 2$ | 23. $\frac{15}{7}$ | 26. $\frac{21}{5}$ |
| 15. $13 \div 6$ | 18. $10 \div 4$ | 21. $\frac{7}{3}$ | 24. $\frac{9}{2}$ | 27. $\frac{17}{4}$ |
| 16. $11 \div 3$ | 19. $100 \div 25$ | 22. $\frac{8}{3}$ | 25. $\frac{17}{6}$ | 28. $\frac{26}{9}$ |

Write *closed* or *not closed* to complete the sentences.

29. The set of rational numbers is ____?____ for addition.
30. The set of rational numbers is ____?____ for multiplication.
31. The set of rational numbers is ____?____ for subtraction.
32. The set of rational numbers is ____?____ for division.

GLOSSARY

Abacus (p. 18)

Rods and beads used to show place value.

Addend (p. 32)

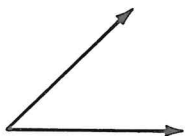
In an addition, the numbers used to name the sum.

Addition (p. 32)

The renaming of two addends as a sum.

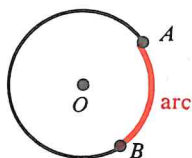
Angle (p. 71)

The union of 2 rays with a common end point.



Arc (of a circle) (p. 80)

Any part of a circle. AB is an arc.



Area (p. 78)

The measure of a region together with the unit used for measurement.

Associative Property (of Addition or Multiplication) (p. 14)

The way in which the addends (or factors) are grouped does not affect the sum (or product).

$$a + (b + c) = (a + b) + c$$

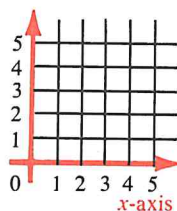
$$a \times (b \times c) = (a \times b) \times c$$

Average (p. 119)

The average of a set of numbers is the number all the numbers would be if they were all the same and their sum did not change.

Axis (p. 94)

One of the two number lines which form a number plane. The axes are usually called the x -axis and the y -axis or the first axis and the second axis.



Base (p. 24)

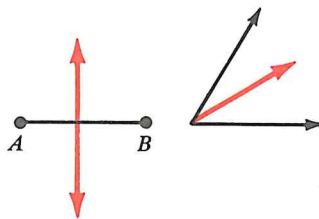
The number on which a place value system is built. The base of our system is ten.

Binary system (p. 184)

A base two system of numeration.

Bisector (p. 282)

A line separating a line segment or an angle into two congruent parts.



Braces { } (p. 2)

Symbols used to enclose members in a set.

GLOSSARY

Cardinal number of a set (p. 6)

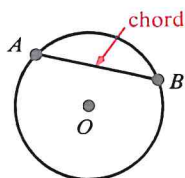
The number that tells how many objects are in a set.

Center (of a circle) (p. 80)

Point in a circular region such that the distance from it to points on the circle is always the same.

Chord (of a circle) (p. 80)

Any line segment that has both end points on the circle.



Circle (p. 80)

A simple closed curve with all its points a fixed distance from a given point.

Circumference (p. 272)

The distance around a circle.

Closed curve (p. 68)

A curve that begins and ends at the same point.

Closed number sentence (p. 130)

A number sentence that is either true or false.

Closed surface (p. 88)

A surface which separates space into three sets of points, those inside, on, and outside the surface.

Closure Property of {Whole Numbers} for Addition (p. 14)

The sum of two whole numbers is always a whole number.

Closure Property of {Whole Numbers} for Multiplication (p. 14)

The product of two whole numbers is always a whole number.

Commutative Property (of Addition or Multiplication) (p. 14)

The order of the addends (or factors) does not affect the sum (or product).

$$a + b = b + a$$

$$a \times b = b \times a$$

Congruent angles (p. 71)

Angles that have the same measure using the same unit.

Congruent figures (p. 80)

Figures whose sides and angles have the same measures using the same units.

Congruent line segments (p. 70)

Line segments that have the same measure using the same unit.

Counting numbers (p. 6)

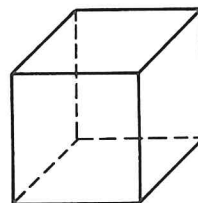
The numbers in $\{1, 2, 3, \dots\}$.

Cross product (p. 8)

The cross product of sets A and B , written $A \otimes B$, is the set of pairs with first members from A , and second members from B .

Cube (p. 88)

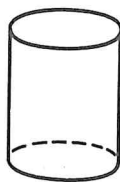
A cube is a closed surface formed by the union of 6 congruent square regions.



GLOSSARY

Cylinder (p. 275)

A closed surface formed by the union of two circular regions and a curved rectangular region.



Curve (p. 68)

A set of connected points that form a path.

Decimal fraction (p. 299)

A numeral that uses place value and a decimal point to name a fractional number.

Decimal point (p. 299)

In the decimal fraction .32, the point is called the decimal point.

Decimal system (p. 16)

A system of numeration based on ten.

Degree (p. 72)

A standard unit for measuring angles. There are 360° (360 degrees) in a circle.

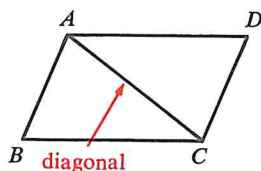
Denominator (p. 199)

9 is the denominator of the fraction $\frac{2}{9}$. It names the number in the total set.

Diagonal (p. 74)

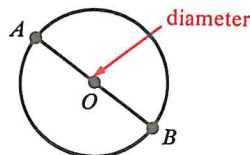
A line segment in a polygon which joins 2 vertices and is not a side.

$\overline{AC}$ is a diagonal of $ABCD$.



Diameter (of a circle) (p. 80)

A diameter of a circle is any line segment that passes through the center, and has both end points on the circle. The diameter is the length of such a line segment.



Difference (p. 36)

The missing addend in a subtraction.

Digit (p. 16)

A symbol used to write numerals. In our system of numeration the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Disjoint sets (p. 4)

Sets that have no common elements.

Distributive Property of Multiplication

To name the sum of two addends with a common factor, for example

$$(3 \times 5) + (4 \times 5),$$

we may multiply, then add

$$(15 + 20 = 35)$$

or add, then multiply

$$(3 + 4 = 7, 7 \times 5 = 35).$$

Dividend (p. 54)

The given product in a division.

Division (p. 54)

The renaming of a product and one factor as the other factor.

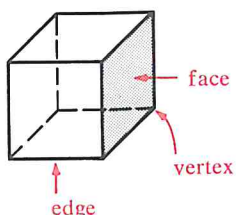
Divisor (p. 54)

The given factor in a division.

GLOSSARY

Edge (p. 88)

In certain space figures, the intersection of the flat surfaces (faces) of a simple closed surface.

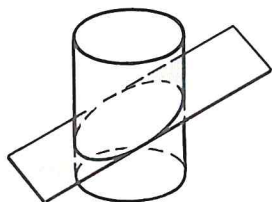


Element (member) of a set (p. 2)

An object or idea in a set.

Ellipse (p. 290)

The intersection of a cylinder and a plane not parallel or perpendicular to the base of the cylinder.



Empty set (p. 2)

The set that has no elements. Its cardinal number is zero.

Equal sets (p. 2)

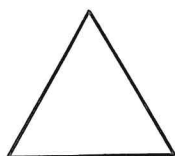
Sets that have exactly the same elements.

Equation (p. 30)

A mathematical sentence stating that 2 numerals name the same number.

Equilateral triangle (p. 75)

A triangle that has 3 congruent sides.



Equivalent Equations (p. 132)

Equations that have the same solution set.

Equivalent Fractions (p. 205)

Fractions that name the same fractional number.

Equivalent sets (p. 6)

Sets with the same cardinal number.

Expanded numeral (p. 212)

A numeral such as $30 + 7$ or $7 + \frac{3}{4}$.

Exponent (p. 17)

A small numeral written above and to the right of another numeral. The exponent tells how many times the base number appears as a factor. For example, in 5^3 , 3 is the exponent; it shows 5 appears as a factor 3 times. $5^3 = 5 \times 5 \times 5$.

Face (p. 88)

(See "Edge.") A plane region of a simple closed surface.

Factor (p. 43)

A number that is multiplied by another number to name a product. In the equation $4 \times 3 = 12$, 4 and 3 are factors.

Finite set (p. 9)

A set whose members can be listed and counted. {All people in the world}, though very large, is finite.

Fraction (p. 198)

A numeral naming a fractional number.

Function (p. 34)

A set of ordered pairs like $\{(0, 2), (1, 3), (2, 4), (3, 5), \dots\}$. In this function we have the rule "add 2."

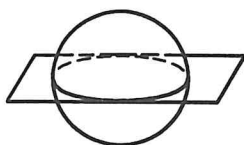
GLOSSARY

Graph (p. 95)

A picture used to show data. Different types of graphs are bar, line, and circle.

Great circle (p. 90)

The intersection of a sphere and a plane passing through its center.

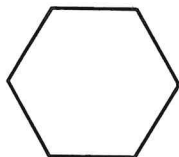


Greatest Common Factor (p. 173)

The greatest member of the set of common factors of two numbers.

Hexagon (p. 74)

A six-sided polygon.



Inequality (p. 30)

A number sentence which states that one number is greater (or less) than another.

Infinite set (p. 7)

A non-empty set which is not finite. All of its members cannot be listed and counted.

Integer (p. 334)

A member of the set $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$.

Intersection (of sets) (p. 4)

The intersection of A and B , written $A \cap B$, is the set of all elements that are common to both A and B .

Isosceles triangle (p. 75)

A triangle which has 2 congruent sides.



Least Common Multiple (p. 174)

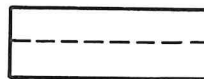
The least nonzero member of the set of common multiples of two numbers.

Line (p. 68)

A straight curve that goes on indefinitely in two directions. A line is named by any 2 points on it. $\overleftrightarrow{AB}$ is read "line AB ."

Line of symmetry (p. 293)

Separates a curve into matching parts.

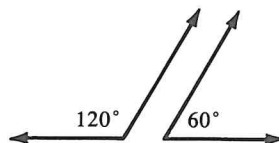


Line segment (p. 68)

Part of a line with 2 end points. $\overline{AB}$ is read "line segment AB ."

Linear pair of angles (p. 276)

Two angles the sum of whose measures in degrees is 180° .



GLOSSARY

Measure (p. 70)

A number that tells how many units match an object for the property being measured: length, area, or volume.

Median (p. 120)

The number which comes in the middle of a set of numbers when they are arranged in order.

Mixed numeral (p. 212)

A numeral with part naming a whole number and part naming a fractional number less than 1.

Mode (p. 120)

The number which appears most often in a set of data.

Multiplication (p. 42)

The renaming of two or more factors as a product.

Negative integers (p. 334)

The set of numbers which are opposites of the positive integers and are to the left of zero on the number line.

Number line (p. 20)

A line with points labeled by numbers.

Number plane (p. 94)

A plane with points labeled by number pairs.

Numeral (p. 12)

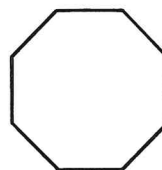
A name for a number.

Numerator (p. 199)

The numerator of the fraction $\frac{2}{9}$ is 2. It names the cardinal number of a subset.

Octagon (p. 74)

An eight-sided polygon.



Open number sentence (p. 130)

A number sentence that cannot be said to be true or false.

Opposite numbers (p. 338)

Two numbers whose sum is zero.

Ordered pair (p. 94)

A pair of numbers in which the order is important. For example (3, 6) is not the same as (6, 3).

Origin (p. 94)

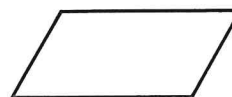
The intersection of the two axes of a number plane.

Parallel lines (p. 69)

Two (or more) lines in the same plane that never intersect. The distance between 2 parallel lines is everywhere the same.

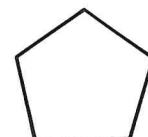
Parallelogram (p. 76)

A quadrilateral with opposite sides parallel and congruent.



Pentagon (p. 74)

A five-sided polygon.



GLOSSARY

Per cent (p. 318)

A numeral for a fractional number. The per cent sign (%) is a numeral for the fraction $\frac{1}{100}$.

Perfect number (p. 177)

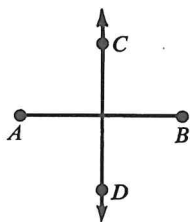
A number whose factors, not counting the number, have a sum equal to the number.

Perimeter (p. 74)

The distance around a plane figure.

Perpendicular bisector (p. 282)

A line that is perpendicular to the line it bisects. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.



Perpendicular lines (p. 73)

Lines that intersect to form right angles (90°).

Plane (p. 68)

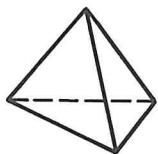
A flat surface that extends without end in all directions.

Point (p. 68)

Undefined term in geometry. It may be thought of as the intersection of 2 lines, and represented by a dot.

Polyhedron (p. 297)

A simple closed surface formed by the union of plane regions.



Polygon (p. 74)

A polygon is a simple closed curve formed as the union of line segments.

Positive integers (p. 334)

The set of counting numbers, $\{1, 2, 3, \dots\}$. On the number line, the numbers to the right of zero are positive integers.

Power (p. 17)

Any product shown by a base and an exponent. $3^2 = 3 \times 3 = 9$ is the second power of 3.

Prime factorization (p. 170)

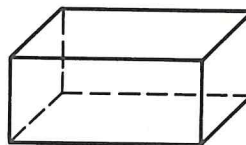
A numeral which names a number as a product of its prime factors.

Prime number (p. 168)

A number with only 2 factors, itself and 1.

Prism (p. 88)

A prism is a simple closed surface formed as the union of 6 rectangular regions.



Probability (p. 226)

Comparing the chance of a particular thing happening with all the possibilities.

Product (p. 43)

A number that results from renaming factors. In the equation $2 \times 5 = 10$, 10 is the product.

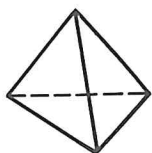
Protractor (p. 72)

An instrument used for measuring and drawing angles. It is usually circular, and uses degrees as units.

GLOSSARY

Pyramid (p. 89)

A pyramid is a simple closed surface made up of 3 or more triangular regions and a base.



Quadrilateral (p. 74)

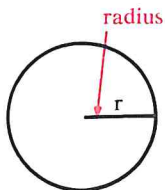
A four-sided polygon.

Quotient (p. 54)

The number resulting from the division of one number by another.

Radius (of a circle) (p. 80)

A radius of a circle is any line segment with one endpoint on the circle, the other the center of the circle. The radius of a circle is the length of such a line segment.



Range (p. 118)

The difference between the greatest and least members of a set of data.

Ratio (p. 286)

A fractional number used to compare two sets of like elements, one of which may be a subset of the other.

Rational numbers (p. 340)

Signed fractional numbers. $-\frac{5}{8}$, $-\frac{1}{3}$, $+\frac{1}{5}$, $+\frac{3}{7}$ are rational numbers.

Ray (p. 68)

Part of a line that has one end point and goes endlessly in one direction. $\overrightarrow{AB}$ is read "ray AB " and A is the end point.

Reciprocals (p. 254)

Two numbers whose product is 1.

Rectangle (p. 76)

A rectangle is a quadrilateral with 4 right angles.

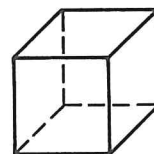


Regular polygon (p. 84)

A polygon with all its sides and all its angles congruent.

Regular polyhedron (p. 297)

A polyhedron all of whose faces are equilateral and congruent.



Remainder (p. 54)

The number remaining in a division when the greatest multiple of the divisor has been subtracted from the dividend.

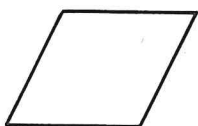
Replacement set (p. 130)

The set from which we name the solution set for an equation.

GLOSSARY

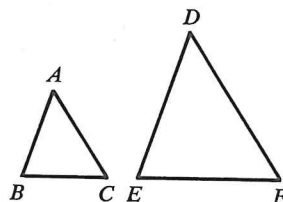
Rhombus (p. 76)

A parallelogram with 4 congruent sides.



Similar triangles (p. 287)

Triangles that have the same shape. Triangles ABC and DEF are similar.



Right angle (p. 72)

The angle formed by the intersection of two perpendicular rays. A right angle has a measurement of 90° .

Right triangle (p. 75)

A triangle which has one angle of 90° .



Simple closed curve (p. 88)

A closed curve in a plane such that if you draw a picture of the curve, your pencil will return to the starting point, it will never leave the paper, and it will not go through any point twice.

Skew lines (p. 87)

Lines which lie in different planes. They neither intersect nor are parallel.

Solution (of a number sentence) (p. 13)

A number whose name makes the sentence true.

Solution set (p. 130)

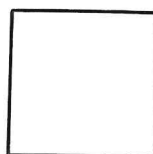
All the solutions of a number sentence.

Sphere (p. 90)

A round closed surface, the points of which are a fixed distance from a fixed point.

Square (p. 76)

A square is a quadrilateral with 4 right angles and 4 congruent sides.



Rounding (p. 104)

Rounding a number to the nearest 10 is naming the multiple of 10 to which it is closest.

Sequence (p. 47)

A set of numbers that form a pattern by which the next number can always be named.

Set (p. 2)

A collection of objects or ideas.

Side (p. 74)

The line segment of a polygon; one of the two rays forming an angle; a plane region of a simple closed surface.

GLOSSARY

Square of a number (p. 122)

The product when the number is used twice as a factor.

Square root of a number (p. 123)

A factor which is used twice to name the number as product.

Subset (p. 3)

A is a subset of B if the members of A are all members of B .

Subtraction (p. 36)

The renaming of a sum and an addend, the opposite of addition.

Sum (p. 32)

The number that is the result of renaming addends.

Superset (p. 13)

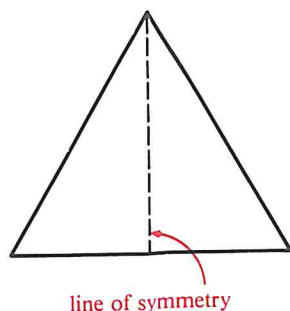
B is a superset of A if A is a subset of B .

Surface area (p. 92)

The combined area of all the faces of a closed surface.

Symmetrical curve (p. 293)

A curve that can be separated into two matching parts.



Term (p. 47)

A member of a sequence.

Theorem of Pythagoras (p. 83)

The square of the measure of the longest side of a right triangle is equal to the sum of the squares of the measures of the other two sides. If c is the longest side, then

$$a^2 + b^2 = c^2.$$

Triangle (p. 74)

A simple closed curve formed as the union of three line segments. A polygon with three sides.

Union (of sets) (p. 5)

The union of A and B , written $A \cup B$, is the set of all the elements that are in A or in B .

Unit (p. 71)

A particular quantity used as a standard of measurement. For instance, an inch is a unit of length, a square inch is a unit of area, and a cubic inch is a unit of volume.

Universal set (p. 3)

The superset for sets we are talking about.

Vertex (p. 71)

(See "Edge.") The common end point of the 2 rays that form an angle or the point in which two sides of a polygon intersect.

Volume (p. 91)

The measure of a closed surface together with the units used for measurement.

Whole Numbers (p. 6)

The numbers in $\{0, 1, 2, \dots\}$.

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